158. *An Algebraic Formulation of K-N Propositional Calculus. III*

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In his paper [1], K. Iséki defined the *KN-algebra*. For the details of the *KN-algebra*, see [1]. The conditions of the *KN-algebra* are as follows:

1) $\sim(p*p)*p=0$.

2) $\sim p*(q*p)=0$.

3) $\sim\sim(\sim\sim(p*r)*\sim(r*q))*\sim(\sim q*p)=0$.

4) Let α, β be expressions in this system, then $\alpha=0$ and $\sim\sim\beta*\sim\alpha=0$ imply $\beta=0$. For the details of *K-N* propositional calculus, see [2]-[4].

In my paper [5], having shown that the *KN-algebra* is characterized by 1), 3), 4), and $p*(\sim p*q)=0$, I do not prove that $p*(\sim p*q)=0$ holds in the *KN-algebra*.

In this paper, we shall show that the *KN-algebra* implies the following theses:

2′) $p*(\sim p*q)=0$,

2″) $p*(q*\sim p)=0$.

In 3), put $p=\beta$, $q=\alpha$, $r=\gamma$, then by 4), we have

A) $\sim\alpha*\beta=0$ implies $\sim\sim(\beta*\gamma)*\sim(\gamma*\alpha)=0$. Then we have the following:

B) $\sim\alpha*\beta=0$, $\gamma*\alpha=0$ imply $\beta*\gamma=0$.

In B), put $\alpha=p*p$, $\beta=p$, $\gamma=\sim p$, then by 1) and 2) we have

5) $p*\sim p=0$.

In A), put $\alpha=p$, $\beta=q*p$, $\gamma=r$, then by 2) we have

6) $\sim\sim((q*p)*r)*\sim(r*p)=0$.

On the other hand, the *KN-algebra* contains the following (For the details, see [1]).

7) $\sim p*p=0$.

In 3), put $p=\alpha$, $q=\alpha$, $r=\beta$, then by 7) we have

8) $\sim\sim(\alpha*\beta)*\sim(\beta*\alpha)=0$, i.e., $\beta*\alpha=0$ implies $\alpha*\beta=0$.

In 6), put $p=\sim p$, $r=p$, then by 5) we have

9) $(q*\sim p)*p=0$.

In 8), put $\beta=q*\sim p$, $\alpha=p$, then by 9) we have

10) $p*(q*\sim p)=0$.

We shall use the following thesis which has been obtained in his paper [1].

11) $\sim(r*p)*(p*r)=0$.

In 3), put $p=\sim p*q$, $q=q*\sim p$, $r=p$, then by 11) and 10) we have

12) $(\sim p*q)*p=0$.

In 8), put $\alpha=p$, $\beta=\sim p*q$, then we have

13) $p*(\sim p*q)=0$.

The theses 10) and 13) are 2′) and 2″). Therefore the proof is complete.

References

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