BOOK REVIEWS

Algebraic projective geometry. By J. G. Semple and G. T. Kneebone. Oxford University Press, 1952. 8+404 pp. $7.00.

This work is similar in style and treatment to Semple and Roth's *Introduction to algebraic geometry*, but more elementary in scope; in fact the latter work, though it appeared earlier, can well be regarded as a sequel and continuation of the present one. On the other hand, it nearly coincides in scope, and in the fundamental point of view, with Todd's *Projective and analytical geometry*, but it uses a much simpler and more elementary technical apparatus.

A couple of introductory chapters deal with the parallel developments in the idea of geometry, on the one hand from descriptions of spatial phenomena, through the applications of algebraic technique, to the concept of geometric entities as defined in algebraic terms, which admits the extensions to any number of dimensions and to an arbitrary ground field (in the book, however, the field of complex numbers is the assumed ground field throughout); and on the other hand by extensions of the invariance group, from Euclidean through affine to projective geometry. This occupies about a tenth of the book. In the rest, complex homogeneous coordinates are the material, and the linear algebra of these the topic.

A chapter is devoted to one dimension, cross ratio, the homographic transformation, etc.; another chapter to the plane and the projective relations between ranges and pencils in it. The conic, and systems of conics, occupy the next four chapters; many metrical interpretations of projective results are pointed out, such as the treatment of confocals as a range or linear family of envelopes. This part of the book reaches its climax in an unusually luminous account of invariants. After a chapter on collineations, correlations, and the most elementary Cremona transformations (little of the last is treated except the quadratic transformations on three points; on the other hand collineations are carefully classified by means of the characteristic roots and vectors of their matrices) we turn to three dimensions.

Here after a preliminary chapter on the general relations between points, lines, and planes, the quadric locus and envelope are treated, classified, like the conics, by the rank of their coefficient matrices, and studied, for the general case, mainly in terms of the two generating reguli, and the appropriate parametrisations. Before considering families of quadrics there is a digression on the twisted cubic, and on the cubic surface, defined as the locus of intersections of corresponding planes of three related stars, which leads to the determinantal equation and the plane mapping—a treatment closely parallel to Reye's but expressed in algebraic terms. The pencil, range, and net of quadrics are then considered, with the cubic transformations determined by the latter, and Hesse's correspondence between its Jacobian sextic and a plane quartic—the last, only to the point of relating the 28 bitangents of the quartic to the 28 pencils of double contact in the net. Linear transformations in space are then treated, though nothing like a complete enumeration is attempted. There is here an interesting treatment of the collineations which leave a quadric invariant, and the applications of this theory to non-Euclidean geometry.

Last of all come a brief—too brief, perhaps—introduction to line geometry; and a final hint at the possibilities of n-dimensional geometry. The workings of duality in four and five dimensions, the general intersections relations in these spaces, the properties of the fifth associated plane in four dimensions, and the representation of conics in the plane and lines in ordinary space by points of five dimensions are offered to whet the appetite; and the student is left, a little abruptly perhaps, mature in outlook, and able to start in earnest on algebraic geometry.

P. Du Val

Real functions. By Casper Goffman. New York, Rinehart, 1953. 12+263 pp. $6.00.

Principles of mathematical analysis. By Walter Rudin. New York, McGraw-Hill, 1953. 9+227 pp. $5.00.

Theory of functions of real variables. By Henry P. Thielman. New York, Prentice-Hall, 1953. 11+209 pp. $6.65.

These three books, each an introductory text on real function theory, have appeared almost simultaneously. This unusual situation has led the reviewer to write a single article comparing the three rather than to write a separate review of each. Thus, the grouping of the three books into one review is not to be taken as an indication that no one of them is of sufficient significance to merit a separate discussion. Rather, it is a recognition of the fact that they will be considered competitively so that a discussion of their relative merits would seem to be the most pertinent.

The following chart gives a brief summary of the contents of the three books. It lists the major topics considered in the union of the