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CORRECTION

BACKFITTING AND SMOOTH BACKFITTING FOR ADDITIVE QUANTILE MODELS

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In Theorem 2.2 on page 2865 of [1] we wrongly stated that the asymptotic biases of the ordinary and of the smooth backfitting estimator were the same. In fact, the bias formulas for the two methods are different. The theorem should be modified as follows.

THEOREM 2.2. *Let* $\alpha_j(u) = m_j'(u)\int(v-u)K_{j,h_j}(u,v)\,dv/\int K_{j,h_j}(u,v)\,dv$ *and* $\mu_{2,K} = \int v^2 K(v)\,dv$. *Assume that* (A1)–(A4), (A8) *and* (A9) *hold*, *that* (A5) *and* (A6) *are satisfied by* $\hat{m}_j^{\mathrm{BF}} = \hat{m}_j^{\mathrm{BF},[0]}$ *and* $\hat{m}_j^{\mathrm{SBF}} = \hat{m}_j^{\mathrm{SBF},[0]}$ $(j = 2, \dots, d)$ *with* $\xi, \Delta_2, \Delta_3, \frac{2}{5} - \frac{1+\rho}{2+3\rho}\frac{4}{5} - \Delta_1 > 0$ *small enough*, *and that* $w_j(a_j + x(b_j - a_j)) \le Cx(1-x)$ *for all* $0 \le x \le 1$ *and for some positive constant* C. *Then*, *we get for* $\hat{m}_j^{l,\mathrm{iter}} = \hat{m}_j^{l,[C_{\mathrm{iter}}\log n]}$ *with an appropriate choice of* $C_{\mathrm{iter}} = C_{\mathrm{iter},l}$ ($l = \mathrm{BF}$ *and* $l = \mathrm{SBF}$) *that for* $a_j < x_j < b_j$

$$\sqrt{nh_j}[\hat{m}_j^{l,\mathrm{iter}}(x_j) - m_j(x_j) - \beta_j^l(x_j)] \to N\left(0, \frac{\alpha(1-\alpha)}{f_{\varepsilon,X_j}(0,x_j)^2} f_{X_j}(x_j) \int K^2(u)\,du\right)$$

in distribution. Here, $\beta_j^l(x_j) = \beta_j^{*,l}(x_j) - \int \beta_j^{*,l}(u_j) w_j(u_j)\,du_j$, *and* $(\beta_j^{*,l} : 1 \le j \le d)$ *for* $l = \mathrm{BF}$ *is the solution of the system of integral equations*

$$0 = \int \left[\alpha_j(x_j) + h_j^2 \mu_{2,K} m_j'(x_j) \frac{\partial f_{\varepsilon,X}(0,x)/\partial x_j}{f_{\varepsilon,X}(0,x)} + \frac{1}{2} h_j^2 \mu_{2,K} m_j''(x_j) - \sum_{k=1}^{d} \beta_k^{*,l}(x_k)\right] f_{\varepsilon,X}(0,x)\,dx_{-j},$$
$$1 \le j \le d,$$

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while, for $l = \mathrm{SBF}$, $\beta_j^{*,l}(x_j) = \frac{1}{2}h_j^2\mu_{2,K}m_j''(x_j) + \mu_{2,K}\beta_j^{**}(x_j)$ *where* $(\beta_j^{**} : 1 \leq j \leq d)$ *is a tuple of functions that minimizes*

$$\int \left[\sum_{j=1}^{d} \left(h_j^2 m_j'(x_j) \frac{\partial f_{\varepsilon,X}(0,x)/\partial x_j}{f_{\varepsilon,X}(0,x)} - \beta_j^{**}(x_j) \right) \right]^2 f_{\varepsilon,X}(0,x)\, dx.$$

In addition, the assumption (A9) on page 2864 should be modified as follows:

(A9) The functions $f^w_{X_j,X_k}(x_j, x_k)$ and $f^w_{X_j}(x_j)$ have second derivatives w.r.t. x_j that are bounded over $x_j \in I_j$, $x_k \in I_k$ for all $1 \leq j \neq k \leq d$. Also, $f^w_{X_j,X_k}(x_j, x_k)$ and its first derivative w.r.t. x_j is continuous in x_k for all $1 \leq j \neq k \leq d$.

The corrected statement of Theorem 2.2 and the modified assumption (A9) do not require a different proof than what is given in [1].

REFERENCES

[1] LEE, Y. K., MAMMEN, E. and PARK, B. U. (2010). Backfitting and smooth backfitting for additive quantile models. *Ann. Statist.* **38** 2857–2883. MR2722458

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