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O-PROPOSITIONS AND OCKHAM'S THEORY OF SUPPOSITION

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1 In their recent article "The Formalization of Ockham's Theory of Supposition" Priest and Read [4] make the following two claims:

1. In propositions[1] of the form of '$(\exists x)(P_1x \mathbin{\&} \sim P_2x)$' the predicate term, in this case P_2, has merely confused supposition, although Ockham mistakenly thought it to have confused and distributive supposition.
2. Contrary to the opinion of those who, like Geach, maintain that merely confused supposition is superfluous, there is at least one type of propositions (viz., negative indefinite exclusive propositions such as 'Only pigs don't fly') in which both the subject and the predicate have merely confused supposition, and in dealing with them the notion of merely confused supposition is ineliminable.

I will try to show in what follows that both of these claims are false. Further, in my discussion of the first claim I will construct an alternative, though Ockhamistic, account of the supposition of the predicate in a particular negative proposition (O-proposition). I will then use this account in constructing an explication of negative indefinite exclusive propositions without recourse to the notion of merely confused supposition. Since my purpose is not explicitly to discuss or evaluate the formalization of Ockham's theory proposed by Priest and Read, I will make use instead of the sort of schematization employed by Loux in "Ockham on Generality" ([3], pp. 23-46). Thus, '$\{AD/'A'\}$' will stand for the appellative domain of A, i.e., for all those things which A can supposit for in a present-tense non-modal proposition. Also, the symbol $\vee$ will be used as both a terminal and propositional connective, with the context of its occurrence making clear which use is intended. Finally, although I agree with Priest and Read that an adequate formalization of Ockham's theory must be given in an infinitary language, that issue is not germane to the points I wish to make here. So I will simplify the following discussion by treating the appellative domains of general terms as finite.

2 According to Priest and Read an O-proposition, schematically represented by

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(1) Some A is not B,

is properly rendered in first-order logic by

(2) $(\exists x)(P_1x \ \& \sim P_2x)$,

substituting P_1 for A and P_2 for B. Now close examination reveals that the predicate of such a proposition, in this case B, cannot as Ockham claims have confused and distributive supposition. For if B has confused and distributive supposition in (1), then (1) is true just in case

(3) (Some A is not $B_1 \wedge$ Some A is not $B_2 \wedge \ldots \wedge$ Some A is not B_n), where B_1 - B_n exhaust $\{AD/\text{'}B\text{'}\}$.

However, on the extralogical assumption that every A is B (1) is clearly false but (3) is true. Therefore, the descensus in question is not truth-preserving and thus B does not have confused and distributive supposition in (1). Nor, consequently, does P_2 have confused and distributive supposition in (2).

Now as I will concede below, problems do indeed arise if we take the predicate of an O-proposition to have confused and distributive supposition. However, Priest and Read have not accurately delineated the problem. The primary reason for this is that their contention that O-propositions have the form of (2) is mistaken. O-propositions are taken by Ockham to be the contradictories of the corresponding A-propositions, i.e., universal affirmative propositions, schematized by

(4) Every A is B.

It is true, of course, that (2) is the contradictory of

(5) $(\forall x)(P_1x \rightarrow P_2x)$,

which Priest and Read take to be the correct rendering in first-order logic of an A-proposition. But this simply leads me to deny that (5) is an A-proposition. For, as Loux ([3], p. 36) points out clearly, A-propositions have existential force on Ockham's view. But (5) does not entail the existence of anything. Correspondingly, on Ockham's view O-propositions do not have existential force, i.e., they are true if the subject term does not actually supposit for anything. But this is not the case with (2).

Perhaps the closest analogue in Ockham's logical theory of a proposition of the form of (2) is a particular affirmative proposition with an infinite term as its predicate. (An infinite term is formed by prefixing the particle 'non-' to a categorematic term.[2]) Thus, (2) might best be rendered by Ockham as

(6) Some A is non-B,

where A is substituted for P_1 and B for P_2. Now if Priest and Read wish to claim that the predicate of (6) does not have confused and distributive supposition, then Ockham, of course, would concur. But neither does the predicate of (6) have merely confused supposition. Rather, as the predicate of a particular affirmative proposition, i.e., an I-proposition, it has determinate supposition. And (6) is an I-proposition, even though it has a negative extreme.

Now in fairness to Priest and Read—and to others who have noted the difficulty[3]—Ockham does seem to be mistaken in claiming that the predicates of O-propositions have confused and distributive supposition. The question arises: why did Ockham make this claim? Let us suppose for now that the lack of existential import of (1) must be made explicit by including in the truth conditions for (1) the disjunct '$\{AD/\text{'}A\text{'}\}$ is empty'. Then, if we descend first under the subject, which has determinate supposition, (1) is true just in case

(7) $\{AD/\text{'}A\text{'}\}$ is empty;

or

$[(A_1 \text{ is not } B) \vee (A_2 \text{ is not } B) \vee \ldots \vee (A_n \text{ is not } B)]$, where A_1 - A_n exhaust $\{AD/\text{'}A\text{'}\}$.

When we now descend under B, it turns out that a conjunctive propositional decensus is, in fact, truth preserving. Thus (1) and (7) are true just in case

(8) $\{AD/\text{'}A\text{'}\}$ is empty;

or

$[(A_1 \text{ is not } B_1) \wedge (A_1 \text{ is not } B_2) \wedge \ldots \wedge (A_1 \text{ is not } B_n)]$
$\vee\ [(A_2 \text{ is not } B_1) \wedge (A_2 \text{ is not } B_2) \wedge \ldots \wedge (A_2 \text{ is not } B_n)]$
$\vdots$
$\vee\ [(A_n \text{ is not } B_1) \wedge (A_n \text{ is not } B_2) \wedge \ldots \wedge (A_n \text{ is not } B_n)]$, where A_1 - A_n exhaust $\{AD/\text{'}A\text{'}\}$ and B_1 - B_n exhaust $\{AD/\text{'}B\text{'}\}$.

Thus, we might imagine Ockham claiming that B has confused and distributive supposition in (1), since the descensus under B from (7) to (8) is both propositional and conjunctive.

However, upon reflection this claim is seen to be misleading. The reason that a conjunctive propositional descensus under B is possible is not that B has confused and distributive supposition in (1), but rather that each occurrence of B in (7) has confused and distributive supposition. Each of the internal disjuncts in (7) is of the form of

(9) A_i is not B.

But (9) is a negative singular proposition—and no one denies that the predicate of such a proposition has confused and distributive supposition. Still, the fact that the predicate of a negative singular proposition has this type of supposition does not entail that the predicate of an O-proposition has it, even though an O-proposition unfolds into propositions like (9) when we descend first under the subject term. For by parity of reasoning one could argue against Ockham, as Geach has, that the fact that in propositions of the form of

(10) A_i is B

the predicate has determinate supposition entails that the predicate of an A-proposition has determinate supposition, since A-propositions unfold into conjunctions of propositions like (10) when we descend first under the subject term. If Ockham is unpersuaded by this argument with respect to A-propositions, he should in all consistency be unpersuaded by an analogous argument with respect to O-propositions.

As we have already seen, moreover, the problem becomes even more acute when we attempt to perform a conjunctive propositional descensus under the predicate of (1) before we descend under the subject. This was done in the move from (1) to (3) above, but that move turned out to be invalid. Thus, it seems that Ockham was indeed mistaken about the supposition of the predicate of an O-proposition.

Now Priest and Read make the helpful (when it is interpreted correctly) suggestion that the predicates of O-propositions have merely confused supposition. On this view when we descend under the predicate of (1) before attending to the subject, we see that (1) is true just in case

(11) $\{AD/\text{'}A\text{'}\}$ is empty;

or

(Some A is not $B_1 \vee B_2 \vee \ldots \vee B_n$), where B_1 - B_n exhaust $\{AD/\text{'}B\text{'}\}$.
When we next descend under 'A', we get

(12) $\{AD/\text{'}A\text{'}\}$ is empty;

or

$[(A_1$ is not $B_1 \vee B_2 \vee \ldots \vee B_n)]$
$\vee\ [(A_2$ is not $B_1 \vee B_2 \vee \ldots \vee B_n)]$
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$\vee\ [(A_n$ is not $B_1 \vee B_2 \vee \ldots \vee B_n)]$, where A_1 - A_n exhaust $\{AD/\text{'}A\text{'}\}$ and B_1 - B_n exhaust $\{AD/\text{'}B\text{'}\}$.

(12), as one should expect, is equivalent to (8). Thus, it seems plausible for us to assume that the predicate of an O-proposition has merely confused supposition. Moreover, it is normally Ockham's practice to assign a given type of supposition to a general term on the basis of what sort of descensus is possible directly under that term, i.e., without a descensus having been first performed under the other extreme of the same proposition. This is surely why he assigns merely confused supposition to the predicate of an A-proposition. And, it seems, on this basis he should have assigned merely confused supposition to the predicate of an O-proposition as well.

Still, there is one inelegant consequence within Ockham's logic of the suggestion made by Priest and Read. Although (8) and (12) are equivalent, there is no procedure enunciated by Ockham for converting (12) into (8). Ockham's theory of common supposition has as its goal the explication of the truth conditions of propositions containing general terms by means of propositions in which both the subject and the predicate have discrete supposition. But it is not clear just how the disjunctive predicate '$B_1 \vee B_2 \vee \ldots \vee B_n$' supposits in a proposition of the form of one of the secondary disjuncts of (12), viz.,

(13) A_i is not $B_1 \vee B_2 \vee \ldots \vee B_n$.

It does not seem to have discrete supposition. In the *Elementarium Logicae* ([2], pp. 175-6) Ockham tells us that discrete supposition occurs when a discrete term supposits significatively. But a disjunctive predicate does not fit the definition of a discrete term, since by one imposition it may be true of more than one thing. Thus, in the absence of a standard procedure for going from (13), which has a disjunctive predicate, to the equivalent

(14) (A_i is not B_1) $\wedge$ (A_i is not B_2) $\wedge \ldots \wedge$ (A_i is not B_n),

(12) is not a wholly satisfactory explication of (1) within the parameters of Ockham's explicit remarks on supposition. What is needed, obviously, is some variant of De Morgan's law which applies to disjunctive predicates. But I have been able to find no such law in Ockham's treatises on supposition.

Now this may not be a devastating objection to the opinion of Priest and Read since, after all, (12) is equivalent to (8), in which every term in the second main disjunct does clearly have discrete supposition. Still, the objection might lead one to search for an alternative account of the supposition of the predicate of an O-proposition—an account which is at least more patently consistent with Ockham's intentions. Such an account, I wish to claim, is in fact available. The first thing to note is that the first main disjunct of both (8) and (12), viz.,

(15) $\{AD/\text{'}A\text{'}\}$ is empty,

is redundant. The reason is that negative singular propositions, schematized by

(9) A_i is not B,

are true, according to Ockham, just in case either nothing is A_i or A_i is non-B ([1] I, chap. 72, ll. 121-126). (Presumably the same holds for a negative singular proposition with a disjunctive predicate, although Ockham does not say so explicitly.) That is, (9) has two exponents, the disjunction of which specifies its truth conditions. If we allow the symbol $\{AD/\text{' '}\}$ to take singular terms on its right-hand side, then we can say that (9) is true just in case

(16) $\{AD/\text{'}A_i\text{'}\}$ is empty or A_i is non-B.

On Ockham's view 'A_i is non-B' is an affirmative proposition which does entail that A_i exists (see [1] II, chap. 12, esp. ll. 22-37).

It is easy to see that this understanding of negative propositions renders (15) redundant in the above explications of the truth conditions of

(1). For if $\{AD/\text{'}A\text{'}\}$ is empty, then each of the internal conjuncts of (8) will turn out to be true, and thus (1) will be true. The same holds for (12), if we assume that Ockham would extend his account of negative singular propositions to those with disjunctive predicates. If $\{AD/\text{'}A\text{'}\}$ is empty, then each of the secondary disjuncts of (12) will be true, and thus (1) will be true. If $\{AD/\text{'}A\text{'}\}$ is not empty, then everything proceeds as before.

One might wonder at this point what it would take to render (15) nonredundant as part of the explication of the truth conditions of (1). The answer is that we merely have to take O-propositions, like other negative propositions, as exponible. Then we can say that (1) has the following exponential analysis:

(17) $\{AD/\text{'}A\text{'}\}$ is empty or some A is non-B.

Since the second disjunct of (17) is an I-proposition, in which each extreme has determinate supposition, a full account of the truth conditions of (1) is

(18) $\{AD/\text{'}A\text{'}\}$ is empty;

or

$[(A_1$ is non-$B_1) \vee (A_1$ is non-$B_2) \vee \ldots \vee (A_1$ is non-$B_n)]$
$\vee\ [(A_2$ is non-$B_1) \vee (A_2$ is non-$B_2) \vee \ldots \vee (A_2$ is non-$B_n)]$
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$\vee\ [(A_n$ is non-$B_1) \vee (A_n$ is non-$B_2) \vee \ldots \vee (A_n$ is non-$B_n)]$, where A_1 - A_n exhaust $\{AD/\text{'}A\text{'}\}$ and non-B_1-non-B_n exhaust $\{AD/\text{'non-}B\text{'}\}$.

(18), it seems clear, is equivalent to both (8) and (12). Furthermore, this account has the advantage of rendering the truth conditions of (1) in terms of propositions in which both the subject and the predicate supposit discretely, no matter which descensus is performed first. For the supposition of terms in I-propositions is unproblematic on that score.

At this point someone might wonder just what type of supposition the predicate of an O-proposition has on this alternative account. The best answer, I think, is that on this account the predicate of an O-proposition is such that its negation has determinate supposition in the exponent which is an I-proposition. This may appear to be an evasion since Ockham normally assigns supposition directly to terms even when they occur in exponible propositions, but this practice leads him into difficulty in at least one case, as we shall see below. In any case, the explication of (1) by means of (17) and (18) is surely consistent with the goals of Ockham's theory of supposition and, moreover, it does not attempt to introduce a fourth kind of supposition alien to his theory. Furthermore, this explication amounts to little more than an extension of his treatment of negative singular propositions to negative particular propositions—and this again seems consistent with his view that the latter do not entail the existence of anything. So perhaps it should not concern us that on this alternative account we cannot assign any type of supposition directly to the predicate of an O-proposition.

3 I will turn now to the second claim made by Priest and Read, viz., that in

(19) Only pigs don't fly

both the subject ('pigs') and the predicate ('fly') have merely confused supposition ineliminably. From their rendition of (19) in first-order logic as

(20) $(\forall x)(\sim P_1 x \rightarrow P_2 x)$,

where P_1 stands for 'fly' and P_2 for 'pig', it is clear that they are taking (19) to be, in Ockham's terminology, the indefinite (or, equivalently, the particular) exclusive proposition

(21) Only a (some) pig does not fly.

For (20) does not imply that no pigs fly, but only that whatever does not fly is a pig. In what follows I will use the schematic

(22) Only an A is not B

for propositions like (21).

Now Priest and Read claim that in (22) both A and B have merely confused supposition. Thus, (22) is true just in case

(23) Only $A_1 \vee A_2 \vee \ldots \vee A_n$ is not $B_1 \vee B_2 \vee \ldots \vee B_n$, where A_1 - A_n exhaust $\{AD/\text{'}A\text{'}\}$ and B_1 - B_n exhaust $\{AD/\text{'}B\text{'}\}$.

It is interesting to note that in making this claim they differ from Ockham, who contended that in a negative indefinite exclusive proposition both the subject and the predicate supposit just as they do in an affirmative indefinite exclusive, i.e., the subject has merely confused supposition and the predicate has confused and distributive supposition ([1] II, chap. 17, ll. 230-233). Thus he holds that the following consequence is valid:

(24) Only a substance is not an accident, therefore only a substance is not this accident.

As the editors of the critical edition of the *Summa Logicae* point out in a footnote, (24) is valid only on the extralogical assumption that there exists just one accident. But, of course, saying this does not salvage Ockham's assertion about the logical or semantic properties of the predicate in a proposition like the antecedent of (24).

Given that Ockham was mistaken on this point, (23) appears to be a plausible account of the truth conditions of (22). However, it suffers from the same defect noted in our previous discussion, i.e., it does not explicate the original proposition, in this case (22), in terms of propositions in which both the subject and the predicate have discrete supposition. Further, and more importantly for the present discussion, it is simply not the case that (22) cannot be explicated without recourse to the notion of merely confused supposition. Contrary to the opinion of Priest and Read, an alternative account is available—an account which, coincidentally, also remedies the defect just alluded to.

According to Ockham the affirmative indefinite exclusive proposition

(25) Only an A is B

is exponible into

(26) An A is B and nothing other than an A is B ([1] II, chap. 17, ll. 39-41).

(26) is plausibly regarded as synonymous with

(27) Some A is B and no non-A is B,

taking an indefinite proposition, as Ockham does, to be equivalent to the corresponding particular proposition. (27) entails the existence of at least one A, since its first conjunct is an affirmative proposition. Along the same lines (22) is exponible into

(28) Some A is not B and no non-A is non-B.[4]

(28) does not have existential force, since both its conjuncts are negative propositions. Now as we saw in our previous discussion, we can treat the particular negative conjunct of (28) as itself exponible. Thus, for (28) we can write

(29) (Either $\{AD/\text{'}A\text{'}\}$ is empty or some A is non-B) and no non-A is non-B.

The truth conditions of (29), which is synonymous with (22) on the reasonable assumption that exponential analyses preserve synonymy, can now be explicated without recourse to the notion of merely confused supposition. For in the I-proposition contained in (29) both A and non-B have determinate supposition, while in the universal negative or E-proposition contained in (29) both non-A and non-B have confused and distributive supposition. The complete explication of the truth conditions of (29) is as follows:

(30) {Either $\{AD/\text{'}A\text{'}\}$ is empty;

or

$[(A_1 \text{ is non-}B_1) \vee (A_1 \text{ is non-}B_2) \vee \ldots \vee (A_1 \text{ is non-}B_n)]$
$\vee\ [(A_2 \text{ is non-}B_1) \vee (A_2 \text{ is non-}B_2) \vee \ldots \vee (A_2 \text{ is non-}B_n)]$
$\vdots$
$\vee\ [(A_n \text{ is non-}B_1) \vee (A_n \text{ is non-}B_2) \vee \ldots \vee (A_n \text{ is non-}B_n)]$, where A_1 - A_n exhaust $\{AD/\text{'}A\text{'}\}$ and non-B_1-non-B_n exhaust $\{AD/\text{'non-}B\text{'}\}\}$;

and

$\{[(\text{non-}A_1 \text{ is not non-}B_1) \wedge (\text{non-}A_1 \text{ is not non-}B_2) \wedge \ldots \wedge (\text{non-}A_1 \text{ is not non-}B_n)]$

$\wedge$ [(non-A_2 is not non-B_1) $\wedge$ (non-A_2 is not non-B_2) $\wedge$ . . . $\wedge$ (non-A_2 is not non-B_n)]

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$\wedge$ [(non-A_n is not non-B_1) $\wedge$ (non-A_n is not non-B_2) $\wedge$ . . . $\wedge$ (non-A_n is not non-B_n)], where non-A_1-non-A_n exhaust {AD/'non-A'} and non-B_1-non-B_n exhaust {AD/'non-B'}}.

Thus, it is not the case that the subject and predicate of (22) must be taken to have merely confused supposition. Rather, on this alternative account, the subject is such that it has determinate supposition in one exponent of (22) and its negation has confused and distributive supposition in the other exponent, and the predicate is such that its negation has determinate supposition in one exponent and confused and distributive supposition in the other exponent. On this account, then, merely confused supposition, *pace* Priest and Read, is nowhere to be seen.

Perhaps the notion of merely confused supposition cannot be entirely eliminated from an Ockhamistic semantics. There are certain intentional contexts, for instance, where no alternative to merely confused supposition is clearly in the offing. (Ockham's favorite example is 'I promise you a horse', assuming that I have no particular horse in mind.) Nevertheless, in propositions like (22) the subject and predicate need not be taken to have merely confused supposition.

NOTES

1. Throughout I will use the term 'proposition' as Ockham does, viz., for any piece of discourse—mental, spoken or written—which is either true or false. Propositions are not eternal entities on his view, but rather exist only when they are formulated. There is a token/type ambiguity in Ockham's use of the term, but I will not attempt to discuss this issue here.

2. For Ockham's treatment of propositions containing infinite terms, see *Summa Logicae* [1], II, chap. 12.

3. *Cf.*, for example, Swiniarski [5], p. 200.

4. This conflicts with what Ockham says at SL II, chap. 17, ll. 41-45. He takes the second exponent to be 'Everything other than an A is B', i.e., 'Every non-A is B'. Thus, on his view the correct exponential analysis of (22) is 'An A is not a B and every non-A is a B'. This entails that there are non-A's. However, it seems clear to me that (22) could be true even if there were no non-A's. For example, 'Only men are not angels' could be true even if there were no non-men. However, even if we were to use Ockham's analysis and concede that one exponent of (22) is an A-proposition, it is generally acknowledged that the use of merely confused supposition in the explication of A-propositions can be eliminated. The point which Priest and Read want to make here is that there are cases in which merely confused supposition cannot be eliminated.

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