ABSTRACTS OF PAPERS

(*Abstract of a paper presented at the Western Regional Meeting, Eugene, Oregon, June 20–21, 1963. Additional abstracts appeared in the June and September, 1963 issues.*)

10. **Representation of Uniform Distributions as Convolutions.** RALPH STRAUCH, University of California, Berkeley.

This paper gives necessary and sufficient conditions for the existence of distributions P and Q such that $H = P * Q$, where H is uniform on $\{0, 1, \cdots, n\}$, as follows. Let $\{d_t:1 \leqq t \leqq \tau\}$ be a sequence of integers such that (i) $d_t > 1, 1 \leqq t \leqq \tau$; (ii) $\prod_{t=1}^{\tau} d_t = n+1$. Let $\{p_t:1 \leqq t \leqq \tau\}$ be distributions such that $p_t(k \prod_{j=1}^{t-1} d_j) = 1/d_t$, for $0 \leqq k \leqq d_t - 1$, where $\prod_{j=1}^{0} d_j = 1$ by convention. If A is a finite set of distributions, let $*A$ denote the convolution of the elements of A. Then there exist distributions P and Q on the nonnegative integers such that $H = P * Q$ if and only if $P = *\{p_t:t \varepsilon T_1\}$ and $Q = *\{p_t:t \varepsilon T_2\}$, where T_1 and T_2 are disjoint sets of integers such that $T_1 \cup T_2 = \{1, \cdots, \tau\}$.

(*Abstracts of papers presented at the Annual Meeting of the Institute, Ottawa, August 27–29, 1963. Additional abstracts appeared in the March, June and September, 1963 issues.*)

26. **The Monotonicity of the Power Functions of Test Procedures for Two Multivariate Problems.** T. W. ANDERSON and S. DAS GUPTA, Columbia University.

Let $\mathbf{X} = [x_{ij}]:p \times m$ and $\mathbf{Y} = [y_{ij}]:p \times n$ be two random matrices $(n \geqq p)$ such that the elements of $\mathbf{X}$ and $\mathbf{Y}$ are mutually independent, and x_{ii} is distributed according to $N(\theta_i, 1)$, $i = 1, \cdots, t = \min(p, m)$, and each of the elements of $\mathbf{Y}$ and the other elements of $\mathbf{X}$ is distributed according to $N(0, 1)$. It is shown that for testing the hypothesis $\theta_1 = \cdots = \theta_t = 0$, the power function of any test, based on the roots of $(\mathbf{XX}')(\mathbf{YY}')^{-1}$ and having the acceptance region convex in each column vector of $\mathbf{X}$ for each set of fixed values of $\mathbf{Y}$ and of the other column vectors of $\mathbf{X}$, is a monotonically increasing function of each θ_i. This result is used to obtain another sufficient condition for a test to have a monotonically increasing power function in each of the invariant parameters for (i) testing a set of multivariate linear hypotheses in the usual linear normal model, and (ii) for testing independence between two sets of normally distributed variates. This result is a generalization of an earlier result by Das Gupta (*Ann. Math. Statist.* **33** (1962) p. 1504).

27. **Adequate Subfields and Almost Sufficiency.** OLE BARNDORFF-NIELSEN and MORRIS SKIBINSKY, University of Minnesota. (By title)

Let $X_1, X_2, \cdots, X_n$ and Θ be random variables and let $T = t(X_1, X_2, \cdots, X_n)$ be a statistic. Suppose that we want to use T as a predictor for Θ. Then the question naturally arises: does T summarize all the "information" concerning Θ, which is contained in the sample $X_1, X_2, \cdots, X_n$? The theory developed in this paper originated in an effort to give a precise meaning to questions like this. Of course the problem is basically similar to that encountered when one wants to estimate a paramter θ by a statistic T. While the relevant notion for the estimation-problem is that of sufficiency, for the prediction-problem it seems to be what we have chosen to call adequacy. Loosely speaking T is adequate (w.r.t. Θ)

if the conditional distribution of Θ given $X_1, \cdots, X_n$ depends on $X_1, \cdots, X_n$ only through T. The theory of adequacy given here follows closely the abstract theory of sufficiency due to Halmos and Savage (*Ann. Math. Statist.* **20** (1949) pp. 225–241), Bahadur (*Ann. Math. Statist.* **25** (1954) pp. 423–462) and others. It is, however, free of some of the pathologies of sufficiency discovered by Pitcher (*Ann. Math. Statist.* **28** (1957) pp. 267–268) and recently by Burkholder (*Ann. Math. Statist.* **32** (1961) pp. 1191–1200). From a Bayesian viewpoint it is not surprising that the concepts of adequacy and sufficiency are closely related and that our main theorem (2.8) establishes the equivalence (under suitable conditions) of adequacy and "almost sufficiency". The definition of adequacy is given in the paper's main section and basic properties derived. The theory is further developed in a product space context. Some illustrative applications to prediction theory are considered and one of Burkholder's examples is discussed.

28. **A Characterization of Multisample DF Statistics** (Preliminary report). C. B. Bell, San Diego State College.

Z. W. Birnbaum (1963) asks whether a SDF (strongly distribution-free) 2-sample statistic is a rank statistic. Consider k independent univariate samples $(X_{1j}), \cdots, (X_{kj})$ with $1 \leqq j \leqq n_i$ and $N = n_1 + \cdots + n_k$. A k-sample statistic T is SDF wrt Ω, if for each borel set B and collection (F_i) of cpfs of Ω, $P(T^{-1}(B) \mid F_1, \cdots, F_k)$ depends only on $F_1F_2^{-1}, \cdots, F_1F_k^{-1}$; and is SWS if it is a symmetric function of each of the k samples individually. One notes (i) that Ω', the class of smc (strictly monotone continuous) cpfs on R_1 is complete, and can be generated by the group $\mathcal{G}'$ of smc mappings of R_1 onto R; (ii) that similarity wrt the power class $\Omega'(N)$ and the SDF property imply almost invariance wrt $\mathcal{G}'(N)$ and $\Omega'(N)$; and (iii) (Scheffé, 1943) if a set W has a null boundary, then it is similar wrt $\Omega'(N)$ iff there exists $0 \leqq b \leqq N!$ such that W has Structure $S(b)$ (i.e. for each point $x = (x_1, \cdots, x_N)$ of R_N, W contains exactly b of the $N!$ points obtained by permuting the coordinates of x) except for a null set. Using these results one obtains: *Theorem.* For a k-sample statistic T (a) if T is a rank statistic, then T is SDF wrt Ω'; (b) if T is SWS and SDF wrt Ω', then T is almost invariant wrt $\mathcal{G}'(N)$ and $\Omega'(N)$; and (c) if T is SWS, SDF wrt Ω', and if for each borel set B, $T^{-1}(B)$ has a null Boundary, then T is equivalent to a rank statistic.

29. **The Non-Linear Regression of Time Series.** David R. Brillinger, Bell Telephone Laboratories and Princeton University.

Let there be given time series $Y(t), X_1(t), \cdots, X_k(t)$ and an error series $e(t)$. Suppose that the series satisfy a non-linear model of the form; $Y(t) = a_0 + \sum_i \int a_i(r)X_i(t-r)\,dr + \sum_i \sum_j \iint a_{ij}(r, s)X_i(t-r)X_j(t-s)\,dr\,ds + \cdots + e(t)$, for some functions a. The Fourier transforms of the functions a and through them the functions themselves, may be estimated by a regression analysis involving the Hilbert transforms of the series involved. The applicability of the model and its order may be inquired into by an examination of the residuals.

30. **A Bayesian Approach to the Importance of Assumptions Applied to the Comparison of Variances.** G. E. P. Box and George C. Tiao, University of Wisconsin. (Invited)

Frequently the distribution of observations **y** depends not only upon a set of parameters ξ_1 of interest, but also on a set of nuisance parameters ξ_2. In judging the sensitivity of inference about the parameters of interest relative to assumptions about the model such as normality and independence, the nuisance parameters can be measures of departure from normality and independence. From a Bayesian point of view, the posterior distribution $p(\xi_1 \mid \xi_2 = \xi_{20}, y)$ indicates the nature of inference about ξ_1 if the corresponding assumptions $\xi_2 = \xi_{20}$ about the model are made, while the posterior density $p(\xi_2 = \xi_{20} \mid y)$ reflects the plausibility of such assumptions. The marginal posterior distribution of ξ_1 obtained by integrating out ξ_2, $p(\xi_1 \mid y) = \int_R p(\xi_1 \mid \xi_2, y)p(\xi_2 \mid y)\, d\xi_2$, thus indicate the overall inference about ξ_1 when proper consideration is given to the various possible assumptions. This approach is applied to the problem of making inference about the variance ratio when two samples are drawn from a class of power distributions characterized by a location parameter, a scale parameter and an non-normality parameter. An actual example is worked out in detail.

31. **Sampling Properties of Tests for Categorical Data.** K. C. CHANDA, Iowa State University.

In employing the standard χ^2-test for testing homogeneity of k samples from k multinomial populations with l classes or for testing independence of two categories with k and l classes, when the total sample size n is small we make two kinds of errors viz., (1) error due to rough approximation as provided by the usual Edgeworth expansion of the distribution function and (2) error due to replacing the sum of discrete terms representing the true probability by an integral. As the sample size becomes larger and larger the two kinds of error become smaller and smaller and tend to zero as $n \to \infty$. But in small samples both these errors may assume significant values. Again, there are large sample tests other than the classical χ^2-tests which are, asymptotically, equivalent to the latter but may have more desirable properties in small samples. An attempt has been made in this article to investigate under the null hypotheses the small sample properties of the various large sample tests employed in this area bearing in mind only the error of the first kind. The procedure is as follows. The first two moments of the test criteria, under the null hypotheses are calculated to order n^{-1}. If A represents the mean of the particular test-criterion T and $2B$ the variance of the same then $\chi_M^2 = AT/B$ is used roughly as a chi-square with A^2/B degrees of freedom. The comparisons of these various tests are then equivalent to the comparisons of these χ_M^2.

Details have been worked out for the particular cases of (i) testing homogeneity of k (assumed large) binomial samples using three different tests viz., the one based on the classical arc sin transformation, the likelihood ratio test and the classical χ^2-test and of (ii) testing independence of two categories with k and l (both assumed large) classes using two different tests viz., the likelihood ratio test and the classical contingency χ^2-test.

In each case the statistic concerned is expanded and proved to be rigorously valid for appropriate expansions for the first two moments considered.

32. **Asymptotically Exact Truncation in Binomial Sequential Analysis** (Preliminary report). HERBERT T. DAVID and RODOLFO M. MENGIDO, Iowa State University.

When $z \equiv \ln(p_1/p_0)/\ln(q_0/q_1)$ is rational ($z = n/m$, n and m relatively prime), a Binomial sequential probability ratio test, when observed only at every $(m + n)$th sampling stage, constitutes an irreducible aperiodic Markov chain with two states absorbing and the remaining states transient. It is shown, for certain values of m and n, that these transient states S_j are geometrically ergodic in the strong sense that there exists a constant λ between zero and one, and a set of positive numbers π_j, with $\lim_{t\to\infty} p_{0,j}(t)/\lambda^t = \pi_j$ for all j. This geometric ergodicity leads to the possibility of constructing truncation rules meeting various criteria asymptotically; for example, a truncation rule, which, for specified p, yields asymptotically the "accept" and "reject" probabilities of the untruncated test, conditionally on no decision up to the point of truncation.

33. **Applications of the Factorial Calculus to General Unequal Numbers Analyses.** W. T. FEDERER and M. ZELEN, Mathematics Research Center, University of Wisconsin; National Institutes of Health.

The necessary elements of the calculus for factorials as developed by Kurkjian and Zelen are described, modified where necessary, and applied to the analysis of unbalanced n-way classifications with fixed effects. Estimators for all main effect and interaction effects parameters are obtained along with the associated variances. The sums of squares for each effect eliminating all other effects is presented in a form suitable for direct computation. This form results in considerable computational saving over the method of fitting constants used in general regression theory. The results are applied to the particular case of proportional frequencies in the subclasses.

34. **Poisson Regression: Confidence Limits and Tests of the Model.** JOHN J. GART, Johns Hopkins University. (By title)

Consider a sequence of mutually independent variates, y_{ij}, each having Poisson distributions with means, βx_i, where $j = 1, 2, \cdots, n_i$ and $i = 1, 2, \cdots, k$. The x's are known fixed constants and the β is to be estimated as in the usual regression situation. It is well known that the maximum likelihood estimator of β is $(\sum \sum y_{ij})/(\sum n_i x_i)$, which is both unbiased and sufficient. An approximation suggested by D. R. Cox (*Biometrika* **40** (1953) pp. 354–360) in a closely related situation enables one to find confidence limits for β by treating the fixed quantity $2\beta \sum n_i x_i$ as if it were a chi-squared variate with $2(\sum \sum y_{ij} + k)$ degrees of freedom. These are compared to the limits found by using the Fisher-Cornish correction for skewness of the type considered by Bartlett (*Biometrika* **40** (1953) pp. 12–19). In investigating the test of the hypothesis of linearity against the general alternative, we find the likelihood ratio test statistic can be identified with Bartlett's test of homogeneity of variances by, as before, treating the $2\beta n_i x_i$'s like chi-squared variates. A test of linearity of regression against the specific alternative of an additional non-zero quadratic term is investigated using the asymptotic test of composite hypotheses proposed by Neyman (*Probability and Statistics, The Harald Cramér Volume*, (1959) pp. 213–234). Finally the various procedures are illustrated by data from pock counting experiments in virology, where this model is sometimes useful.

35. **Posterior Odds for Multivariate Normal Classifications.** SEYMOUR GEISSER, National Institutes of Health.

It is sometimes of interest to an investigator to assess in some way the posterior odds or probability that a particular observation z belongs to one of k multivariate normal populations Π_i, given the prior probability $q_i, i = 1, \cdots, k$. We assume that the parameters of Π_i are unknown but estimates of them are available, based on samples of size N_i. A prior density, with an adjustable constant, is assigned to the parameters of each of the Π_i's thus enabling us to compute the posterior probability that z belongs to any of the Π_i's when the q_i's are known. This is extended, via a Bayes approach, to the case where the q_i's are also unknown.

This approach is in contradistinction to the usual classification method which divides up the observational space into mutually exclusive and exhausitive regions by minimizing the average loss of misclassification. The latter approach, while useful for classifying large numbers of observations subject to fixed error frequencies of misclassification, however, makes no statement as to the probability of a particular recorded observation belonging to one or another of the populations.

36. Minimax Character of Hotelling's T^2-Test in the Simplest Case. N. GIRI, J. KIEFER and C. STEIN, Cornell University, Cornell University, Stanford University. (By title)

Let $X_1, X_2, \cdots, X_N$ be independent normal (column) p-vectors with common mean vector ξ and common nonsingular covariance matrix Σ. Write $N\bar{X} = \sum_{i=1}^N X_i$ and $S = \sum_{i=1} (X_i - \bar{X})(X_i - \bar{X})'$. We need consider only procedures which depend on the sufficient statistic $(\bar{X}, S)$. Let $\delta > 0$ and $0 < \alpha < 1$ be specified. For testing the hypothesis $H_0: \xi'\Sigma^{-1}\xi = 0$ against $H_1: \xi'\Sigma^{-1}\xi = \delta$, a commonly employed procedure is Hotelling's T^2-test, which rejects H_0 if $\bar{X}'S^{-1}\bar{X} > c$ where c is chosen to yield a test of level α. This test is well known to be best invariant under the full linear group, but the Hunt-Stein theorem is not valid for that group. Reducing the problem instead by the group of lower triangular matrices, one obtains a p-dimensional maximal invariant and $(p - 1)$-dimensional reduced parameter space Δ. In the first nontrivial case, $p = 2$, $N = 3$, a somewhat lengthy calculation verifies the existence of a prior distribution on Δ relative to which the T^2-test is Bayes; hence, this test maximizes the minimum power on H_1 among all level α tests.

37. Minimax Character of the R^2-Test in the Simplest Case. N. GIRI and J. KIEFER, Cornell University.

Let $X_1, X_2, \cdots, X_N$ be independent normal (column) p-vectors with common mean vector ξ and common nonsingular covariance matrix Σ. Write $N\bar{X} = \sum_{i=1}^N X_i$ and $S = \sum_{i=1}^N (X_i - \bar{X})(X_i - \bar{X})'$ and partition S and Σ as

$$\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$$

respectively where S_{11} and Σ_{11} are 1×1. Let $\delta > 0$ and $0 < \alpha < 1$ be specified. For testing the hypothesis $H_0: \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}/\Sigma_{11} = 0$ against $H_1: \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}/\Sigma_{11} = \delta$, a commonly employed procedure is the R^2-test, which rejects H_0 if $R^2 = S_{12}S_{22}^{-1}S_{21}/S_{11} > c$ where c is chosen so as to yield a test of level α. Using a development parallel to that of the previous (T^2) abstract, it is shown that the R^2-test maximizes, among all level α tests, the minimum power under H_1, in the first nontrivial case, $p = 3$, $N = 4$, for each possible choice of δ and α.

38. A Note on Classical and Bayesian Prediction Intervals for Location, Scale and Regression Models. W. J. HALL and MELVIN R. NOVICK, University of North Carolina.

Consider a random sample from a distribution with location and scale parameters, λ and σ, and a prediction is to be made about a future observation Y. It is proved, under certain conditions, that if λ and σ are taken to have the prior probability element $d\lambda\, d\sigma/\sigma$, then the *Bayesian prediction intervals* based on the posterior distribution of Y coincide with the *classical prediction intervals* (sometimes called *confidence intervals* or *β-expectation tolerance intervals*). Analogous results obtain if, in addition to location and scale parameters, there are one or more regression parameters, distributed uniformly a priori. Hence, in these cases, if one observes a sample of size n, determines a β-content Bayesian interval based on the posterior distribution of Y, and predicts that the next observation will fall in it, he will be correct with long-run frequency equal to β. These results are consistent with and lend credence to the concept that $d\lambda\, d\sigma/\sigma$ reflects the absence of prior information, and support Bayesian prediction methods which incorporate prior information through natural conjugate methods proposed by the authors. Normal, exponential and uniform examples are given.

39. Some Results on Estimating the Number of Classes in a Discrete Uniform Population. BERNARD HARRIS, Mathematics Research Center, University of Wisconsin.

Assume that a random sample of size N has been drawn from a multinomial population with an unknown but finite number of equiprobable classes, θ. We further require that the classes do not have a natural ordering. Let n_r be the number of classes occurring r times in the sample, $r = 0, 1, \cdots, N$ and let $d = \sum_{r=1}^{N} n_r$. It is easily seen that d is a complete sufficient statistic; the insufficiency of d was established by Lewontin and Prout (*Biometrics*, 1956). It is shown that there is no unbiased estimator of θ, which is a function of d. However, if θ can be restricted to a set of N possible values Θ^* there is a unique unbiased estimator, which depends however on the N specified choices for θ. If $\Theta^* = \{1, 2, \cdots, N\}$, $\hat{\theta} = \alpha_{d,N}/\alpha_{d,N+1}$ where $\alpha_{d,N}$ are the Stirling Numbers of the Second Kind, and in the case θ is a consistent estimator. For arbitrary Θ^*, an explicit but complicated expression can be obtained and in fact, $\hat{\theta}$ may be negative. Alternative estimators are investigated, including an estimator derived from results of Harris (*Ann. Math. Statist.*, 1959) and a suggestion by David and Johnson (*Biometrics*, 1952).

40. Large Deviations in Multinomial Distributions. WASSILY HOEFFDING, University of North Carolina.

Let $N\nu = N(\nu_1, \cdots, \nu_k)$ have the multinomial distribution $\Pr\{N\nu_1 = N_1, \cdots, N\nu_k = N_k\} = N!(\Pi N_i!)^{-1}\Pi p_i^{N_i}$, $\sum N_i = N$, $\sum p_i = 1$. The asymptotic behavior as $N \to \infty$ of $\Pr\{\nu \,\varepsilon\, A\}$ is considered, where A is a fixed set of points $x = (x_1, \cdots, x_k)$ with $x_i \geqq 0$ and $\sum x_i = 1$, k and $p = (p_1, \cdots, p_k)$ are fixed, $p_i > 0$, and p is not in the closure of A. Sanov (*Mat. Sb.* **84** (1957)) has shown that under mild restrictions on A, $\Pr\{\nu \,\varepsilon\, A\} = \exp\{-N \inf_{x \varepsilon A} I(x, p) + o(N)\}$ where $I(x, p) = \sum x_i \ln (x_i/p_i)$. Here it is shown that under certain conditions $\Pr\{\nu \,\varepsilon\, A\} = c_N N^r \exp\{-N \inf_{x \varepsilon A} I(x, p)\}$, where $0 < c' < c_N < c'' < \infty$ and $r \leqq (k - 3)/2$. In "ordinary" cases $r = -\frac{1}{2}$. In the special case $A = \{x \mid I(x, p) \geqq c\}$ with c positive and not too large $r = (k - 3)/2$. The results are used to compare the asymptotic performance of certain tests such as the chi square test and the likelihood ratio test.

41. The Parametric and Non-Parametric Compound Decision Problems in the Sequence Case. M. V. JOHNS, JR., Stanford University. (By title)

Consider a compound decision problem consisting of a sequence of finite-action component problems having a common loss structure. Nature selects a sequence of sampling distributions $\{F_n\}$ from a specified (parametric or non-parametric) family $\mathfrak{F}$. The unknown distribution of the observations for the nth component problem is F_n. A (compound) decision rule for the nth component problem may depend on the observations obtained in the previous $n - 1$ problems. The loss functions are piecewise polynomials in certain functionals defined on $\mathfrak{F}$, and generalize those considered by H. Robbins and E. Samuel in forthcoming papers and by the author (*Ann. Math. Statist.* **28** 649–669) in connection with the two-action empirical Bayes problem. For certain problems of this type it is shown that, under mild restrictions, decision rules analogous to those suggested for the corresponding empirical Bayes problems produce an average risk for the first n component problems which is asymptotically (for large n) the same as the Bayes risk for a single hypothetical component problem in which nature selects one of $F_1, F_2, \cdots, F_n$ randomly with equal probabilities. Generalizations are also discussed.

42. Goodness Criteria for 2-sample Distribution-Free Tests. J. M. Moser and C. B. Bell, San Diego State College.

Using the notation of and extending the work of the preliminary report (*Ann. Math. Statist.* **33** (1962) p. 1486) one redefines a test function T to be mono if $T((X_i), (Y_j)) \leqq T((X_i^*), (Y_j^*))$ whenever $X_i^* \leqq X_i$ and $Y_j^* \geqq Y_j$ for all i and j. Theorems 1 and 2 of the preliminary report are valid. Using Chapman's (1958) one-sample admissibility result one proves that for $H_0: F = G$ vs $H_1: F > G$ (F, G continuous and strictly increasing) *Theorem 3*. A monotone 2-sample DF test is admissible. From invariance considerations one proves *Theorem 4*. Each rank test is DF, SDF, and has structure (d).

Using the results of Chernoff and Savage (1958) one finds *Theorem 5*. For statistics S which are asymptotically normal both under H_0 and H_1, and, in particular, for the statistics of Fisher-Yates, Van der Waerden, Mann-Whitney, Doksum, and Epstein-Rosenbaum-Moses, the Chapman-Large-Sample-Power bounds are,

$$\bar{\beta}(\Delta) = \Phi[\{\sigma(n_1, n_2, \Delta)\}^{-1}\{\sigma(n_1, n_2)\mathfrak{z}_\alpha + \mu(n_1, n_2, \Delta) - \mu(n_1, n_2)\}]$$

and $\underline{\beta}(\Delta) = \inf \Phi[\{\sigma(n_1, n_2, \Delta, u_0)\}^{-1}\{\sigma(n_1, n_2)\mathfrak{z}_\alpha + \mu(n_1, n_2, \Delta, u_0) - \mu(n_1, n_2)\}]$ where Φ is the standard normal cpf, $\Phi[\mathfrak{z}_\alpha] = \alpha$, the infimum is taken over the set $0 \leqq u_0 \leqq 1 - \Delta$; and the indicated means and standard deviations are those of S when $F = U$; and $G = U$, $\bar{G}$, and $\underline{G}(u_0; 1)$, resp.

43. A Class of Procedures With Monotonicity Properties for Four Problems in Multivariate Normal Statistical Analysis. G. S. Mudholkar, University of North Carolina and University of Rochester.

The problems considered are those of (i) MANOVA, (ii) testing independence between two sets of variates, (iii) testing equality of two dispersion matrices and (iv) testing equality of a dispersion matrix with a given matrix, well known in multivariate normal statistical analysis. It is known that each of these problems can be reduced to an extent by invariance considerations, characteristic roots of certain matrices being the maximal invariants. All the known test procedures for these problems are based on these characteristic roots. Roy and Mikhail (*Ann. Math. Statist.* **32** (1961) pp. 1145–1151) and Mikhail (*Ann. Math. Statist.* **33** (1962) pp. 1463–1465) have shown that the power functions of the union-intersection procedures, for the above problems, based on the maximum and the minimum of the aforementioned characteristic roots have certain monotonicity properties. If $\lambda_1, \lambda_2, \cdots, \lambda_u$ denote the characteristic roots and $e_1, e_2, \cdots, e_u$ their elementary symmetric function then it is the purpose of this paper to show that the power functions of procedures characterized by acceptance regions of the form $a_1\lambda_1 + \cdots + a_p\lambda_u \leqq$ constant ($a_1, \cdots, a_{u-1} \geqq 0$, $a_u > 0$) and $a_1e_1 + \cdots + a_ue_u \leqq$ constant ($a_1, \cdots, a_u \geqq 0$) have the monotonicity properties similar to those discussed in the papers by Roy and Mikhail and Mikhail.

44. Estimation by Order Statistics When the Censored Minimum of Random Variables is Observed. Charles DeWitt Roberts, University of North Carolina.

Let X be a random variable with mean m and variance v^2. For $X_1, X_2, \cdots, X_k$ independently distributed as X define $Y = \min(X_1, X_2, \cdots, X_k)$. Further let $Y_1, Y_2, \cdots, Y_n$ be n independent observations on Y and $Y_{(1)} \leqq Y_{(2)} \leqq \cdots \leqq Y_{(n)}$ their ordered values.

Best linear estimates are derived for m and v when censored observations $Y_{(r+1)}$, $Y_{(r+2)}$, $\cdots$, $Y_{(n-s)}$ where $1 \leqq r + s \leqq n - 1$ are available. It is shown that for some distributions of Y (exponential included) the estimates for $k = 1$ easily yield estimates if $k \geqq 1$. Tables to facilitate estimation if $k = 1$ are available for several distributions (for example, exponential, Sarhan and Greenberg, *J. Amer. Statist. Assoc.* **52** (1957), 58–87). Consideration is also given to the estimation problem when the distribution of X is specified.

45. **Idemvariant Transformations of a Random Variable** (Preliminary report). Norman C. Severo and Paul J. Schillo, State University of New York at Buffalo.

Let X be a random variable with distribution function F. If h and k are transformations of X such that the distribution functions of the random variables $h(X)$ and $k(X)$ are the same, then h and k are said to be *idemvariant transformations* of X (or h is said to be *idemvariant to k with respect to X*). We show that a class of transformations, each member of which is idemvariant to the given transformation h with rsepect to X can be constructed in two steps: (i) by decomposing the real line, R_1, into three sets A, B, and C, which depend on three essential features of F (*i.e.*, A is the set of points of discontinuity of F; B is the set of all points of R_1 that lie in open intervals over which F is constant; and $C = R_1 - (A \cup B)$), and (ii) by distorting h over A, B, and C, separately, in such a way that the various resulting distributions of the modified transformed random variable is the same as the distribution of $h(X)$.

46. **Economic Partially Balanced 2^n Factorial Fractions.** J. N. Srivastava and R. C. Bose, University of North Carolina.

Partially balanced factorial fractions, defined earlier by Bose and Srivastava, are broadly those fractions in whose analysis the properties of the linear algebras of PBIB designs are useful. Such fractions, though irregular, lend themselves to a relatively much easier analysis. In this paper a number of them from the 2^n factorials ($5 \leqq n \leqq 10$), both with and without blocks, have been considered (for estimating main effects and two factor interactions). They have the following further properties: (i) They are economic; i.e. do not involve an undesirably large number of assemblies, (ii) the correlations are small, those involving any main effect being negligible. An illustration of the method of construction used is given by a 2^9 factorial fraction in 64 assemblies. These are divided into 4 blocks, the 16 assemblies in them being those which respectively satisfy the equations: (i) $(U_1, U_2, U_3, U_{10}, U_{11}) = (0, 0, 0, 0, 0)$, (ii) $(U_1, U_2, U_3, U_4, U_5) = (1, 1, 1, 1, 1)$, (iii) $(U_{10}, U_{11}, U_{12}, U_7, U_8) = (1, 1, 1, 1, 1)$, and (iv) $(U_5, U_6, U_7, U_8, U_9) = (0, 0, 0, 0, 0)$, where (i) $U_1 = x_1 + x_2 + x_3$, $U_2 = x_4 + x_5 + x_6$, $U_3 = x_7 + x_8 + x_9$, $U_4 = x_1 + x_6 + x_8$, $U_5 = x_2 + x_4 + x_9$, $U_6 = x_3 + x_5 + x_7$, $U_7 = x_1 + x_4 + x_7$, $U_8 = x_2 + x_5 + x_8$, $U_9 = x_3 + x_6 + x_9$, $U_{10} = x_1 + x_5 + x_9$, $U_{11} = x_3 + x_4 + x_8$, $U_{12} = x_2 + x_6 + x_7$; (ii) x_i represents the level (0, and 1) of the ith factor; (iii) all calculations are done (mod 2).

47. **The Distribution of the Goodness-of-Fit Statistic U_N^2. 1.** Michael A. Stephens, University of Toronto.

The goodness-of-fit statistic W_N^2 has recently been supplemented (Watson, *Biometrika* 1961, 1962) by U_N^2 defined by $U_N^2 = N \int_{-\infty}^{\infty} \{F_N(x) - \int_{-\infty}^{\infty} [F_N(y) - F(y)]\, dF(y)\}^2\, dF(x)$. In this expression, $F(x)$ is the cumulative distribution function being tested, and $F_N(x)$ the sample c.d.f. for a sample of size N. This statistic is of special value in circular problems where W_N^2 cannot be used. The paper gives the first four numbers of the distribution of U_N^2

on the null hypothesis that the sample comes from $F(x)$, together with the exact distribution for $N = 2, 3$ and some results for higher N. U_N^2 is somewhat easier to examine than W_N^2, and the convergence of the distribution to the asymptotic distribution given by Watson is quite rapid.

48. **Unbalanced Limit Theorems** (Preliminary report). Henry Teicher, Purdue University.

Let $X_n = (X_{n1}, X_{n2}, \cdots, X_{nm})$, $n = 1, 2, \cdots$ constitute a sequence of independent identically distributed random vectors on some probability space with partial sums $S_n^{(j)} = \sum_{i=1}^n X_{ij}$, $1 \leqq j \leqq m$, $n = 1, 2, \cdots$ and suppose that $b(n)$, $n = 1, 2, \cdots$ is an increasing sequence of positive members tending to infinity such that the distribution of $(S_n^{(1)}/b(n), \cdots, S_n^{(m)}/b(n))$ tends to a limiting cumulative distribution function (c.d.f.), $F(x_1, \cdots, x_m)$. The limiting c.d.f. of $(S_{N_{n1}}^{(1)}/b(N_{n1}), \cdots, S_{N_{nm}}^{(m)}/b(N_{nm}))$ is given in terms of F when (i) N_{nj}, $1 \leqq j \leqq m$, $n = 1, 2, \cdots$ are m increasing sequences of positive integers satisfying certain conditions, (ii) N_{nj}, $1 \leqq j \leqq m$, $n = 1, 2, \cdots$ are m sequences of positive integer-valued random variables converging in probability to constants a_j, $1 \leqq j \leqq m$.

49. **On Head-of-the-Line Priority Queues.** Peter D. Welch, IBM Thomas J. Watson Research Center. (By title)

The following queueing process is considered. The input is the superposition of r independent Poisson processes, each process corresponding to a priority level. Associated with each priority level there is a distinct arbitrary service time distribution function $G_k(x)$; $k = 1, \cdots, r$. A single server operates under a head-of-the-line priority service discipline. For this process, with $r = 2$, Miller (*Ann. Math. Statist.* **31** (1960) 86–103) characterized the asymptotic behavior of the number of customers of each priority level in the queue. We determine the transient and asymptotic behavior of the number of customers of each priority level in the queue for general r.

(Abstract of a paper to be presented at the Annual Meeting Amherst, Massachusetts, August 30 to September 4, 1964.)

1. **Sequential Life Tests With Piecewise Constant Failure Rates.** L. A. Aroian and D. E. Robison, Space Technology Laboratories, Redondo Beach, California.

Sequential life testing involving the Wald sequential probability ratio test (SPR) has been widely developed for the exponential case. A common prlblem is to test $\lambda = \lambda_0$ against an alternative $\lambda = \lambda_1$, $\lambda_1 > \lambda_0$. These results are well known.

In the present paper we extend the SPR ideas for the exponential case to include testing of hypotheses in which λ and its alternatives change value a finite number of times during the test. The operating characteristic functions and the average time to termination of the test are derived.

Additionally, SPR tests are derived for the normal and Weibull distributions. The OC function and the average time to termination of the test in these cases are approximated by applying the previous results, and assuming that the failure function, $\lambda(t)$, of these distributions may be approximated in a finite number of intervals in which the failure rate is constant. Replacement and non-replacement of items are treated. Two numerical examples illustrate the theory.

(Abstracts not connected with any meeting of the Institute.)

1. Optimal Asymptotic Tests of Linear Hypotheses. B. R. BHAT and S. R. KULKARNI, Karnatak University.

Reiersøl, Bhapkar, Mitra, and others (Cf. Bhapkar, *Ann. Math. Statist.* **32** (1961) p. 72) have applied Neyman's minimum modified χ^2 method (χ_1^2 method) to test linear hypotheses (H) in binomial and multinomial experiments. Neyman has made an assumption (A) that $p_{ij} = f_{ij}(\theta_1, \theta_2, \cdots \theta_k) > 0$ and $\sum_j f_{ij}(\theta_1 \cdots \theta_k) \equiv 1$ for the whole range of variation of the parameters $\theta_1, \theta_2, \cdots, \theta_k$, where p_{ij}'s are probabilities of the ith multinomial experiment and f_{ij}'s are functions of known form. It was not noticed by these authors that (A) does not hold for testing linear hypotheses. In view of this objection and other optimality considerations, in this paper, it is proposed to use Neyman's ideas of Optimal Asymptotic Tests (OAT) (H. Cramér Volume, Wiley (1959)), to test linear hypotheses in these experiments. OAT criteria to test a linear hypothesis for arbitrary distributions have also been given. Further, Neyman's idea of OAT has been extended to test a multi-parametric composite hypotheses.

2. A Non-Parametric Test Based on U-Statistics for Several Samples. JAYANT V. DESHPANDE, University of Poona. (Introduced by V. P. Bhapkar)

A non-parametric test is offered for testing the equality of location parameters of c populations under the assumption that they are of the same form, given c samples with $n_1, n_2, \cdots, n_c$ observations rsepectively. c-plets are formed by taking one observation from each sample. Let v_1^i be the number of c-plets in which the observation from the ith sample is the least and v_c^i be the number of c-plets in which the observation from the ith sample is the largest. Let $u_j^i = v_j^i/\prod n_i$. The V-test (Bhapkar, *Ann. Math. Statist.* **32** (1961) 1108–1117) is based on the statistics u_1^i only; the test being proposed now is based on u_1^i and u_c^i. Let $l_i = -u_1^i + u_c^i$. It is then shown that as $n \to \infty$, with $n_i = ns_i$, s_i a positive integer and $p_i = n_i/\sum n_i$, $N = \sum n_i$,

$$L = \left[N(2c-1)(c-1)^2\binom{2c-2}{c-1}\bigg/ 2c^2\left\{\binom{2c-2}{c-1} - 1\right\}\right]\left[\sum_{i=1}^{c} p_i l_i^2 - \left\{\sum_{i=1}^{c} p_i l_i\right\}^2\right]$$

has in the limit χ^2 distribution with $c - 1$ d.f. under the null hypothesis, and non-central χ^2 under the alternative hypothesis of shift. It is found that this L-test is more efficient (twice as much, as number of samples tends to infinity) than V-test for all symmetric distributions. It is more efficient than Kruskal's test for some distributions and at least equally efficient for $c \leqq 7$ for normal distributions.

3. Some Considerations of Stochastic Processes in Estimating Weibull Parameters. SATYA D. DUBEY, Proctor & Gamble Co., Cincinnati.

Two stochastic models conforming to stochastic processes are formulated to answer some problems arising from the areas of industrial products, management science, etc. Model I deals with the *aging* behavior of industrial products. Let $R(t)$ be the probability that an item drawn randomly from a lot at the time point t is in good state. Considering a two-state irreversible process, a general solution of $R(t)$ is obtained. This approach enables one to derive several different estimators of the scale and the shape parameters of the Weibull Law. The asymptotic properties of these estimators are investigated. They are *consistent* and *asymptotically normal*. The covariance matrices of these estimators are derived too. Model II is concerned with a problem of management science. Here practical considerations are made to derive similar estimators of the scale and the shape parameters of the Weibull Law. Their asymptotic properties are also investigated and essential results are obtained. Some applications of the results, derived in this paper, are discussed. Finally, the results applicable to the special cases of these models are given and some new results are obtained in this situation.

4. **Extremal Processes.** MEYER DWASS, Northwestern University.

If $\{X_i\}$ is a sequence of independent and identically distributed r.v.'s, $M_n = \max X_1, \cdots, X_n$, then n.a.s.c.'s are well known for $(M_n - a_n)/b_n$ to converge in law. The limiting distribution is one of three well-known types. We define a stochastic process $Y(t)$ as the "limit" of $Y_n(t) = (M_{[nt]} - a_n)/b_n$, $t \geqq 0$, as $n \to \infty$. Specifically, the joint distribution of $(Y(t_1), \cdots, Y(t_k))$ is for every finite sequence $t_1, \cdots, t_k$ defined to be the limiting distribution of $(Y_n(t_1), \cdots, Y_n(t_k))$. There are three possible types of processes $Y(t)$ which we call extremal processes of types I, II, III; our purpose is to determine their structure. For $0 < a < t < b < \infty$, $Y(t)$ is with probability 1 a step function with a finite number of jumps. This number is Poisson distributed with parameter $\log(b/a)$. If (c, d) is in the range of $Y(t)$ then the number of jumps of $Y(t)$ necessary to bring it from a height below c to one above d $(c < d)$ is also Poisson distributed. The parameter of this distribution depends of the type of process. $Y(t)$ is Markovian and a simple explicit representation is given for each of the three types.

5. **Uniform Approximation for Minimax Point Estimates** (Preliminary report). M. N. GHOSH, Institute of Agricultural Research Statistics, New Delhi.

The problem of estimation of a bounded function $g(\theta)$ of the parameter θ, for a class of distributions $F(x, \theta)$ is considered in this paper, when the loss function has the form $W(u, \theta) = [u(x) - \theta]^p (p > 1)$ and sufficient conditions are given so that a minimax sequence of estimates converges with respect to the metric

$$\rho(u_1, u_2) = \sup_\theta \{E[|u_1(x) - u_2(x)|^p \mid \theta]\}^{1/p}.$$

This in many cases will imply that no almost subminimax (ϵ-minimax) estimates can exist for sufficiently small ϵ, which would give a sizable reduction in risk for some value of θ. Sufficient conditions are also given under which the minimax estimate $u(x)$ in the function space $L^{(p)}(F)$ can be approximated to any extent by minimax estimates $\bar{u}_n(x)$ in a finite dimensional linear space spanned by basis vectors $u_1(x), \cdots, u_n(x)$ of $L^{(p)}(F)$.

6. **On the Complex Analogues of T^2- and R^2-Tests** (Preliminary report). N. C. GIRI, Cornell University.

Let X be a p-variate complex Gaussian random variable with mean α and positive definite Hermitian covariance matrix Σ. Let Σ be partitioned as

$$\begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}^* & \Sigma_{22} \end{pmatrix},$$

where Σ_{22} is a $(p-1) \times (p-1)$ submatrix of Σ and Σ_{12}^* is the adjoint of Σ_{12}. The maximum likelihood estimates of Σ and $\Sigma_{12}\Sigma_{22}\Sigma_{12}^*/\Sigma_{11}$ and their distributions for $\alpha = 0$ have been obtained by Goodman (1963). In this paper the complex analogues of T^2- and R^2-tests of the real multivariate case have been obtained. Some optimum properties of these tests which are counterparts of the real case are also discussed.

7. A Condition for Optimality in Sequential Signal Detection (Preliminary report). K. B. GRAY, Hughes Research Laboratories, Malibu, California.

A received signal has the form $X_t = \theta s(t) + N_t$, where $s(t)$ is a known function of time and $t = 1, 2, \cdots$. Assume that N_t is a "white," gaussian noise process, i.e., the elements of any finite subset of $N_1, N_2, \cdots$ are normal, independent, and identically distributed with mean zero. We consider sequential procedures for testing between the alternative hypotheses $H_0:\theta = \theta_0$ and $H_1:\theta = \theta_1$. It is shown that the condition

$$\liminf_{n\to\infty} n^{-1}\sum_{t=1}^{n} \exp\left[-1/s^2(t)\right] > 0$$

is sufficient to insure that the sequential probability ratio test is optimal in the sense of the Wald-Wolfowitz theorem.

8. A New Method for Estimation of the Correlation Coefficient in Contingency Tables With Non-Metrical Characters. H. O. LANCASTER and M. A. HAMDAN, University of Sydney.

Pearson (*Philos. Trans. Roy. Soc. London* 195A (1900) 1–47) estimated the coefficient of correlation by the tetrachoric series in the fourfold table. Pearson (Drapers Company Research Memoirs, *Biometric Series No. 1* (1904)) equated $\phi^2 \equiv \chi^2/N$ to the series $\sum_1^\infty \rho^{2i} \equiv \rho^2/(1 - \rho^2)$, a special case of the Parseval equation, and in (*Biometrika* **9** (1913) 116–139) modified the method by subtracting the degrees of freedom, namely $(m - 1)(n - 1)$ from the χ^2 of the $m \times n$ table. It has never been clear how the broadness of the partition of the marginal distributions affects the estimate. An approach to this problem has been made as follows: orthonormal bases have been defined on the two marginal distributions, for convenience, using functions obtained from Helmert matrices. Then the orthogonal functions have been expanded in terms of the Hermite-Chebyshev polynomials. It is now possible to determine, under the hypothesis that ρ is not null, the parameter of noncentrality of each of the $(m - 1)(n - 1)$ variables, which are asymptotically normal (0, 1) and mutually independent under the null hypothesis. Then $N\phi^2 \equiv \chi^2 - (m - 1)(n - 1)$ is equated to the sum of these parameters, giving an equation in ρ^2 which can be arbitrarily terminated (at ρ^6 say). It is now evident that this method is superior to the usual Pearson method. For example, in Table XXV of Pearson and Lee (*Biometrika* **2** (1903) 357–462) the coefficient of correlation is 0.5157. In contingency tables constructed from this table, the estimates by the present method are given below with the Pearsonian estimates in brackets: for an 18×18 table 0.5170 (0.5062); for an 8×8 table 0.5041 (0.4735); for a 3×3 table 0.4973 (0.3814); for a 2×8 table 0.5382 (0.3854). In the case of pooling non-adjacent classes the difference is much wider.

9. On the Axioms of Information Theory. P. M. LEE, University of Cambridge. (Introduced by David G. Kendall)

In this paper it is proved under less restrictive conditions than previously, that Shannon's measure of information is unique. More exactly, it is shown that if the requirement of symmetry and the usual relation between the functions H_k and H_{k+1} measuring the information provided by the performance of experiments with, respectively, k and $k + 1$ possible outcomes, is assumed, and if $h(t) = H_2(t, 1 - t)$ is assumed measurable, then the H_k's are uniquely determined by the resulting functional equations for $h(\cdot)$. This result improves on various previous papers which assumed in addition continuity, integrability, or monotony of $h(\cdot)$.

10. Spacing Problems in Minimum Variance Extrapolation. ARNOLD LEVINE, University of California, Los Angeles. (Introduced by Paul G. Hoel)

When the variables $y_{t_1}, y_{t_2}, \cdots, y_{t_n}$ in polynomial regression are uncorrelated, the predicted value of $E(y_t)$ for t beyond the interval of observations is shown to possess minimum variance among all Markov predictors when the k points at which observations are to be taken in the interval $(-1, 1)$ are the points at which the Chebychev polynomial assumes its maximum and minimum values in that interval and when the number of observations to be taken at each such point is made proportional to the value of the corresponding Lagrange interpolation polynomial at the selected extrapolation point. The weighting of observations depends upon the selected t value but the spacing does not.

11. The Distribution of the Ranges in Multi-Variate Normal Samples in Terms of the Probability Contents of Regions Under Spherical Normal Distribution (Preliminary report). K. V. MARDIA, University of Rajasthan. (Introduced by B. D. Tikkiwal)

Harold Ruben (*Ann. Math. Statist.* **31** (1960) 1113–1121) has expressed the distribution of range in normal samples as the product of the sample size and the probability contents of a certain parallelotope relative to a hyperspherical normal distribution. We know the exact distribution function of ranges from k-variate normal population (Mardia, a paper under consideration in *Ann. Math. Statist.*). In this paper, it is expressed as $\sum_{i=1}^{k} \{n(n-1) \cdots (n-i+1)\} P(R_i)$ where $P(R_i)$ is the probability contents of region R_i in $k(n-1)$-Euclidean space relative to a hyperspherical normal distribution and R_i is a parallelotope bounded by $2k(n-1)$-flats where each flat is of dimensionality $k(n-1)-1$. For $k = 2$, the flats in R_1 make angle either $\cos^{-1} \pm \frac{1}{2}$, $\cos^{-1} \pm \rho/2$ or $\cos^{-1} \pm \rho$ while the angles between flats in R_2 are complicated expressions.

12. On the Extremities of One-Sided Distribution Functions Having Analytic Characteristic Functions. B. RAMACHANDRAN, Institute of Mathematical Sciences, Madras.

It is known (see, for instance, E. Lukacs (1960), *Characteristic Functions*, Griffin, London) that, if $F(x)$ is a distribution function (d.f.) having an analytic characteristic function (c.f.), $f(t)$, and is, further, bounded to the left, then its left extremity is given by: lext $F = -\limsup_{y\to\infty} [\ln f(iy)/y]$. We wish to point out here that, in this formula, "lim sup" can be replaced by "lim"; for $\phi(y) = \ln f(iy)$ is a convex function of y in $y \geqq 0$ such that $\phi(0) = 0$, and, consequently, $[\phi(y)/y]$ is a nondecreasing function in $y > 0$, so that its limit as $y \to \infty$ exists. Similarly, if $F(x)$ be a d.f., having an analytic c.f. $f(t)$ and bounded to the right, then its right extremity is given by: rext $F = \lim_{y\to\infty} [\ln f(-iy)/y]$. In particular, if $F(x)$ be a "finite" d.f. (i.e., bounded both above and below), then, since its c.f. is an entire function, its two extremities are given by the above formulas. The sharpened form of the result on the left extremity finds application in the establishing of a "denumerable α-decomposition" theorem for the Poisson law, for which purpose the less precise form is found inadequate. Details are published elsewhere.

13. On Selecting a Subset of Normal Populations Containing the Population Whose Mean Has the Largest Absolute Value. M. HASEEB RIZVI, University of Minnesota and Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio.

Let $\mu_i (i = 1, 2, \cdots, k)$ be the means of $k \geqq 2$ normal populations all having a common unit variance and let $\theta_i = |\mu_i|$. The problem of selecting a small non-empty subset containing the population whose mean has the largest absolute value is considered here. With correct selection (CS) defined in an obvious manner, a procedure R is required to satisfy the condition $P\{\text{CS} \mid R\} \geqq P^*$ for a pre-assigned P^* and regardless of true unknown θ-values. Let $\bar{x}_i$ be the ith sample mean based on a common number n of independent observations and let $w_i = |\bar{x}_i|$ $(i = 1, 2, \cdots, k)$. The proposed procedure R is: Retain the ith population in the selected subset if and only if $w_i \geqq w_{\max} - d$, where $d \geqq 0$ is determined subject to R satisfying the probability condition. The solution $\delta = n^{\frac{1}{2}}d$ of the probability condition is tabulated in Bechhofer (*Ann. Math. Statist.* **25** (1954) 16–39). The expected size of the selected subset is derived and its supremum obtained. Two secondary problems of determining n are discussed. The related problems of unknown common variance and the one of selecting the population with the smallest absolute value of the mean are also treated and directions for generalizations indicated.

14. Ranking Normal Populations by the Absolute Values of Their Means: Fixed Sample Size Case. M. Haseeb Rizvi, University of Minnesota.

Consider $k \geqq 2$ normal populations with unknown means μ_i $(i = 1, 2, \cdots, k)$ and a common unit variance; let $\theta_{[1]} \leqq \theta_{[2]} \leqq \cdots \leqq \theta_{[k]}$ be the ordered values of $\theta_i = |\mu_i|$. This paper studies the problem of selecting $t\,(<k)$ populations with t largest θ-values. With the obvious definition of a correct selection (CS) a procedure R_t is required so as to satisfy the condition $P\{\text{CS} \mid R_t, \theta_{[k-t+1]} - \theta_{[k-t]} \geqq \delta^*\} \geqq P^*$, where P^* and $\delta^* > 0$ are specified constants. Define $w_i = |\bar{x}_i|$ $(i = 1, 2, \cdots, k)$, where $\bar{x}_i$ is the ith sample mean based on a common pre-determined number n of independent observations. The proposed procedure R_t ranks w_i and selects the populations with t largest w_i as populations with t largest θ-values. Then n is determined so that R_t satisfies the probability condition. Certain bounds on $P\{\text{CS} \mid R_t\}$ are obtained. Tables for special cases, $t = 1$ and $t = k - 1$, give values of the infimum of $P\{\text{CS} \mid R_t\}$ for $k = 2(1)10$ and $\lambda = n^{\frac{1}{2}}\delta^* = 0(0.1)7{\cdot}0$. In addition the values of λ are given for $k = 2(1)10$ and several P^*'s. Finally the decision rule R_t is shown to be most economical by demonstrating its minimax and admissible nature respective to a simple loss function.

15. A Test for "Instrinsic" Correlation in the Theory of Accident Proneness (Preliminary report). K. Subrahmaniam, Johns Hopkins University.

In "Contributions to the theory of accident proneness. I. An optimistic model of the correlation between light and severe accidents," Bates and Neyman (Univ. California, 1952) have studied the correlation between the pair (X, Y) of light and heavy accidents. This correlation is the outcome of the model used; viz., that the average number μ and λ of light and heavy accidents for any individual are perfectly correlated and have a Pearson Type III distribution. This we refer to as the extrinsic correlation between X and Y. They do not, however, consider the possibility of X and Y being "intrinsically" correlated i.e., X and Y for any given individual being correlated. In this paper a test for the "intrinsic" independence of X and Y under the assumption of X and Y having a bivariate Poisson distribution is given; i.e., the p.g.f. of (X, Y) for a given individual is $\Pi(z_1, z_2) = \exp\{a_1(z_1 - 1) + a_2(z_2 - 1) + b(z_1z_2 - 1)\}$ with $a_1 = \lambda$, $a_2 = \gamma_1$, and $b = \gamma_2\lambda$. Under this assumption, the coefficient of correlation, ρ, between X and Y for any given individual is $\rho = \gamma_2/\{(1 + \gamma_2)(\gamma_1 + \gamma_2)\}^{\frac{1}{2}}$ and is independent of λ; i.e., the coefficient of "intrinsic" correlation between X and Y is the same for all the individuals in the population. If we assume for λ a gamma distribution $\{r/m; r\}$, we obtain for the unconditional distribution of (X, Y) a bivariate negative binomial distribution with p.g.f.

$$[1 - (m/r)\{(z_1 - 1) + \gamma_1(z_2 - 1) + \gamma_2(z_1z_2 - 1)\}]^{-r}.$$

A test for $\rho = 0$ is developed and the properties of this distribution are studied.

CORRECTION TO ABSTRACT

Professor Seiden has informed the Editor that the proof of the result she announced in the Abstract "On the Non-Existence of Balanced Incomplete Block Designs BIBD, With Parameters (46, 69, 9, 6, 1) and (51, 85, 10, 6, 1)," these *Annals* **34** 685, is false, and that the problem remains open.