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ISOMETRIC IMMERSIONS OF SASAKIAN MANIFOLDS IN SPHERES

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Introduction. A Sasakian manifold M^m which is isometrically immersed in a Riemannian manifold $*M^{m+1}$ of constant curvature $C\neq 1$ is of constant curvature 1, as was shown by Takahashi [7]. The case where the constant curvature of $*M^{m+1}$ is 1 is in a very different situation. In this paper we study the case. The rank of the second fundamental form is called the type number of the immersion.

THEOREM. *Let M^m be a Sasakian manifold which is isometrically immersed in a Riemannian manifold $*M^{m+1}$ of constant curvature* 1. *Then*

(i) *the type number $k\leqq 2$, and*

(ii) *M^m is of constant curvature* 1 *if and only if the scalar curvature S is equal to $m(m-1)$.*

In an η-Einstein space for any point p the Ricci curvature $R_1(X, X)$ is constant for any unit vector X at p such that $\eta(X)=0$. In [7] it was also proved that an η-Einstein Sasakian manifold M^m $(m\geqq 5)$ which is isometrically immersed in a Riemannian manifold $*M^{m+1}$ of constant curvature 1 is of constant curvature 1. We generalize this in the following form.

THEOREM. *Let M^m $(m\geqq 5)$ be a Sasakian manifold which is isometrically immersed in a Riemannian manifold $*M^{m+1}$ of constant curvature* 1. *Assume that at any point p of M^m we have a subspace F_p of the tangent space at p to M^m such that*

(i) $\dim F_p=m-2$,

(ii) $\eta(X)=0$ *for any* $X\in F_p$,

(iii) $R_1(X, X)=$*constant for any unit* $X\in F_p$.

Then M^m is of constant curvature 1.

In § 2 we study some properties of contact Riemannian manifolds which satisfy some conditions on the Ricci tensor or the Riemannian curvature tensor, for example, $R(X, Y)\cdot R=0$ or $R(X, Y)\cdot R_1=0$.

In the last section, we consider invariant submanifolds of Sasakian manifolds. We see that they are minimal. As a special case, we have invariant submanifolds M^m of a unit sphere S^{2r+1} considered as a Sasakian manifold, which are shown to be unit spheres if and only if $S=m(m-1)$.

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