refrain from cautioning them against imagining that they are becoming acquainted with symbolic logic, or the deductive system in general, as mathematicians know it and use it. It may be hoped that logicians, too, will see fit to consult the article in the encyclopedia just mentioned or some other source of similar character, such as Pieri's inaugural address, before they permit themselves to form an opinion on the accomplishments, value, and recent advances of symbolic logic.

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SHORTER NOTICES.

H. Durège, Elemente der Theorie der Funktionen einer komplexen veränderlichen Grösse, in fünfter Auflage, neu bearbeitet, von LUDWIG MAURER, mit 41 Figuren im Text. Leipzig, B. G. Teubner, 1906. 397 + x pp.

THE present work, although styled the fifth edition of Durège's well-known "Elements," is in reality a new treatise. The title and the short historical introduction have been retained ; aside from these, we have not remarked a trace of the original work. Nor could it be otherwise. The theory of functions has grown enormously since the days of Riemann. On the one hand, new fields have been opened up and explored, on the other the old tools of research have been given a greater refinement and many new ones have been added. The present author, in preparing a new edition, quite rightly decided not to patch up the old edifice, but to tear it down completely and erect a new one in harmony with the needs and tendencies of the present day. The result is an up-to-date treatise of moderate proportions, clearly and attractively written, which will surely have a widespread and well-deserved popularity.

The book starts out with an introductory chapter on real variables. Dedekind's theory of irrational numbers is sketched; such notions as simple and multiple limits, upper and lower limits, uniform convergence, also a few notions from the theory of point aggregates are briefly treated. The subject of integration is developed more fully and terminates with Gauss's relation between line and double integrals, which is later used to prove Cauchy's fundamental theorem.

After a short chapter on the arithmetic of complex numbers and their geometric representation, we begin on page 75 the study of the theory of functions of a complex variable. The definition of an analytic function chosen by the author, is Cauchy's, viz.: $f(z)$ shall have a continuous derivative. One hundred pages, divided into three chapters, suffice for the author to give the reader a tolerably well rounded survey of the general theory of one valued functions. The selection of topics is felicitous, the treatment is fresh and individual, and finally a happy mean is held between laxity and extreme rigor in the proofs.

The author has given Cauchy's integral relation

$$f(z) = \frac{1}{2\pi i}\int \frac{f(t)dt}{t - z}$$

a use we have not noticed elsewhere. He shows very readily that if f is a continuous function of x, y and satisfies this relation, it is analytic. By the aid of this simple property, he shows that a uniformly convergent series or product of analytic functions is analytic. In this connection the reviewer may be permitted to express his surprise that Herr Maurer has not made use of the theorem of page 118 relative to the termwise integration of a series, to shorten very much the proof of Taylor's and Laurent's theorems. With its aid the lengthy discussion of the remainders is unnecessary.

Now follows a chapter on doubly periodic functions. This break in the general theory serves a two-fold purpose. First, it gives the reader a chance to rest after a rather arduous march; secondly, he is given an opportunity to apply his new instruments to the study of an interesting and important class of functions. We know there is nothing to sustain a healthy interest like doing things.

We may be allowed to call attention to the pretty way the $\wp$ function is introduced. As simplest elliptic function, we choose one of order two. We take its pole double, as the order of its derivative is then three instead of four. For convenience we suppose this pole to be at the origin. The development of the function is thus

$$\frac{a_{-2}}{z^2} + a_0 + a_1 z + a_2 z^2 + \cdots$$

The choice of a_{-2} and a_0 completely fixes the function. We take now $a_{-2} = 1$ and $a_0 = 0$. It is then easy to show that this function is Weierstrass's $\wp$ function. Let us note in passing that the author has broken away from the traditional symbol $\wp$, which he replaces by p. The reviewer has never been able to understand the vogue of this fetish, unless it be that mathematicians, like children, take pleasure occasionally in some strange and outlandish novelty.

Leaving the elliptic functions, the reader is introduced to the theory of many valued functions. Here the notions analytic continuation, Riemann surfaces and Schwarz's principle of reflection are briefly treated. As an instructive example, the representation afforded by the elliptic function is studied in detail.

A noteworthy chapter on algebraic functions now follows. It would be difficult to imagine a better choice of material than that which compactly fills its fifty pages. Puiseux's cycles are determined and the corresponding Riemann surface is constructed. No attempt is made to reduce the general surface to one simply connected; the author has wisely restricted the discussion to the all important case where its branch points are of the first order. The chapter closes with a study of functions which are one valued on a given Riemann surface.

The last chapter, occupying just 100 pages, is devoted to the theory of homogeneous linear differential equations of the second order. This is an innovation in text-books on the function theory. We believe, however, it is one which will win many friends for the book.

The first problem is to determine the form of the solution in the vicinity of its singular points. The presence of certain singular points called points of indetermination causes great complication. Criteria are therefore established which characterize equations free from such points. They are the equations first studied by Fuchs and have been named in his honor. Simple examples of these are the equations satisfied by the hypergeometric function and the P function of Riemann. These are now studied in detail. When the independent variable describes a circuit about a branch point the fundamental integrals are subjected to a linear substitution. The coefficients of the latter are now determined, and the result finds at once an application in the study with Riemann and Schwarz of the representation afforded by the quotient of two independent solutions of the hypergeometric equation. We are thus led directly to the theory of automorphic functions. Space is found for a short discussion of the dreieck functions, *i. e.*, functions which map the half plane on a triangle whose sides are circles.

We hardly need to add in closing that we recommend Herr Maurer's book most heartily to the student. Even the instructor will, we doubt not, find fresh inspiration in perusing its pages, and find here and there a mode of treatment which he will be tempted to incorporate into his own lectures.

JAMES PIERPONT.

Elliptische Funktionen. Von HEINRICH BURKHARDT. Zweite, durchgesehene und verbesserte Auflage mit zahlreichen Figuren im Text. Leipzig, Veit and Company, 1906. 373 + xvi pp.

THE present edition is essentially a reproduction of the first, except that here and there a proof has been improved or a typographical error has been corrected. Numerous friends of the book have sent the author lists of errata. Thanks to their cooperation, the author hopes the formulas are now entirely reliable. This is certainly a most important feature in a subject which almost suffers from its inexhaustible wealth of formulas.

For a detailed account of the contents and tendencies of this superior work, the reader may consult an extended review in this BULLETIN for July, 1900, pages 452–463.

JAMES PIERPONT.

Quadratic Forms and their Classification by Means of Invariant Factors. By T. J. I'A. BROMWICH. Cambridge University Press, 1906. viii + 100 pp.

THE theory of elementary divisors, with which this book deals, is one of the most useful and perfect of algebraic theories. Although it is now nearly forty years since Weierstrass's fundamental paper was published, no treatment of the subject appeared in English until the year 1904, when a brief discussion was included in Mathews's revision of Scott's Determinants. This treatment is far from being suited to the needs of one wishing to penetrate for the first time into the theory, and the same is true of the only treatise on the subject which exists in any language, that of Muth. The appearance of Mr. Bromwich's book is therefore to be hailed with satisfaction as affording the