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Precise informations on the poles of the scattering matrix for two strictly convex obstacles

By

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1. Introduction.

In the previous papers [3, 4] we considered the scattering matrix for two strictly convex obstacles. To say more precisely, let $\mathcal{O}_j$, $j=1, 2$, be bounded and strictly convex open sets in $\boldsymbol{R}^3$ with smooth boundary Γ_j. Suppose that

$$\bar{\mathcal{O}}_1 \cap \bar{\mathcal{O}}_2 = \emptyset.$$

Set $\mathcal{O}=\mathcal{O}_1 \cup \mathcal{O}_2$, $\Omega=\boldsymbol{R}^3-\bar{\mathcal{O}}$, $\Gamma=\Gamma_1 \cup \Gamma_2$. Consider an acoustic problem

$$\left\{\begin{array}{lll}\square u=\dfrac{\partial^2 u}{\partial t^2}-\Delta u=0 & \text{in} & \Omega\times(-\infty, \infty) \\ u=0 & \text{on} & \Gamma\times(-\infty, \infty),\end{array}\right. \tag{1.1}$$

where $\Delta=\sum_{j=1}^{3} \frac{\partial^2}{\partial x_j^2}$. Denote by $\mathscr{S}(z)$ the scattering matrix for this problem. About the definition of the scattering matrix see for example Lax and Phillips [7, page 9].

We showed in [3, 4] the following facts:

(i) There exist positive constants c_0 and c_1 such that for any $\varepsilon>0$

$$\{z;\ \operatorname{Im} z \leqslant c_0+c_1-\varepsilon\}-\bigcup_{j=-\infty}^{\infty} B_j$$

contains only a finite number of poles of $\mathscr{S}(z)$, where

$$B_j=\{z;\ |z-z_j| \leqslant C(1+|j|)^{-1/2}\},$$

$$z_j=ic_0+\frac{\pi}{d}j, \quad d=\operatorname{dis}(\mathcal{O}_1, \mathcal{O}_2).$$

(ii) For large $|j|$, B_j contains at least one pole.

The purpose of this paper is to give very precise informations on the poles in B_j. Namely, we shall show the following

Theorem 1. *For large* $|j|$

(a) *every* B_j *contains exactly one pole of* $\mathscr{S}(z)$,

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