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THE ENTAILMENT-PRESUPPOSITION RELATIONSHIP

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Interest in entailment and presupposition and in their relationship has recently been shown not only in philosophical writings,[1] but also in much recent work in theoretical linguistics.[2]

The notion of entailment has been introduced in the literature (in the literatures) both by example and by the presentation of an analysis which putatively captures the notion expressed in the examples given. Many recent attempts to determine the relationship between entailment and presupposition have relied on this analysis of entailment, and what I will propose is that we are in need of a new analysis of this notion.

Before weighing the case too heavily in any direction, let me at this point, pre-theoretically, introduce what I take to be rather clear and noncontroversial examples of entailment. We will then try to fit the analysis to this body rather than attempt to girdle this body into some preselected analysis (as I believe is necessary with the old treatment of

1. E.g., J. L. Austin, "Performative-Constative," originally in French as "Performatif-Constatif," in *La Philosophie Analytique,* pp. 271-304, reprinted in C. Caton, ed., *Philosophy and Ordinary Language,* pp. 22-54; P. F. Strawson, "On Referring," *Mind,* vol. 59 (1950), pp. 320-344, also reprinted in Caton, ed., *op. cit.,* pp. 162-193; P. T. Geach, "Russell's Theory of Descriptions," *Analysis,* vol. 10 (1950), pp. 84-88; G. C. Nerlich, "Presupposition and Entailment," *American Philosophical Quarterly,* vol. 2 (January 1965), pp. 32-42; Roger Montague, "Presupposing," *The Philosophical Quarterly,* vol. 19 (April 1969), pp. 97-110; and many more.

2. E.g., A. Kraak, "Presupposition and the analysis of adverbs," mimeographed, M.I.T. (1964); Noam Chomsky, "Deep Structure, Surface Structure, and Semantic Interpretation," Reprint by the Indiana University Linguistics Club (January 1969); Laurence Horn, "A Presuppositional Analysis of *only* and *even,*" *Papers from the Fifth Regional Meeting of the Chicago Linguistic Society, April 18-19, 1969* (*PFRMCLS*), pp. 98-107; Jerry Morgan, "On the Treatment of Presupposition in Transformational Grammar," also in *PFRMCLS,* pp. 167-177; Robin Lakoff, "Some reasons why there can't be any *some-any* rule," *Language,* vol. 45 (September 1969), pp. 608-615; and many more.

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entailment). In the examples and discussion which follow, each A will entail the corresponding B.

Example 1.	$A1$:	Bob has five cats.
	$B1$:	Bob has two cats.
Example 2.	$A2$:	At least three of Bob's five cats are grey.
	$B2$:	At least two of Bob's cats are grey.
Example 3.	$A3$:	Vita, whose husband is a lawyer, is an artist.
	$B3$:	Vita is an artist.
Example 4.	$A4$:	Karel has a son.
	$B4$:	Karel is a parent.
Example 5.	$A5$:	Yvonne still longs to return to her homeland.
	$B5$:	Yvonne longs to return to her homeland.

In its usual formulation, to be questioned below, it is held that a statement (proposition) A entails a statement (proposition) B if and only if the following two conditions hold:

(i) A's being true is a sufficient condition for B's being true

(ii) B's being false is a sufficient condition for A's being false.

This formulation works well in simpler cases, but in more complex instances, it can be seen to be inadequate. In examples one and four, above, for example, every (possible) instance in which $A1$ ($A4$) is true is such that $B1$ ($B4$) is true also. And, every one in which $B1$ ($B4$) is false is such that $A1$ ($A4$) is also false.

This, however, is not the case in the second, third, and fifth examples given above. The reason for this is that there is a difference in presupposition involved, independently of the fact that entailment does hold between each A and the corresponding B.

In these cases and in indefinitely many others, condition (i) above *does* hold. However, condition (ii) does *not*, and it is here that a reformulation of the analysis of entailment is required.

If, that is (to state this in its general form), a given A entails some B, such that A has a presupposition P which B does not have, then it will be possible for A to be without truth value (be neither true nor false), for example when P is false, while B might be either true *or* false.

To illustrate this point with one of the preceding examples, let us suppose that Vita's husband is not a lawyer but an electric piano player in a rock band, and either that (a) Vita herself is an artist, or that (b) Vita is not an artist but a worker in a factory. Since $A3$ presupposes $P3$, viz., that Vita's husband is a lawyer, and $P3$ is false, $A3$ has no truth value, is neither true nor false. But $B3$ in sub-case (a) is true, and in sub-case (b), false. (Basically: a given truthvalueless statement might entail a statement which has a truth value.)

Correlatively, if Vita has no husband, $P3$ (that her husband is a lawyer) has a presupposition, viz., that Vita *has* a husband, which is false, and $P3$ is truthvalueless. In *this* case, $A3$ (with a truthvalueless presupposition) is also neither true nor false, while $B3$, as above, is true in sub-case (a), false in sub-case (b).

This example shows that *B*'s being false is not, contra the analysis of entailment given above, a sufficient condition for *A*'s having a truth value, and, a fortiori, not a sufficient condition for *A*'s being false.

What I now want to propose is that condition (ii) given above be replaced by condition (iii):

(iii) *B*'s being false is a sufficient condition for *A*'s being not true. This condition (iii) thus states that *B*'s being false establishes that *A* is either false or without truth value.

I take it that conditions (i) and (iii) are what is central to the concept of entailment, and that on the basis of this modified position, one can propose the following, where '∴' is read "therefore":

A entails *B* if and only if the following are valid:

(a) *A* is true ∴ *B* is true

(b) *A* is false ∴ *B* is not truthvalueless

(c) *B* is false ∴ *A* is not true

(d) *B* is truthvalueless ∴ *A* is truthvalueless, and,

where nothing can be validly concluded about the other's being true, false, or truthvalueless given only either (e) or (f):

(e) *A* is truthvalueless

(f) *B* is true.

This new analysis shows, I think, both how the usual analysis of entailment almost works yet why it nonetheless fails. What has happened is that the instances in which one or the other of the statements in question is neither true nor false have been overlooked. Taking such cases into consideration, it can be seen that the claim "If *A* entials *B*, then if *B* is false, *A* is false also" is a somewhat oversimplified (distorted) way of stating the (correct) claim that "If *A* entails *B*, then if *B* is false, *A* is not true". Overlooking the case in which *A* is without a truth value, this "*A* is not true" was expressed not as something equivalent to "*A* is either false or truthvalueless", but as something equivalent to "*A* is false".

With this more adequate analysis of entailment at our disposal, I think that we will be able to better determine what relationship entailment and presupposition do have.

To see this, let us look at the notion of presupposition. For *A* to presuppose *B* is for *B*'s being true to be a necessary condition for *A*'s being true and for *A*'s being false. Differently stated, if *A* presupposes *B*, then if *B* is not true, *A* is truthvalueless ("*A* has a truth value gap" as it's sometimes said.)

Notice that this notion of presupposition is such that it would *not* be a particular kind of entailment, *if* we were to take the traditional analysis of entailment to be adequate (which by now we do not). For, one would argue, if *A entails B*, then if *B* is false, *A* is too, while if *A presupposes B*, then if *B* is false, *A* is not false but rather truthvalueless.[3] Since, however, we do not accept this traditional analysis, the question remains (for the time being) an open one.

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3. This is essentially the position of Geach, *op. cit.*, p. 86; Horn, *op. cit.*, p. 98; etc.

Below I will maintain that presupposition is a particular kind of entailment. In addition, I will state precisely why it is simply a particular subspecies of entailment and not identical to the species itself.

I propose to exhibit the relationships between these two notions by what (following Wittgenstein's "truth tables") I will call validity tables. A validity table states what it is valid to conclude given certain information about the truth value of given statements and certain relationships between them.

The validity table is to be interpreted as expressing a function. As a schema, the table

	X
Y	Z

represents Z as a function of X and Y. ($Z = f(X, Y)$.) According to this table, the information stated in Z is validly derivable from that given in X and Y. Such a table is a convenient way of presenting an analysis of the relationship in question.

In what follows, a blank ("__") is to be interpreted as stating that no information about the truth value or lack thereof of the given statement is validly determinable. An "X__" will occur, therefore, only in place of the above schema's 'Z'. In addition, I will use the following abbreviations:

abbreviation	*for*
Xt	X is true
Xf	X is false
Xo	X is truthvalueless (= X is neither true nor false)
-Xt	X is not true (= X is either false or truthvalueless)
-Xf	X is not false (= X is either true or truthvalueless)
-Xo	X is not truthvalueless (= X is either true or false)

To illustrate: We might have the following as *part* of the matrix for the relationship Q:

	A Q's (bears the relationship Q to) B
⋮	⋮
-Ao	*Bf*
⋮	⋮

This matrix subsection states that when A Q's B and when A is not truth-valueless, B is false. That is, if A Q's B, then B is false in all cases in which A has a truth value.

Below I will give a table representing in collapsed form the validity tables for presupposition and for entailment, combined for the purpose of easier comparison. This table is basically a graphic representation in a rather explicit form of the analyses presented above.

	A presupposes B	*A entails B*	
At	*Bt*	*Bt*	(*line 1*)
Af	*Bt*	- *Bo* (=*Bt or Bf*)	(*line 2*)
Ao	*B__*	*B__*	(*line 3*)
Bt	*A__*	*A__*	(*line 4*)
Bf	*Ao*	-*At* (=*Ao or Af*)	(*line 5*)
Bo	*Ao*	*Ao*	(*line 6*)
-*At*	*B__*	*B__*	(*line 7*)
-*Af*	*B__*	*B__*	(*line 8*)
-*Ao*	*Bt*	-*Bo* (=*Bt or Bf*)	(*line 9*)
-*Bt*	*Ao*	-*At* (=*Ao or Af*)	(*line 10*)
-*Bf*	*A__*	*A__*	(*line 11*)
-*Bo*	*A__*	*A__*	(*line 12*)

From this representation of the relationships of presupposition and entailment, we can see a great similarity between the two. In fact, there are only four lines in which there are any differences: those numbered *2*, *5*, *9*, and *10*. Since the lower half of this matrix (that below the double horizontal line) is a function of the upper half,[4] the differences between presupposition and entailment expressed in line *9* are a function of the differences in line *2*, and those in line *10*, a function of those in line *5*.

The non-derivative differences between presupposition and entailment, then, amount to the following: Given that *A* is false its presupposition *Bp* is true, and its entailment *Be* is either true or false (but not truthvalueless); given that *B* is false, its presupposer *Ap* is truthvalueless, its entailer *Ae* is either truthvalueless or false (but not true).

This establishes that presupposition is, in fact, a kind of entailment. Specifically, it is that kind of entailment in which, where *A* presupposes (not simply entails) *B*:

(1) when *A* is false (and, derivatively, not truthvalueless), *B* is true (not simply true *or* false), and,

(2) when *B* is false (and, derivatively, not true), *A* is truthvalueless (not simply truthvalueless *or* false).

Thus, presupposition is a particular case of entailment, and not all entailments are presuppositions.

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4. This is so because -Xt = (Xf or Xo), -Xf = (Xt or Xo), and -Xo = (Xt or Xf). Thus, line *7* is a function of lines *2* and *3*; *8* of *1* and *3*; *9* of *1* and *2*; *10* of *5* and *6*; *11* of *4* and *6*, and *12* of *4* and *5*.