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ON SURFACES OF FINITE TOTAL CURVATURE

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Abstract

We consider surfaces M immersed into $\mathbf{R}^n$ and we prove that the quantity $\int_M |A|^2$ (where A is the second fundamental form) controls in many ways the behaviour of conformal parametrizations of M. If M is complete, connected, noncompact and $\int_M |A|^2 < \infty$ we obtain a more or less complete picture of the behaviour of the immersions. In particular we prove that under these assumptions the immersions are proper. Moreover, if $\int_M |A|^2 \leq 4\pi$ or if $n = 3$ and $\int_M |A|^2 < 8\pi$, then M is embedded. We also prove that conformal parametrizations of graphs of $W^{2,2}$ functions on $\mathbf{R}^2$ exist, are bilipschitz and the conformal metric is continuous. The paper was inspired by recent results of T.Toro.

1. Introduction

Let M be a complete , connected, noncompact, oriented two-dimensional manifold immersed in $\mathbf{R}^n$. If the second fundamental form A satisfies $\int_M |A|^2 < +\infty$ then a well-known result of Huber implies that there exists a conformal parametrization $f : S \setminus \{a_1, \dots, a_q\} \to M \hookrightarrow \mathbf{R}^n$, where S is a compact Riemannian surface. One of our aims in this paper is to study f (viewed as a map into $\mathbf{R}^n$) in a neighbourhood of the "ends" a_i. We shall see that f resembles (in a rather weak sense, cf. Proposition 4.2.10) the function $(z - a_i)^{-m_i}$ in that neighbourhood. We can call the integer m_i the multiplicity of the end at a_i. One con-

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