been misled in trying to extend the circular argument of cos x, sin x. This, it is true, may be taken as an arc, but it has long been known that in order to extend the analogy to the hyperbolic functions it is necessary to take as argument the ratio of the *sector* to one half the square of the radius. The baneful influence of considering trigonometric functions as lines depending on a linear argument is evidently not yet extinguished.

However, the author makes no use of his erroneous statement, but depends solely on series and De Moivre's formula, so that his formulas are correct enough.

The purpose of the work, scarcely realized in its treatment, is stated thus:

"Nous nous proposons de rechercher si le nombre imaginaire a des lignes trigonométriques: sinus, cosinus, tangentes, circulaires ou hyperboliques. . . . Nous en établissons la trigonométrie. Nous montrons de nouveaux moyens pour résoudre certains problèmes."

JAMES BYRNIE SHAW.

Vorlesungen über bestimmte Integrale und die Fourierschen Reihen. Von J. THOMAE. Leipzig, Teubner, 1908. 8vo. vi + 182 pp. 7.80 Marks.

This book undertakes to give a rather general view of the subjects mentioned in its title. It is somewhat more on the order of a descriptive course than either a systematic development or a practical handbook. Thus, in the first fifty-seven pages the student will see unfolded before him a view of the main theorems with some regard to the dangerous places near them. He will learn that there are such functions as Euler's $E(x)$, Dirichlet's function, Riemann's classic function (x), written here $r(x)$, Riemann's convergent series with an infinity of discontinuities

$$f(x) = r(x) + \frac{r(2x)}{2^2} + \frac{r(3x)}{3^2} + \cdots.$$

He will find that there is a definition of integral as the limit of a sum, which indeed is suggested by the inversion of a differentiation, and that under this definition many functions become integrable, for example those of Riemann mentioned above. The first mean value theorem he finds to hold equally for such functions, and the second mean value theorem is developed. He also finds that sometimes the variable may be changed, sometimes not. Differentiation as to a parameter may be possible, but an example is given showing that this is not always the case. No indication is given of the conditions under which such differentiation is permissible.

The reader will be disappointed if he looks for a systematic treatment, even an elementary one. The classes of integrable functions are not clearly indicated, even if the two important ones are given. The tests for integrability of improper integrals are rather left to be surmised than clearly set forth; no really practical exemplification is given. The classes of functions which have improper integrals are not distinguished. In any particular case the student would feel that in a country known to have precipices he has neither guide nor map.

A systematic development of the properties of definite integrals, with the modifications necessary in improper integrals, is nowhere given; nor are the methods of evaluating forms more than hinted at by a few standard examples. The whole question of inversion of differentiation and integration, and of inversion of iterated integration is barely touched on. This is the more disappointing as the author says in his preface: "Aber als Hauptziel der Vorlesungen ist die wirkliche Auswertung bestimmter Integrale anzusehen."

As applications Fourier's series and Euler's integral are considered. Thirty-eight pages are devoted to Fourier's series, including twelve pages on vibrating cords. Illustrations are given of the determination of the coefficients for certain developments, among them the Riemann series mentioned above. The student is shown by examples that a series cannot always be integrated term by term, but is integrable term by term when uniformly convergent. It is shown that the sine series is differentiable term by term. What happens in the case of the cosine series is not mentioned. The conditions of convergence of Dirichlet are proved, and Schwarz's example is given to show that continuity merely is not a sufficient condition. Other questions of convergence, particularly uniform convergence, are barely hinted at, so that the matter of integrability is left open. No attempt is made to set forth the modern widening of Dirichlet's conditions.

The only example given is the solution of the equation of a vibrating string

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}.$$

The usual particular cases are stated and rather fully discussed, in connection with musical strings.

Again the student desiring more than a cursory view of the subject would be disappointed. The development of the cosine series is awkward. The essential difference between representing x, say by a sine series, or a cosine series, or a mixed series, is not clearly exhibited. This is always a point of confusion to the student. The manner in which the various approximation curves give the graph of the function is not intimated. The classes of developable functions are left under the narrow Dirichlet conditions. No mention is made of Fourier's classic developments in the theory of heat, where indeed the really practical value of Fourier series shows at its best.

Twenty-eight pages are devoted to double integrals, followed by ten pages on Fourier's integrals. The practical value of the latter is barely mentioned. Eighteen pages are given to the elementary formulas of the B and the Γ functions.

Thirty-two pages are given to the integration of expressions in two independent variables,

$$dw = pdx + qdy.$$

This introduces naturally the functions of a complex variable. It is shown that these lead to solutions of Laplace's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

It is regrettable that no modern text-book exists which presents the subject of definite integrals and their applications in a complete and practical way. Of course treatises on the theory of functions of a real variable usually contain the general theory, but they also contain much extraneous matter. With the growing use and importance of harmonic analysis particularly, such a text becomes more necessary. The present volume, interesting and quite smoothly carried out as it is, does not meet this demand. It is for the general reader rather than the student who desires to become skilful with these useful tools. Students of mathematical physics must look elsewhere for what they want.

JAMES BYRNIE SHAW.