duced in Chapter III. The properties of general, distributive and modular lattices are discussed and some basic theorems demonstrated.

An operator which corresponds to implication in the propositional interpretation is introduced in Chapter IV. Implicative lattices are those for which detachment and exportation hold. Four formulations are proposed, one relational (with detachment and exportation taken as postulates) and three logistic. The latter are *TA* (Gentzen's natural formulation), *HA* (the familiar calculus construction) and *LA* (Gentzen's *L*). Some theorems are proved about the implicative lattices and their duals (the subtractive lattices). *HA* and *TA* are shown to be implicative lattices equivalent to each other, to Bernays' positive logic, and to intuitionistic logic without negation.

Where, in the propositional interpretation, implication is construed as deducibility, the above systems (called the positive systems) are applicable. For a truth functional definition of implication, they must be strengthened to accommodate some form of Pierce's law. An implicative lattice so strengthened is a classic implicative lattice. Appropriate modifications of *TA*, *HA*, and *LA* are made. *TC*, *HC*, and *LC* (the classic positive systems) are shown to be classic implicative lattices. Classic subtractive lattices (Boolean rings) are treated in some detail.

Four kinds of negation are considered in Chapter V. *M*, minimal (refutability); *N*, intuitionist (absurdity); *D*, strict (refutability with excluded third); *K*, classic (absurdity with excluded third). Algebras with each type of negation are discussed and some results established about their interrelation including an extension of Glivenko's theorem. The larger part of Chapter V is devoted to the classic (Boolean) algebra.

In conclusion, extensions beyond the classical are briefly considered, particularly in the direction of modalities.

Following each chapter are notes of a bibliographical, historical, and occasionally expository nature which add greatly to the value of the book. The preface by R. Feys is oriented toward the student of philosophy. (On page 99, line 23, add "Nous employerons pour cette espéce la lettre "*K*"." at the end of the line. On page 136, line 18, for "§7" read "§1.")

R. Barcan Marcus

Theory of perfectly plastic solids. By W. Prager and P. G. Hodge. New York, Wiley, 1951. 10+264 pp. $5.50.

This book is an important and valuable contribution to the literature of mechanics, because it presents in consecutive and easily readable form the views and the main results of a considerable body of investigators. Accordingly, the emphasis is on the solution of specific examples in the simplest rate theories. There is little discussion of the foundations or of the nature of plastic phenomena, and there is no development of the incremental or accumulative theories. There is compromise, however, between the Russo-English school of pure calculation of special cases and the American tendency to seek general properties of solutions. In an era of hasty and unsound books, the reader will welcome the dignified, easy exposition, even though at times he may find it a little tedious, but most of all the scholarly but unobtrusive references to the basic papers from which the material was taken. The problems for the student are moderate in number, interesting in content, and sometimes difficult.

The book is designed for engineers, not mathematicians. Not only are mathematical points either altogether disregarded, as may well be only proper in a field whose mechanical foundations are still insecure, or even skirted in mystifying terms (e.g. p. 66), but also the engineering method of dealing with particular examples concerning frameworks before formulating the general theory, and of sometimes reverting thereafter to "physical" methods rather than using the basic equations to solve special problems, will occasion some difficulty. For example, I could not follow the analysis of bending of a beam in §7, or the discussion of elastic-plastic torsion in §10. In my opinion a much stronger case can be made out for the mathematical method than the explanations offered by the authors on p. 1: one has only to cite rigid body mechanics, elasticity, and fluid dynamics, taking due note of the manner in which those theories were discovered and developed originally.

The entire book deals with the Prandtl-Reuss theory, which may be described as postulating that in an incompressible body subject to infinitesimal strain the stress is the sum of that given by classical elasticity plus that given by the classical theory of viscous fluids, except that the viscosity instead of being a material constant is a certain function of the stress deviator—the nature of this function being specified by the yield condition, which for an isotropic material is a single scalar equation connecting the stress invariants. The Tresca condition of constant maximum shear stress and the v. Mises condition of constant intensity of the stress deviator are the only ones used. Much attention is paid to the simpler theory of v. Mises, which results from that of Prandtl and Reuss when the elastic shear modulus is set equal to infinity. Not all readers will agree with the authors' distinction between the two theories on p. 30 and pp. 82–84, particularly when they insist that the Mises theory regards the material as rigid under stresses below the yield stress and when they imply that that theory cannot be used in treating elastic-plastic problems. A welcome reassurance comes at last on p. 121 when the authors explain that the v. Mises equations are applicable to deformations of any magnitude, provided what the authors have called strain rates be interpreted as the instantaneous rates of deformation. The student of rational mechanics would have been able to take a happier part in the developments of Chap. I if this remark, along with a proper discussion of strain, had been inserted there; but, as the authors notice on p. 120, there are difficulties in finite strain problems if a part of the work of deformation is recoverable even during plastic flow. The modification of the Prandtl-Reuss formulae which the authors suggest leads to tensorially illegitimate equations if their components (18.14) are used. Indeed, a strong advantage of the v. Mises equations lies in their meaningfulness and reasonableness in problems of large deformation, and in this respect they are a partial generalization, rather than a special case, of the Prandtl-Reuss equations.

The special problems treated in the book are without exception one-dimensional or two-dimensional. There is a chapter on torsion and four chapters on plane strain. I shall not attempt to detail their contents, which includes or refers to most of the particular solutions now known. A special gem is Sokolovsky's inverse method for the elastic-plastic torsion problem (§11). The mathematically inclined reader will find in Chap. 5 many ideas already familiar in gas dynamics. The authors make a conscientious attempt to deal with the problem of contained plastic flow, which is of course the major problem to be solved. After some cases in torsion and axially-symmetric plane strain, however, they frankly give up the attempt as hopeless (§19), dealing thenceforward almost entirely with unrestricted plastic flow. For this they are not to blame: it has been the natural tendency of the field to find problems which can be solved, rather than to solve problems which are important.

The student of other branches of mechanics, accustomed to complete solutions in special cases, may learn with surprise from the examples given here that in the theory of plasticity not so much is expected. Although there is a great deal of discussion of plastic flow, very few flow fields are exhibited, and a problem is apparently regarded as solved when stresses satisfying the equations have been found. From the practical side it can be said that indeed determination of the stresses is often the main objective. However, the reader may grow a little queasy about the whole matter if he asks himself in a particular case how a specimen suffering the stresses in question will deform. In other parts of continuum mechanics exact solutions obtained by inverse or semi-inverse methods are extremely valuable. In plasticity, too, I hazard the conjecture that such methods may soon yield some *complete* solutions which can illuminate the true nature of plastic flow as embodied in the governing differential equations. To add to the present unsettled nature of the subject, there appear to be no existence or uniqueness theorems or even any plausible arguments in this connection (cf. pp. 169–182). These remarks may explain why plasticity has so far been avoided by many students of other branches of mechanics, being cultivated mainly by specialists.

An almost completely new body of material is contained in Chapter 7 on "limit analysis," a field to a great extent created by the senior author and his school. The principal result is contained in two beautiful theorems of Drucker, Greenberg, and Prager, for which the printing of the work was held up for a year. Suppose all surface loads be written with a multiplicative factor which is allowed to increase from 0. The ratio of surface traction at the instant of impending plastic flow to a given value is called the *safety factor*. A number is a *statically admissible multiplier* if there exists for this multiple of the loads a stress field satisfying the equations of equilibrium, the boundary conditions, and the yield inequality; a *kinematically admissible multiplier* is defined in a somewhat more complicated way. Then the two new theorems are:

THEOREM 1. *The safety factor is the largest statically admissible multiplier.*

THEOREM 2. *The safety factor is the smallest kinematically admissible multiplier.*

The proofs require considerable manipulation. These theorems are applied by the authors to justify use of the St. Venant-Mises theory for unrestricted plastic flow, despite its rejection in the preceding period of contained plastic flow.

The book closes with a chapter on the extremum principles of Sadowsky-Hill-Markov and Greenberg, and their application to prove the safety factor theorems in three dimensions.

This book will be read and used intensively in the immediate future. For a definitive treatment of the mathematical theory of plasticity we must wait, however, for substantial improvements in the theory itself.

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