Deconvolution of time series. Huber has described a class of projection pursuit procedures wherein the segments of length d from a univariate time series are treated as the basic d-dimensional observations. Projections which give rise to least normal univariate distributions are candidates for the desired filter or inverse filter to be applied to the time series. While considerable success is claimed for such procedures, their rationale seems to depend in part on supposing that the deconvolved series should be an i.i.d. sequence. In many geophysical problems the deconvolved series is expected to look like a step function corresponding to stratigraphy. This suggests that the projection index should pay some attention to the time order of the deconvolved series. For example, one might consider an index based on the scaled total variation of the deconvolved series.

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As always, it is a pleasure to see the carefully thought through and neatly organized result of Huberizing a field.

I shall confine my detailed comments to Section 19, where the $x_{p,t}$ are essentially the column sums of a Buys-Ballot table (e.g. Whittaker and Robinson, 1924).

This approach was once more conventional than the periodogram (not then yet invented). We can improve its behavior somewhat by replacing equally weighted sums, of X_{t+kp} over p, by windowed sums, where the (data) window tends finitely to zero at the nearest points which would have appeared in the sum if their values had been observable, but which were not observed.

The difficulty with harmonics and subharmonics can be minimized by beginning with the largest Fourier amplitude $|c(p)|^2$, which will also be better calculated with a data window (and, further, if more refined assessment of periods is desired, padded rather extensively with zeroes), and then using Buys-Ballot technique to identify—and then subtract—a general periodic constituent whose period is sufficiently close to the Fourier-selected period. A new set of Fourier amplitudes can then be found (cheaply by an FFT), and the cycle repeated.

Notice that

$$\sum (1/m^2)\cos 2\pi(2^{-m}f_0)t$$

shows that we cannot hope, whatever our approach, to always avoid selecting a harmonic of a frequency also present. So we must be prepared to also have a revision process, in which, once we have a good finite sum of periodic terms, we look for harmonic relations among their periods and corresponding reductions of the number of periodic terms. This is needed for the approach suggested above, as well as for any other approach.

REFERENCE

WHITTAKER, E. T. and ROBINSON, G. (1924). *The Calculus of Observations.* Blackie, London.

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I would like to thank Professor Huber for a most thought-provoking paper.

I will restrict myself to discussion of projection pursuit regression, in particular to describing an approach to PP regression different than and possibly complementary to that of linear combinations of ridge functions as introduced in Section 9. This approach may be called partial spline modelling. One models a function of, say, $k + d$ variables, parametrically in k variables and as a (thin plate) spline function in the remaining d variables. The role of projection pursuit is to determine which d of the $k + d$ variables must be splined. Partial spline modelling can also be extended to the context of GLIM models, whereby again the dependency on some variables is via the usual GLIM approach while the dependency on other variables is only "smooth." It will turn out that partial spline estimates are linear combinations of (uniquely determined) polynomials and shifted versions of certain spherically symmetric functions (in the d splined variables). These splines are known to nicely model in a nonparametric way the interaction effects among a small number of variables (provided there is enough data), so they in some sense have properties that are complementary to ridge function approximation, and thus may be expected to do well where ridge functions do not. The structure of these estimates hopefully allows the benefits of the lesser data requirements of parametric modelling where that is warranted, while doing smooth nonparametric regression where it is not. By analogy with Huber's definition of "interesting" as "nonnormal" in multivariate density estimation, one could define "interesting" in this context as having a dependence more complicated than that modellable by a low-degree polynomial. With that definition, projection pursuit with the models being proposed here would, if successful, identify the "interesting" directions, particularly if the choice of d is part of the "pursuit."

Several authors have proposed partial spline models with one splined variable, with notable success (Engle, Granger, Rice and Weiss, 1983; Green, Jennison and Seheult, 1983; Anderson and Senthilselvan, 1982; Shiller, 1984). In all of these interesting applications the choice of the (single) splined variable was dictated by the context, so that projection pursuit is not necessary. Gaver and Jacobs (1983), however, consider the problem of predicting low level stratus conditions at Moffet field using surface meteorological measurements of east and north wind velocity, temperature, dew point and existence or nonexistence of low level stratus on preceding days. They use subset selection combined with logistic