NOTE ON THE NON-EXISTENCE OF ODD PERFECT NUMBERS OF FORM $p^\alpha q_1^2 q_2^2 \cdots q_{t-1}^2 q_t^4$

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A paper under this title appears in Bull. Amer. Math. Soc. vol. 49 (1943) pp. 712–718. On October 2, 1943, I received a photoprint of a paper of H. J. Kanold, *Verschärfung einer notwendigen Bedingung für die Existenz einer ungeraden vollkommenen Zahl*, J. Reine Angew. Math. vol. 184 (1942) pp. 116–124, for reviewing in the Mathematical Reviews. This paper was not available to me previously. In this paper the author, too, proves my theorem. But his proof is so much more difficult than my proof that I believe that the publication of my paper is also now not superfluous. The new method of both papers is the proof of case I. While my proof consists of only 8 lines, Kanold needs almost 5 pages for the proof of this case. In the proof of II, well known methods can be applied, and therefore it is not surprising that both proofs are similar. Difficulties arise here only if we obtain divisors which are greater than the range of D. N. Lehmer's prime factor tables.

Thus in the proof of II c, I had to consider $\sigma(30941^2)$. I checked the small primes and found that every prime factor must be greater than 151. Using this fact, and Lemma 1 and 2, I could prove this case without knowing the factorization of $\sigma(30941^2)$. I learned from Kanold's paper that just the next prime, namely 157, is a divisor,

$$\sigma(30941^2) = 957{,}376{,}423 = 157 \cdot 433 \cdot 14083.$$

Now II c can be proved in a simpler, but perhaps less interesting way. Since $s = 13$ and $k = 2$ one of the three factors of $\sigma(30941^2)$ equals p and the others must be q_1 and q_2. But then $(p+1)/2$ and hence n must be divisible by 79 or by 7. This contradicts $k = 2$.

Moreover, I learned from Kanold's paper that he proved the special cases $\alpha = 1$ and $\alpha = 5$ already in an earlier paper *Über eine notwendige Bedingung für die Existenz einer ungeraden vollkommenen Zahl*, Deutsche Mathematik vol. 4 (1939) pp. 53–57, not available here and not mentioned in his paper cited in footnote 5.

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