instruction is needed. This might be gained by more abundant discussion of the problems involved, by helpful suggestion and mutual visitation of classes, and occasionally even by the joint conduct of a class.

Finally, the amount of routine instruction, at least for the most promising of our younger college and university investigators, should be so regulated as to further rather than suppress their individual development. On the other hand, the man with a gift for teaching, though not for investigation, should be encouraged to obtain a broad and comprehensive knowledge of mathematics, and, obtaining this, should not be cashiered for failure to "produce" nor be spoiled as a teacher by being trimmed into a very mediocre investigator. The difference between the two kinds of gifts — the power to teach and the ability to extend the frontiers of our science — should be more clearly recognized. It is for a free and not a standardized development that we plead; and, above all, for greater freedom and leisure for the most able of our younger instructors that they may achieve the best that is in them.

VECTOR ANALYSIS.

Vector Analysis. By JOSEPH G. COFFIN. New York, John Wiley and Sons, 1909. xvii + 248 pp.

Einführung in die Vektoranalysis mit Anwendung auf die mathematische Physik. Zweite Auflage von Prof. Dr. RICHARD GANS. Leipzig, Teubner, 1909. x + 125 pp.

Die Vektoranalysis und ihre Anwendung in der theoretischen Physik. Von Dr. W. v. IGNATOWSKY. Theil I: *Die Vektoranalysis,* 1909, vi + 112 pp. Theil II: *Anwendung der Vektoranalysis in der theoretischen Physik,* 1910, iv + 123 pp. Leipzig, Teubner.

A GLANCE at books of the above type calls attention to the unsatisfactory position that vector analysis still occupies in our courses of study. We find here discussions of the most elementary kind mixed with other discussions requiring a considerable knowledge of mathematics and physics. To the student unacquainted with the branches of science involved, vector analysis is hardly intelligible. To the student acquainted with them, it seems superfluous as leading merely to results already known. It seems that this duplication might be avoided. In fact, vector analysis is not a separate subject but is part of many other subjects. Vectors and their addition could be given in elementary geometry, their multiplication in trigonometry, the differentiation and integration in calculus, and the physical applications in connection with the various scientific subjects to which they apply. It is through some such fusion that vector analysis will ultimately assume its proper everyday place in mathematics and cease to be an ornamental means of stating results already known.

The first of the above books will no doubt appeal strongly to students who desire a simple yet thorough treatment of vector analysis and who do not have the time or inclination to read a larger book such as that of Gibbs-Wilson. The notation is that of Gibbs except that the cross of the cross product is made smaller and placed above the line. Thus the cross product is $\mathbf{a} \times \mathbf{b}$ instead of $\mathbf{a} \times \mathbf{b}$. Vectors are printed in black type, scalars in italics. This distinction is easy enough in print but it would interest many of us to know how it is to be effected in writing on paper or blackboard.

At the beginning of each chapter the theory is developed. This is followed by applications to mechanics and physics and the chapter ends with examples for solution. The first four chapters contain the addition and multiplication of vectors and the differentiation with respect to a scalar, with application to centers of gravity, work, moments, geometrical loci, curvature, and the dynamics of a particle. The following chapter contains the differential operators and their physical interpretations with applications to conservative systems and perfect differentials. The remaining two chapters contain integration theorems, applications to electrical theory, the dynamics of a system, and hydromechanics. The book ends with an appendix containing a comparison of systems of notation and a summary of formulas.

The examples for solution are ample and without being puerile are simple enough to be solved by the average reader interested in vector analysis rather than the particular example. The proofs are given evidently with the idea of convincing the reader that the results are true. Definitions of technical terms are given and physical facts clearly stated so that the book might be read by students unfamiliar with the subjects discussed.

On two points the author is rather enthusiastic. He frequently repeats that "all quantities representable by vectors obey the parallelogram law." The incautious reader might apply this to finite rotations. Again he asserts that all natural phenomena are continuous and hence it is not necessary to complicate the discussion by consideration of discontinuities. Avoiding perhaps the realm of molecular mechanics, the point may be well taken. Unfortunately however we must represent physical phenomena by mathematical expressions which are sometimes discontinuous and which give by mathematical processes correct results only in regions for which those processes are valid. Hence many theorems, stated without restriction, require in their application that some attention be given to considerations of continuity. Yet to the thinking student such remarks are perhaps unnecessary, and vector analysis is not likely to interest those altogether averse to the process of thought.

The book of Professor Gans is a second edition of a work too well known to require detailed discussion. The principal addition is a chapter on strains. These are discussed by means of the ellipsoid and no use is made of the dyadic.

In the preface to his book Ignatowsky expresses his belief in the value of presenting a scientific subject in as many ways as possible. That he has adopted a viewpoint somewhat out of the ordinary is shown by the fact that he scarcely mentions coordinates in the mathematical part of his work. And when they are finally introduced in the discussion of strains it is excused by the existence in the field of a natural set of directions. We must admit that some of the discussions are not as simple as the corresponding ones usually given in coordinates. Yet that may be a criticism of the book and not of the method. It is a question whether the apparent advantage of using in demonstrations concepts not contained in the final theorems indicates more than that the best method of demonstration has not yet been devised.

The scalar product of $\mathbf{a}$ and $\mathbf{b}$, written $\mathbf{ab}$, is defined in the usual way. The vector product, written $[\mathbf{ab}]$ is introduced by integration. A directed surface element $d\mathbf{f}$, i. e., an element of surface whose border is described in a given direction, is represented by a normal vector whose length equals the area of the element and whose direction is determined by the corkscrew rule. Two vectors $\mathbf{a}$ and $\mathbf{b}$ are placed so that they form a parallelogram with the arrows pointing in the same direction around it. The vector product is then defined as the integral of $d\mathrm{f}$ over the parallelogram, the sense of $d\mathrm{f}$ being indicated by the arrows on the border. By integration over surfaces of particular forms the laws of combination of vectors are deduced.

The operations d/dt and ∇ are introduced as corresponding linear and space processes. If A is a function of position, scalar or vector, and $\nabla(\ )A$ represents any one of the operations $\nabla \cdot A$, $\nabla \times A$ or ∇A, which under the circumstances has a sense, the definition of ∇ used by Ignatowsky may be written

$$\nabla(\ )A = \lim_{V=0} \frac{\int_F A(\ )d\mathrm{f}}{V},$$

where F is the surface surrounding V. This is Maxwell's view of ∇ as an average over the surface referred to the volume as a unit. From this definition by integration over regions of special form the properties of gradient, divergence, etc., are derived.

The method of obtaining integration theorems from this definition of ∇ is very crudely shown by writing the above equation

$$\nabla(\ )A = \frac{\int_{dF} A(\ )d\mathrm{f}}{dv}.$$

We may write this

$$\nabla(\ )A dv = \int_{dF} A(\ )d\mathrm{f}$$

and by integration get

$$\int_V \nabla(\ )A dv = \int_F A(\ )d\mathrm{f},$$

where F is the exterior surface, since integration over the interior surfaces surrounding volumes dV occurs twice with normals in opposite directions.

The book contains an interesting chapter on muliply connected surfaces and another on lamellar and solenoidal fields. Then after the theory is practically developed, rectangular coordinates are introduced and used in the discussion of dyadics and strains.

The second part, on applications to physics, begins with a very interesting discussion of the general principles of dynamics, from which it is interesting to note the absence of Lagrange's equations and Hamilton's principle. This is followed by applications to electrical theory, a short chapter on the Lorentz theory of electrons, and one on the transmission of light through crystals.

H. B. Phillips.

NOTES.

The fourth regular meeting of the Southwestern Section of the American Mathematical Society will be held at the University of Nebraska on Saturday, November 26. Titles and abstracts of papers should be in the hands of the Secretary of the Section as early as November 10.

The seventeenth annual meeting of the American Mathematical Society will be held in New York December 28–29. The winter meeting of the Chicago Section will be held at Minneapolis, Minnesota, in affiliation with the sixty-third meeting of the American Association for the advancement of science, on December 29–30. Titles and abstracts of papers to be presented at these meetings should be in the hands of the respective secretaries by December 10.

At the meeting of the London mathematical society held on June 9, the following papers were read: By W. H. Young, "A new method in the theory of integration," and "On semi-integrals and oscillating successions of functions"; by G. T. Bennett, "The composition of finite screw displacements"; by M. J. M. Hill, "Note on the theory of linear differential equations"; by P. Milne, "The generation of cubic curves by apolar pencils of lines."

At a meeting of the International Commission on the Teaching of Mathematics held at Brussels on August 8, 1910, thirty delegates were present. These included Professor Klein, of Göttingen, President; Sir George Greenhill, of London, Vice-President; and Professor Fehr, of Geneva, Secretary.