

Erratum

On the Euclidean Version of Haag's Theorem in $P(\varphi)_2$ Theories

(Commun. math. Phys. 36, 133—136 (1974))

R. Schrader

Institut für Theoretische Physik, Freie Universität Berlin,
 Berlin

Received May 16, 1974

As communicated to the author by Newman, estimate (7) does not seem to follow from the results of [7], or at least not easily. (We use the literature quoted in our paper and continue the numeration). As Newman pointed out, however, the weaker result holds

$$E(|E_l(X_{2l,\lambda} - X_{l,\lambda})|) \leq O(e^{-\varepsilon l}) \quad (12)$$

for $l \geq 2l$ and some $\varepsilon > 0$.

With this we may indeed prove

$$E(X_{l',\lambda} A_l) \leq O(e^{-\varepsilon l}) \quad (l' \geq 2l) \quad (13a)$$

and by the $\lambda \leftrightarrow \lambda'$ symmetry

$$E(X_{l',\lambda'} C A_l) \leq O(e^{-\varepsilon l}) \quad (l' \geq 2l). \quad (13b)$$

By the previous arguments, these estimates are of course sufficient for the proof of the theorem. To prove estimate (13a), by Newman's estimate (12), it is sufficient to consider $E(X_{2l,\lambda} A_l)$. Let $V_D = V_{2l,\lambda} - V_{l,\lambda}$ then by the definition of A_l :

$$E(X_{2l,\lambda} A_l) \leq E(e^{-V_{2l,\lambda}})^{-1} (E(e^{-V_{l,\lambda}}) E(e^{-V_{l,\lambda'}})^{-1})^\delta E(e^{-V_D - V_{l,\lambda}(\delta)}) \quad (14)$$

for any $0 < \delta < 1$ with $\lambda(\delta) = \delta\lambda' + (1 - \delta)\lambda$.

By the Feynman-Kac-Nelson formula, the last factor on the r.h.s. of inequality (14) is

$$K = (\Psi_0, e^{-\frac{1}{2}H_{2l,\lambda}} e^{-l\hat{H}} e^{-\frac{1}{2}H_{2l,\lambda}} \Psi_0) \quad (15)$$

in the standard notation with

$$\hat{H} = H_0 + \int_{x \in \mathbb{R}} g(x) : P(\varphi(0, x)) : dx$$

$$g(x) = \begin{cases} \lambda & -l \leq x \leq -\frac{l}{2}; \quad \frac{l}{2} \leq x \leq l \\ \lambda(\delta) & |x| \leq \frac{l}{2} \\ 0 & \text{otherwise.} \end{cases}$$

Let \hat{E} be the groundstate energy of \hat{H} and $E_{2l, \lambda}$ the groundstate energy of $H_{2l, \lambda}$. Then

$$\begin{aligned} K &\leq (\Psi_0, e^{-lH_{2l, \lambda}} \Psi_0) e^{-l\hat{E}} \\ &\leq e^{-lE_{2l, \lambda} - l\hat{E}}. \end{aligned} \tag{16}$$

By Theorem 1.7 in [3]

$$-\hat{E} \leq l(\alpha_\infty(\lambda) + \alpha_\infty(\lambda(\delta))) \tag{17}$$

and

$$-E_{2l, \lambda} \leq 2l\alpha_\infty(\lambda). \tag{18}$$

Combining these estimates with estimate (1) and choosing δ as in (2) gives

$$E(X_{2l, \lambda} A_l) \leq O(e^{-\varepsilon l^2 + o(l)})$$

q.e.d.

Meanwhile a simpler proof exists, giving essentially the same but not identical results. It is based on the strong mixing (i.e. cluster) and hence ergodicity property of the measure. It has or will have appeared in preprints by Fröhlich, J. Rosen and Simon and by Hegerfeldt.

Acknowledgement. The author has profited from a correspondence with C. Newman.

Communicated by A. S. Wightman

R. Schrader
 Institut für Theoretische Physik
 der Freien Universität Berlin
 D-1000 Berlin 33
 Arnimallee 3