In the special case $m = k$ and for a symmetrical initial distribution with mean zero, the following equations hold

$$(13) \qquad {}_m\alpha = \alpha_k = \alpha_m; \qquad {}_mu = -u_k = -u_m.$$
$$(13') \qquad {}_m\Phi = 1 - \Phi_k = 1 - \Phi_m; \qquad {}_m\varphi = \varphi_k = \varphi_m.$$

and the bivariate distribution of the mth values from the bottom ${}_mx$, and from the top x_m, is

$$(14) \qquad \mathfrak{w}_n({}_mx, x_m) = {}_mf({}_mx)\cdot f_m(x_m),$$

where

$$(14') \qquad {}_mf({}_mx) = f_m(-x_m)$$

is the expression used in the beginning of article [1]

It follows from (11) that the mth observation in ascending order, and the kth observation in descending order, may be dealt with as independent variates provided that n is large, the ranks m and k are small, and that the initial continuous unlimited distribution is of the exponential type as defined by equations (3).

REFERENCES

[1] E. J. Gumbel, "Ranges and midranges," *Annals of Math. Stat.*, Vol. 15 (1944), pp. 414–422.
[2] E. J. Gumbel, "Les valeurs extrêmes des distributions statistiques" *Annales de l'Institut Henri Poincaré*, Vol. 4 (1935), pp. 115–158.
[3] Maurice G. Kendall, *The advanced theory of statistics*, London, 1945, p. 219.

A NOTE ON SAMPLING INSPECTION

By Paul Peach and S. B. Littauer

North Carolina State College and *Newark College of Engineering*

In designing an industrial sampling plan conformable to the Pearson-Neyman approach, the operating characteristic is made to pass as nearly as possible through two predetermined points. Wald [1] has used this method for setting up sequential sampling plans.

A similar type of single sampling plan can be designed by using tables of the incomplete Beta function. Unfortunately, tables of this function are not generally available, and the existing tables do not cover the range for large sample sizes.

An approximate solution of the problem for single sampling can be based on the widely available tables of percentage points of the chi-square distribution. This is equivalent to assuming a Poisson distribution of defectives in the sample, utilizing the well known fact that for even degrees of freedom the chi-square distribution gives the summation of a Poisson series.

We use the following well established notation:

n = sample size

c = acceptance number

p_1 = acceptable fraction defective

p_2 = objectionable fraction defective

α = risk of rejecting a lot if $p = p_1$.

β = risk of accepting a lot if $p = p_2$.

There seems little to be gained by using a large assortment of possible risk values, since the necessary adjustment to secure a desired effect can be made on the p's. We suggest the adoption of .05 as a standard value for both α and β. This convention conforms to much existing statistical practice, in particular to some existing inspection tables.

We propose also the use of

$$R_0 = p_2/p_1,$$

which we call the "operating ratio," as a measure of the power of discrimination of an inspection scheme. Dodge and Romig [2] used what is essentially the reciprocal of R_0 as a basis for the construction of sampling plans. Now, assume a binomial distribution of defectives in samples and a series of single sampling plans with the same c but different n. As n increases, the effective values of p_1 and p_2 clearly decrease. Their ratio R_0 is not constant, but it does not change very much after n has got beyond the range of very small samples—say $5(c + 1)$. The value obtained from the chi-square table is the upper limit of R_0 for a fixed c and increasing n. Since R_0 is to a first approximation a function of c alone, provided n is not very small, it is a useful index for the construction of tables, and gives great compactness.

Using the chi-square approach, we note that

$$D.\,F. = 2c + 2$$

$$np_1 = \tfrac{1}{2}\chi^2_{2c+2,1-\alpha}$$

$$np_2 = \tfrac{1}{2}\chi^2_{2c+2,\beta}$$

$$R_0 = \frac{\chi^2_{2c+2,\beta}}{\chi^2_{2c+2,1-\alpha}}.$$

Table I gives R_0, c, and np_1 over a considerable range, with $\alpha = \beta = .05$. Given p_1 and p_2, we calculate R_0 and use it to enter the table; c is read off directly, and the sample size is $n = np_1/p_1$.

Sample sizes obtained in this way will be too large when the true distribution of defectives follows the binomial or hypergeometric laws. There is, however, a gain in protection due to the extra inspection. For the binomial case the exact

TABLE I

Single sample inspection plans

$\alpha = \beta = .05$

R_0	c	np_1
58.	0	.051
13.	1	.355
7.5	2	.818
5.7	3	1.366
4.6	4	1.970
4.0	5	2.61
3.6	6	3.29
3.3	7	3.98
3.1	8	4.70
2.9	9	5.43
2.7	10	6.17
2.63	11	6.92
2.53	12	7.69
2.44	13	8.46
2.37	14	9.25
2.30	15	10.04
2.24	16	10.83
2.19	17	11.63
2.14	18	12.44
2.10	19	13.25
2.07	20	14.07
2.03	21	14.89
2.00	22	15.72
1.92	25	18.22
1.81	30	22.44
1.71	37	28.46
1.61	47	37.20
1.51	63	51.43
1.335	129	111.83
1.251	215	192.41

In view of the approximate nature of this table due to the Poisson distribution, it is suggested that when the calculated value of R_0 does not appear, the table be entered with the next larger value. This rule will result in partial compensation for the approximation.

values p_1 and p_2 for a given n and c can be calculated, using a table of the 5 per cent points of the F (variance ratio) distribution. We may take

$$n_1 = 2(n - c)$$
$$n_2 = 2(c + 1)$$
$$F_1 = F(n_1, n_2)$$
$$F_2 = F(n_2, n_1).$$

Then
$$p_1 = \frac{n_2}{n_2 + n_1 F_1}$$
and
$$p_2 = \frac{n_2 F_2}{n_1 + n_2 F_2},$$

utilizing a property of the F distribution pointed out in [3], page 2.

REFERENCES

[1]. A. Wald, "Sequential tests of statistical hypotheses", *Annals of Math. Stat.*, Vol. 16 (1945), pp. 117-186.

[2]. H. F. Dodge and H. G. Romig, "A method of sampling inspection", *Bell System Technical Journal*, Vol. 8 (1929), pp. 613-631.

[3]. R. A. Fisher and F. Yates, *Statistical Tables*, 2nd edition, Oliver and Boyd, 1943.

ON AN EQUATION OF WALD

By David Blackwell

Howard University

Let $X_1, X_2, \cdots$ be a sequence of independent chance variables with a common expected value a, and let $S_1, S_2, \cdots$ be a sequence of mutually exclusive events, S_k depending only on $X_1, \cdots, X_k$, such that $\sum_{k=1}^{\infty} P(S_k) = 1$. Define the chance variables $n = n(X_1, X_2, \cdots) = k$ when S_k occurs and $W = X_1 + \cdots + X_n$. We shall consider conditions under which the equation

$$E(W) = aE(n), \tag{1}$$

due to Wald [3, p. 142], holds.

This equation has various interpretations:

A. n may be considered as defining a sequential test on the X_i. If a and $E(W)$ are known, (1) may be used to determine $E(n)$, the expected number of observations required by the sequential test, [3, p. 142 et seq].

B. n may be considered as representing a gambling system, i.e. it represents the point at which a player decides to stop. W then represents his winnings,