the  $m^{th}$  power of a transformation of the special linear homogeneous group. In particular if m is relatively prime to n, the number of variables, A is the  $m^{th}$  power of a transformation of the special linear homogeneous group.

If A cannot be generated by the repetition of an infinitesimal transformation of the special linear homogeneous group, the  $m^{th}$  power of A can be generated thus, provided m contains  $\delta/\delta_1$  (the quotient of  $\delta$  by the greatest common divisor of  $\delta$  and k), but not otherwise.

March 4, 1896.

## NOTES.

A REGULAR meeting of the American Mathematical Society was held in New York on Saturday afternoon, February 29, at three o'clock, the President Dr. Hill, in the chair. There were eleven members present. On the recommendation of the Council, Mr. Harry Brown Evans, University of Pennsylvania, Philadelphia, Pa., was elected to membership. The following papers were read:

(1) Professor M. I. Pupin: "On the integral of the differential equation

$$a\frac{d^2y}{dt^2} + b\frac{dy}{dt} = c\frac{d^2y}{dx^2}$$

under certain boundary conditions occurring in the theory of long electrical waves."

(2) Professor E. H. MOORE: "A two-fold generalization of Fermat's theorem."

In the absence of Professor Moore his paper was read by the Secretary.

Among the courses for the summer semester at Berlin are the following—Professor Fuchs: Introduction to the theory of differential equations; Introduction to the theory of functions;—Professor Frobenius: Theory of numbers (second course); Theory of determinants;—Professor Schwarz: Analytical geometry; Applications of the theory of the elliptic functions; Hyper-geometric series; Mathematical colloquium;—Professor Knoblauch, Theory of the elliptic functions; Theory of curved surfaces; Theory of space curves;—Professor Hettner: Introduction to the theory of infinite series, products, and continued fractions;—ProfessorHensel: Concerning the transcendentalism of e and  $\pi$ ; Higher alge-

bra (group and substitution theories);—Dr. Schlesinger: Geometry of three dimensions (synthetic and analytie); Differential calculus and introduction to analysis; Exercises in the differential calculus;—Dr. Kötter: Integral calculus; Focal properties of surfaces of the second order; Exercises in the integral calculus;—Dr. Hoppe: Integral calculus; Analytical mechanics;—Dr. Arons: Application of the differential and integral calculus to physical and chemical problems;—Dr. Glan: Quaternions.

Among the courses for the summer semester at Göttingen are the following:—Professor Schering: Mechanics;—Professor Klein: Special topics in the theory of numbers; Concerning problems in applied mechanics;—Professor Schur: Numerical calculation and applications to the method of least squares;—Professor Burkhardt; Elliptic functions;—Professor Hilbert; Theory of ordinary differential equations; Concerning the squaring of the circle;—Professor Schönflies: Analytical geometry; Special topics in the theory of functions;—Dr. Sommerfeld: Theory of surfaces, Exercises in the theory of surfaces;—Dr. Bohlmann: Differential calculus.

The French Association for the Advancement of Science holds its annual meeting this year at Tunis, April 1-4; E. Collignon is Chairman of Sections I and II (mathematics, mechanics, and astronomy).

A WORK of more than ordinary interest and originality has just been issued from the press of B. G. Teubner. It is the first volume of the Geometrie der Berührungstransformationen, dargestellt von Sophus Lie und G. Scheffers. The authors give the following synopsis of the subjects treated in this volume:

"During the years 1869–71, Lie developed a geometrical theory of contact transformations and explained by means of it the mutual connection and interrelation between projective and metrical geometry. At the same time he showed the fertility of this theory by numerous applications to partial differential equations of the first and second orders. The present work is intended to give a full presentation of these theories.

It has been attempted to present the subject in such a form as to make it intelligible not only to the professional mathematician but even to students who are acquainted merely with the more elementary branches of higher mathematics.

The first volume is divided into three parts. In the first part it is shown how, in the course of time, the idea of

point transformations was introduced and applied in geometry. Particular attention is here given to the first contact transformation ever introduced into mathematics, viz., dilatation, which really lies at the basis of Huygens's theory of waves. Next the transformations by means of pedals and reciprocal polars are treated. After these and other examples, the general idea of the contact transformation in the plane is defined both analytically and geometrically and its theory is fully developed.

The importance of the theory of *infinitesimal* contact transformations is finally illustrated by an elegant and difficult problem: to determine the form of the element of arc for all those surfaces on which the system (Schaar) of geodesic circles admits a continuous system of contact transformations or a discrete number of point transformations.

While the first part contains a theory of the transformations of line elements in the plane, the second part is intended to give a geometry of line elements in space. This geometry comprises the Pfaffian equations and expressions closely connected with the null-systems, and the Mongian equations  $\Omega(x, y, z; dx, dy, dz) = 0$  to which Plücker's line complexes are intimately related. All these theories appear here from a new point of view, as parts of the same whole. To illustrate the wide applicability of these views a theory of the tetrahedral complexes is given and a number of important categories of partial differential equations of the second order are integrated.

The central point of the discussions of the second part will again be found in its last chapter where an important polar relation between two spaces is investigated, each space containing a certain line complex whose lines correspond to the points of the other space. This establishes a correspondence between the surface elements of the two spaces which is really a contact transformation in space transforming straight lines into spheres and asymptotic curves into lines of curvature. This at once transforms Plücker's line geometry into a sphere geometry.

The third part introduces the geometry of surface elements. First, Lagrange's theory of partial differential equations of the first order is developed geometrically, according to Monge. It is then shown how much this theory gains in simplicity through the introduction of the surface element as a fundamental idea. Then follows the consideration of partial differential equations of the first order admitting infinitesimal point transformations; finally, a series of categories of such equations, of special interest in geometry, is investigated."