STIRLING'S NUMBERS AS POLYNOMIALS

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Professor Charles Jordan, of Budapest, in his admirable treatment of Stirling's Numbers of the First and Second Kinds, in Vol. 37, First Memorial Volume, "The Tohoku Mathematical Journal", pp. 259 and 265,[1] shows that these numbers are *polynomials*, and gives in each case an auxiliary table of values for their calculation.

It is interesting to note that by employing Mr. E. T. Frankel's[2] method for determining the general formula for a polynomial, a single table can be compiled which will do the work of Jordan's two tables referred to. The terms of this single table are also in general numerically smaller than those of Jordan's tables.

The notation used will be:-

$$n_{(r)} = \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!}$$

$$n_{[r]} = \frac{n(n+1)(n+2)\cdots(n+r-1)}{r!} \qquad ^{3)}$$

$$r! = 1\cdot2\cdot3\cdots r.$$

The auxiliary table referred to, from $s=0$ to $s=5$, and from $m=1$ to $m=6$ is given below.

TABLE OF $C_{s.\,m}$

$s=$	0	1	2	3	4	5
$m=$						
1	1					
2	1	2				
3	1	8	6			
4	1	22	58	24		
5	1	52	328	444	120	
6	1	114	1452	4400	3708	720

The numbers $C_{s.\,m}$ in the fore-going table are given by the difference equation:-

1) See also "Calculus of Finite Differences", by Charles Jordan, Röttig and Romwalter, Budapest, 1939, Chapter IV, pp. 142-229.
2) "A Calculus of Figurate Numbers and Finite Differences", by Edward T. Frankel, American Mathematical Monthly, Vol. LVII, No. 1, January, 1950.
3) This "square bracket" notation is due to Professor Lancelot Hogben, F.R.S.

$$C_{s.\,m} - (2m - s - 1)\cdot C_{s-1.\,m-1} - (s + 1)\cdot C_{s.\,m-1} = 0.$$

The following will serve as a check when constructing the table:-

$$\sum_{s=0}^{m-1} C_{s.\,m} = 1\cdot 3\cdot 5 \cdots (2m - 1).$$

We now have, for Stirling's Numbers of the First Kind:-

$$S_n^r = (-1)^{n+r} \sum_{s=0}^{\infty} C_{s.\,n-r}(n + s)_{(2n-2r)}.$$

And, for Stirling's Numbers of the Second Kind:-

$$\mathfrak{S}_n^r = \sum_{s=0}^{\infty} C_{s.\,n-r}(r - s)_{[2n-2r]}.$$