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Equivariant completion

By

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0. Introduction.

Let k be an algebraically closed field of arbrtrary characteristic. Let G be a linear algebraic group and let X be an algebraic vairety on which there is given a regular action, i.e. there is a morphism $\sigma: G\times X \ni (g, x) \to \sigma(g, x) = gx \in X$ satisfying $ex=x$ (e being the unit element of G), $(g_1 g_2)x = g_1(g_2 x)$ for every point x of X and any elements g_1, g_2 of G. We are assuming that G, X and σ are defined over k. In this paper, we shall show the following three results.

a) If G is a connected linear algebraic group (resp. a torus group) and if X is a normal variety on which there is given a regular action of G, then X has an open covering which consists of G-stable quasi-projective (resp. affine) open subsets of X (cf. Lemma 8 and Corollary 2). Furthermore, if X is a normal quasi-projective variety on which G acts regularly, then we may assume that the action is linear, i.e. there exist a projective embedding $\varphi: X \to \mathbf{P}^n$ and a projective representation $\rho: G \to \mathrm{PGL(n)}$ such that $\varphi(gx) = \rho(g)\varphi(x)$ for every g of G and every x of X (cf. Theorem 1).

Therefore, combining these results, we see that every regular action of connected linear algebraic group (resp. a torus group) on a normal variety is obtained by patching finitely many linear actions on normal quasi-projective (resp. affine) varieties.

b) Let X be a variety on which connected linear algebraic group G acts regularly. Then X has an equivariant Chow cover, i.e. there exist a quasi-projective variety $\tilde{X}$ on which G acts regularly, a G-