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On the poles of the scattering matrix for two strictly convex obstacles

Dedicated to the memory of Prof. Hitoshi Kumano-go

By

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§1. Introduction

Let $\mathcal{O}$ be a bounded open set in $\boldsymbol{R}^3$ with sufficiently smooth boundary Γ. We set $\Omega=\boldsymbol{R}^3-\bar{\mathcal{O}}$. Suppose that Ω is connected. Consider the following acoustic problem

$$(1.1)\qquad \begin{cases} \square u(x, t)=\dfrac{\partial^2 u}{\partial t^2}-\triangle u=0 & \text{in} \quad \Omega\times(-\infty, \infty) \\ u(x, t)=0 & \text{on} \quad \Gamma\times(-\infty, \infty) \end{cases}$$

where $\triangle=\sum_{j=1}^{3}\dfrac{\partial^2}{\partial x_j^2}$. Denote by $\mathscr{S}(\sigma)$ the scattering matrix for this problem. Concerning the definition of the scattering matrix see, for example, Lax and Phillips [8, page 9]. It is known that $\mathscr{S}(\sigma)$ is a unitary operator from $L^2(S^2)$ onto itself for all $\sigma\in\boldsymbol{R}$ and

Theorem 5.1 of Chapter V of [8]. *$\mathscr{S}(\sigma)$ extends to an operator valued function $\mathscr{S}(z)$ analytic in* $\operatorname{Im} z\leq 0$ *and meromorphic in the whole plane.*

Concerning how the scattering matrix $\mathscr{S}(\sigma)$ is related to the geometric properties of obstacles

Theorem 5.6 of Chapter V of [8]. *The scattering matrix determines the scattering.*

About a question as to a concrete correspondance of geometric properties of $\mathcal{O}$ to analytic properties of $\mathscr{S}(\sigma)$, Majda and Ralston [10], Petkov [14] and Petkov and Popov [15] made clear relationships between $\mathcal{O}$ and the asymptotic behavior of the scattering phase of $\mathscr{S}(\sigma)$ for $\sigma\to\pm\infty$ when $\mathcal{O}$ is non-trapping. But concerning relationships between $\mathcal{O}$ and the poles of $\mathscr{S}(z)$ we know a few facts. The results we want to show in this paper are

Theorem 1. *Let $\mathcal{O}=\mathcal{O}_1\cup\mathcal{O}_2$, $\bar{\mathcal{O}}_1\cap\bar{\mathcal{O}}_2=\emptyset$. Suppose that $\mathcal{O}_1$ and $\mathcal{O}_2$ are strictly convex, that is, the Gaussian curvatures of the boundary Γ_j of $\mathcal{O}_j$, $j=1, 2$ never vanish. Then there exist positive constants c_0 and c_1 such that*