Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2014, Article ID 786128, 11 pages
http://dx.doi.org/10.1155/2014/786128

Research Article

H_∞ Control for Network-Based 2D Systems with Missing Measurements

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Received 3 April 2014; Revised 19 April 2014; Accepted 20 April 2014; Published 11 May 2014

Academic Editor: Jun Hu

The problem of H_∞ control for network-based 2D systems with missing measurements is considered. A stochastic variable satisfying the Bernoulli random binary distribution is utilized to characterize the missing measurements. Our attention is focused on the design of a state feedback controller such that the closed-loop 2D stochastic system is mean-square asymptotic stability and has an H_∞ disturbance attenuation performance. A sufficient condition is established by means of linear matrix inequalities (LMIs) technique, and formulas can be given for the control law design. The result is also extended to more general cases where the system matrices contain uncertain parameters. Numerical examples are also given to illustrate the effectiveness of proposed approach.

1. Introduction

Two-dimensional (2D) systems have attracted considerable research interest over the past few decades due to their wide applications in the areas such as multidimensional digital filtering, linear image processing, signal processing, process control, and iterative learning control [1–5]. Thus the stability and stabilization of 2D systems have attracted a lot of interests; see, for example, [6–13] and the references therein. H_∞ optimization is a powerful tool that can be used to design a robust controller or filter, which has been proved to be one of the most important strategies. Recently, such problems on 2D systems have stirred a great deal of research attention. For example, several versions of 2D bounded real lemma have been established in [2, 6, 14]. The problem of H_∞ control for 2D systems with state delays has been considered in [15]. The problem of H_∞ filtering for 2D systems has been studied extensively in [16]. An H_∞ controller is designed for a class of 2D nonlinear discrete systems with sector nonlinearity in [17, 18].

Notice that all the above-mentioned results are based on an implicit assumption that the communication between the physical plant and controller is perfect; that is, the signals transmitted from the plant will arrive at the controller simultaneously and perfectly. However, in many practical situations, there may be a nonzero probability that all the signals can be measured during their transmission. In other words, the systems may have missing measurements. Moreover, networked systems are becoming more and more popular for the reason that they have several advantages over traditional systems, such as low cost, reduced weight and power requirements, simple installation and maintenance, and high reliability [19–22]. If network is introduced to controller design, the data packet dropout phenomenon, which appears in a typical network environment, will naturally induce missing measurements from the plant to the controller. In 1D system, the problems of stability, stabilization, filtering, or state estimation for networked control systems have been widely researched [23–30]. However, in the network-based 2D system case, only few results have been available. For instance, the problem of robust H_∞ filtering for 2D systems described by the Fornasini-Marchesini (FM) second model with missing measurements is considered in [31]. To the best of the authors' knowledge, the H_∞ control for 2D systems represented by the Roesser model which is structurally quite distinct from FM second model has not been addressed in the literature so far.

Motivated by the aforementioned observations, this paper considers the problem of H_∞ control for network-based 2D systems presented by the Roesser model with missing measurements. We also notice that some dynamical processes such as gas absorption, water stream heating, and air drying can be described by a 2D Roesser model. In practical, these systems are often implemented by distribute control systems (DCSs) or field-bus control systems (FCSs), where control loops that are closed over a communication network. Hence, the considered topic in this paper is of practical significance. Compared with existing results, this paper proposes a state feedback controller design method for 2D systems in the framework of networked control systems. Meanwhile, the introduction of the random missing measurements renders the 2D system to be stochastic instead of a deterministic one. To analyze the stability, we introduce the stochastic mean-square asymptotically stable and stochastic H_∞ disturbance attenuation level. The controller is also designed under the framework of 2D stochastic systems.

The remainder of this paper is organized as follows. In Section 2, the mathematical description and design objectives of this paper are presented. In Section 3, a sufficient condition of mean-square asymptotic stability with H_∞ performance for such 2D stochastic systems is derived by means of LMI technique, and then formulas can be given for the control law design. In Section 4, the design result is extended to the 2D systems with uncertain parameters. Numerical examples are given in Section 5 and conclusions are drawn in Section 6.

Notation 1. The superscript "T" denotes the matrix transposition, R^n denotes the n-dimensional Euclidean space, I denotes the identity matrix, 0 denotes the zero vector or matrix with the required dimensions, and diag$\{\cdot\}$ denotes the standard (block) diagonal matrix whose off-diagonal elements are zero. In symmetric block matrices, an asterisk $*$ is used to denote the term that is induced by symmetry. The notation $\|\cdot\|$ refers to the Euclidean vector norm and $\lambda_{\min}(\cdot), \lambda_{\max}(\cdot)$ denote the minimum and the maximum eigenvalue of the corresponding matrix, respectively. $E\{x\}, E\{x \mid y\}$ mean the expectation of x and the expectation of x conditional on y. Matrices, if the dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem Formulation

Consider the following 2D discrete system given by

$$\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = A \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + Bu(i,j) + B_1 w(i,j), \quad (1)$$

where $x^h(i,j) \in R^{n_1}$, $x^v(i,j) \in R^{n_2}$, and $u(i,j) \in R^m$ represent the horizontal state, vertical state, and control input, respectively. $w(i,j) \in R^p$ denotes the noise input, which belongs to ℓ_2. A, B, B_1 are real matrices with appropriate dimension.

We make the following assumption on the boundary condition.

Assumption 1. The boundary condition is assumed to satisfy

$$\lim_{N\to\infty} E\left\{\sum_{k=0}^{N}\left(\left|x_{k,0}\right|^2 + \left|x_{0,k}\right|^2\right)\right\} < \infty. \quad (2)$$

Now, we consider the following state feedback control law:

$$u(i,j) = G\tilde{x}(i,j), \quad (3)$$

where $\tilde{x}(i,j)$ is the measurement of state signals, G is appropriately dimensioned controller matrix to be determined. When the feedback control is implemented via a networked control system, the data $x^h(i,j), x^v(i,j)$ are transferred as two separate packets from the remote plant to the controller. In this process, the data may be missed due to the network transmission failure, resulting in what we call missing measurement. It is assumed that the data packet dropout can be described by a stochastic variable; that is,

$$\tilde{x}(i,j) = \alpha_{i,j} x(i,j), \quad (4)$$

where $x(i,j) = \left[\begin{smallmatrix} x^h(i,j) \\ x^v(i,j) \end{smallmatrix}\right]$, the stochastic parameters $\{\alpha_{i,j}\}$ is a Bernoulli distributed white sequence taking the values of 0 and 1 with

$$\begin{aligned} \text{Prob}\left\{\alpha_{i,j} = 1\right\} &= E\left\{\alpha_{i,j}\right\} = \alpha, \\ \text{Prob}\left\{\alpha_{i,j} = 0\right\} &= 1 - E\left\{\alpha_{i,j}\right\} = 1 - \alpha, \end{aligned} \quad (5)$$

and $0 \le \alpha \le 1$ is a known constant.

Notice that the introduction of the stochastic variable $\{\alpha_{i,j}\}$ renders the 2D system to be stochastic instead of a deterministic one. Before proceeding further, we need to introduce the following definition of stochastic stability for the 2D system, which will be essential for our derivation.

Definition 2 (see [31]). The 2D system (1) is said to be mean-square asymptotically stable if under the zero input and for every bounded initial condition $x^h(i,0)$, $x^v(0,j)$, the following is satisfied

$$\lim_{i+j\to\infty} E\left\{\left\|x(i,j)\right\|^2\right\} = 0. \quad (6)$$

Definition 3. Given a scalar $\gamma > 0$, the 2D system (1) is said to have an H_∞ disturbance attenuation level γ, if it is mean-square asymptotically stable and under zero initial conditions, $\|x\|_E < \gamma\|w\|_2$ is satisfied for any external disturbance $w(i,j) \in \ell_2$, where

$$\begin{aligned} \|x\|_E &= \sqrt{E\left\{\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left\|x(i,j)\right\|^2\right\}}, \\ \|w\|_2 &= \sqrt{E\left\{\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left\|w(i,j)\right\|^2\right\}}. \end{aligned} \quad (7)$$

To this end, the design objective of this paper can be described as follows.

Problem Statement. For any initial condition satisfying Assumption (1) and missing measurements described as (5), design a state feedback law (3) such that the closed-loop 2D system is mean-square asymptotically stable and has an H_∞ disturbance attenuation level γ.

3. Main Results

In this section, we assume that the system matrices A, B, B_1 and controller gain matrix G are known, and then we study the condition under which the closed-loop 2D system is mean-square asymptotically stable with a guaranteed H_∞ performance. Then, a feasible controller gain matrix can be given based on the condition.

From system (1), (3), and (4), we can obtain the following closed-loop system:

$$\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = \left(A + \alpha_{i,j} BG\right) \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + B_1 w(i,j). \quad (8)$$

Define $\tilde{\alpha}_{i,j} = \alpha_{i,j} - \alpha$; it is obvious that

$$E\left\{\tilde{\alpha}_{i,j}\right\} = 0, \qquad E\left\{\tilde{\alpha}_{i,j}\tilde{\alpha}_{i,j}\right\} = \alpha(1-\alpha). \quad (9)$$

Then, the 2D closed-loop system can be rewritten as

$$\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = \left(A + \alpha BG + \tilde{\alpha}_{i,j} BG\right) \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + B_1 w(i,j). \quad (10)$$

Theorem 4. *The 2D closed-loop system* (10) *is mean-square asymptotically stable with a given H_∞ disturbance attenuation level γ, if there exists a positive define symmetric matrix $P = \mathrm{diag}\{P_h, P_v\} > 0$, satisfying*

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} \\ * & \Xi_{22} \end{bmatrix} < 0, \quad (11)$$

where

$$\begin{gathered} \Xi_{11} = (A+\alpha BG)^T P (A+\alpha BG) + \theta^2 (BG)^T PBG + I - P, \\ \Xi_{12} = (A+\alpha BG)^T PB_1, \qquad \Xi_{22} = B_1^T PB_1 - \gamma^2 I, \\ \theta^2 = \alpha(1-\alpha). \end{gathered} \quad (12)$$

Proof. We first prove the mean-square asymptotically stability of 2D system (10) with zero disturbance $w(i,j) = 0$. In this case, the system becomes

$$\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = \left(A + \alpha BG + \tilde{\alpha}_{i,j} BG\right) \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}. \quad (13)$$

Define

$$\begin{gathered} W_1 = E\left\{ \begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix}^T P \begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} \mid \tilde{x} \right\}, \\ W_2 = \tilde{x}^T P \tilde{x}, \end{gathered} \quad (14)$$

where $\tilde{x} = \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}$.

Consider the following index

$$J \triangleq W_1 - W_2. \quad (15)$$

Substituting (13) into the above index, we can obtain

$$\begin{aligned} J &= E\left\{ \begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix}^T P \begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} \mid \tilde{x} \right\} - \tilde{x}^T P \tilde{x} \\ &= E\left\{ \begin{matrix} \left[\left(A + \alpha BG + \tilde{\alpha}_{i,j} BG\right)\tilde{x}\right]^T P \\ \left[\left(A + \alpha BG + \tilde{\alpha}_{i,j} BG\right)\tilde{x}\right] \mid \tilde{x} \end{matrix} \right\} - \tilde{x}^T P \tilde{x} \\ &= \tilde{x}^T \left[(A+\alpha BG)^T P (A+\alpha BG) + \theta^2 (BG)^T PBG - P\right] \tilde{x} \\ &= \tilde{x}^T \Psi \tilde{x}, \end{aligned} \quad (16)$$

where

$$\Psi = (A+\alpha BG)^T P (A+\alpha BG) + \theta^2 (BG)^T PBG - P. \quad (17)$$

From (11), it is easy to see that $\Psi < 0$. Hence, for all $\tilde{x} \neq 0$, we have

$$\frac{W_1 - W_2}{W_2} = -\frac{\tilde{x}^T(-\Psi)\tilde{x}}{\tilde{x}^T P \tilde{x}} \le -\frac{\lambda_{\min}(-\Psi)}{\lambda_{\max}(P)} = \delta - 1, \quad (18)$$

where $\delta = 1 - (\lambda_{\min}(-\Psi)/\lambda_{\max}(P))$.

Notice that $(\lambda_{\min}(-\Psi)/\lambda_{\max}(P)) > 0$; we have $\delta < 1$. From (18), it is also easy to see that

$$\delta \ge \frac{W_1}{W_2} > 0. \quad (19)$$

Hence, $\delta \in (0,1)$ and it is independent of $\tilde{x}$. Thus, we obtain $W_1 \le \delta W_2$, and taking expectation of both sides yields

$$\begin{aligned} &E\left\{ \begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix}^T P \begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} \right\} \\ &\quad \le \delta E\left\{ \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}^T P \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} \right\}. \end{aligned} \quad (20)$$

That is,

$$\begin{aligned} &E\left\{x^v(k+1,0)^T P_v x^v(k+1,0)\right\} \\ &\quad = E\left\{x^v(k+1,0)^T P_v x^v(k+1,0)\right\} \\ &E\left\{x^h(k+1,0)^T P_h x^h(k+1,0) + x^v(k,1)^T P_v x^v(k,1)\right\} \\ &\quad \le \delta E\left\{x^h(k,0)^T P_h x^h(k,0) + x^v(k,0)^T P_v x^v(k,0)\right\} \\ &E\left\{x^h(k,1)^T P_h x^h(k,1) + x^v(k-1,2)^T P_v x^v(k-1,2)\right\} \\ &\quad \le \delta E\left\{x^h(k-1,1)^T P_h x^h(k-1,1) \right. \\ &\qquad \left. + x^v(k-1,1)^T P_v x^v(k-1,1)\right\} \end{aligned}$$

$$
\begin{aligned}
&E\left\{x^h(k-1,2)^T P_h x^h(k-1,2)\right.\\
&\qquad \left.+x^v(k-2,3)^T P_v x^v(k-2,3)\right\}\\
&\qquad \leq \delta E\left\{x^h(k-2,2)^T P_h x^h(k-2,2)\right.\\
&\qquad\qquad \left.+x^v(k-2,2)^T P_v x^v(k-2,2)\right\}\\
&\quad \vdots\\
&E\left\{x^h(1,k)^T P_h x^h(1,k)+x^v(0,k+1)^T P_v x^v(0,k+1)\right\}\\
&\qquad \leq \delta E\left\{x^h(0,k)^T P_h x^h(0,k)+x^v(0,k)^T P_v x^v(0,k)\right\}\\
&E\left\{x^h(0,k+1)^T P_h x^h(0,k+1)\right\}\\
&\qquad =\left\{x^h(0,k+1)^T P_h x^h(0,k+1)\right\}.
\end{aligned}
\tag{21}
$$

Adding both sides of the inequality system (21) yields

$$
\begin{aligned}
&E\left\{\sum_{j=0}^{k+1}\left[x^h(k+1-j,j)^T P_h x^h(k+1-j,j)\right.\right.\\
&\qquad\qquad \left.\left.+x^v(k+1-j,j)^T P_v x^v(k+1-j,j)\right]\right\}\\
&\quad \leq \delta E\left\{\sum_{j=0}^{k}\left[x^h(k-j,j)^T P_h x^h(k-j,j)\right.\right.\\
&\qquad\qquad \left.\left.+x^v(k-j,j)^T P_v x^v(k-j,j)\right]\right\}\\
&\qquad +E\left\{x^v(k+1,0)^T P_v x^v(k+1,0)\right\}\\
&\qquad +E\left\{x^h(k+1,0)^T P_h x^h(k+1,0)\right\}.
\end{aligned}
\tag{22}
$$

Using this relationship iteratively, we can obtain

$$
\begin{aligned}
&E\left\{\sum_{j=0}^{k+1}\left[x^h(k+1-j,j)^T P_h x^h(k+1-j,j)\right.\right.\\
&\qquad\qquad \left.\left.+x^v(k+1-j,j)^T P_v x^v(k+1-j,j)\right]\right\}\\
&\quad \leq \delta^{k+1} E\left\{x^h(0,0)^T P_h x^h(0,0)+x^v(0,0)^T P_v x^v(0,0)\right\}\\
&\qquad +E\left\{\sum_{j=0}^{k}\delta^j\left[x^v(k+1-j,0)^T P_v x^v(k+1-j,0)\right.\right.\\
&\qquad\qquad \left.\left.+x^h(0,k+1-j)^T P_h x^h(0,k+1-j)\right]\right\}\\
&\quad =E\left\{\sum_{j=0}^{k+1}\delta^j\left[x^v(k+1-j,0)^T P_v x^v(k+1-j,0)\right.\right.\\
&\qquad\qquad \left.\left.+x^h(0,k+1-j)^T P_h x^h(0,k+1-j)\right]\right\},
\end{aligned}
\tag{23}
$$

which implies

$$
\begin{aligned}
&E\left\{\sum_{j=0}^{k+1}\|x(k+1-j,j)\|^2\right\}\\
&\quad \leq \kappa\sum_{j=0}^{k+1}\delta^j E\left\{\|x^v(k+1-j,0)\|^2\right.\\
&\qquad\qquad \left.+\left\|x^h(0,k+1-j)\right\|^2\right\},
\end{aligned}
\tag{24}
$$

where

$$
\kappa := \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}.
\tag{25}
$$

Now, denote $\chi_k := \sum_{j=0}^{k}\|x(k-j,j)\|^2$; then, upon the inequality (24) we have

$$
\begin{aligned}
&E\{\chi_0\} \leq \kappa E\left\{\|x^v(0,0)\|^2+\left\|x^h(0,0)\right\|^2\right\}\\
&E\{\chi_1\} \leq \kappa\left[\delta E\left\{\|x^v(0,0)\|^2+\left\|x^h(0,0)\right\|^2\right\}\right.\\
&\qquad\qquad \left.+E\left\{\|x^v(1,0)\|^2+\left\|x^h(0,1)\right\|^2\right\}\right]\\
&E\{\chi_2\} \leq \kappa\left[\delta^2 E\left\{\|x^v(0,0)\|^2+\left\|x^h(0,0)\right\|^2\right\}\right.\\
&\qquad\qquad +\delta E\left\{\|x^v(1,0)\|^2+\left\|x^h(0,1)\right\|^2\right\}\\
&\qquad\qquad \left.+E\left\{\|x^v(2,0)\|^2+\left\|x^h(0,2)\right\|^2\right\}\right]\\
&\quad \vdots\\
&E\{\chi_N\} \leq \kappa\left[\delta^N E\left\{\|x^v(0,0)\|^2+\left\|x^h(0,0)\right\|^2\right\}\right.\\
&\qquad\qquad +\delta^{N-1} E\left\{\|x^v(1,0)\|^2+\left\|x^h(0,1)\right\|^2\right\}\\
&\qquad\qquad \left.+\cdots+E\left\{\|x^v(N,0)\|^2+\left\|x^h(0,N)\right\|^2\right\}\right].
\end{aligned}
\tag{26}
$$

Adding both sides of the inequalities yields

$$
\begin{aligned}
&\sum_{k=0}^{N} E\{\chi_k\}\\
&\quad \leq \kappa\left(1+\delta+\cdots+\delta^N\right) E\left\{\|x^v(0,0)\|^2+\left\|x^h(0,0)\right\|^2\right\}\\
&\qquad +\kappa\left(1+\delta+\cdots+\delta^{N-1}\right) E\left\{\|x^v(1,0)\|^2+\left\|x^h(0,1)\right\|^2\right\}\\
&\qquad +\cdots+\kappa E\left\{\|x^v(N,0)\|^2+\left\|x^h(0,N)\right\|^2\right\}
\end{aligned}
$$

$$\begin{aligned}
&\leq \kappa\left(1+\delta+\cdots+\delta^{N}\right) E\left\{\left\|x^{v}(0,0)\right\|^{2}+\left\|x^{h}(0,0)\right\|^{2}\right\} \\
&\quad+\kappa\left(1+\delta+\cdots+\delta^{N}\right) E\left\{\left\|x^{v}(1,0)\right\|^{2}+\left\|x^{h}(0,1)\right\|^{2}\right\} \\
&\quad+\cdots+\kappa\left(1+\delta+\cdots+\delta^{N}\right) E\left\{\left\|x^{v}(N,0)\right\|^{2}\right. \\
&\qquad\qquad\left.+\left\|x^{h}(0,N)\right\|^{2}\right\} \\
&=\kappa \times \frac{1-\delta^{N}}{1-\delta} E\left\{\sum_{i=0}^{N}\left[\left\|x^{v}(k,0)\right\|^{2}+\left\|x^{h}(0,k)\right\|^{2}\right]\right\}.
\end{aligned} \tag{27}$$

Then, under Assumption 1, the right side of this inequality is bounded, which means that $\lim_{k\to\infty} E\{\chi_k\} = 0$; that is, $\lim_{i+j\to\infty} E\{\|x(i,j)\|^2\} = 0$. Then the 2D stochastic system (10) is mean-square asymptotically stable.

Now, the H_∞ performance for the 2D stochastic system (10) will be established. To this end, assume zero initial boundary conditions; that is, $x^h(0,i)=0, x^v(i,0)=0$ for all i. In this case, the index J becomes

$$\begin{aligned}
J &= E\left\{\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1)\end{bmatrix}^T P \begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1)\end{bmatrix} \mid \tilde{x}\right\} - \tilde{x}^T P \tilde{x} \\
&= E\left\{\begin{array}{c}\left[\left(A+\alpha BG+\tilde{\alpha}_{i,j}BG\right)\tilde{x}+B_1 w(i,j)\right]^T P \\ \left[\left(A+\alpha BG+\tilde{\alpha}_{i,j}BG\right)\tilde{x}+B_1 w(i,j)\right] \mid \tilde{x}\end{array}\right\} - \tilde{x}^T P \tilde{x}.
\end{aligned} \tag{28}$$

Another index is introduced as

$$\begin{aligned}
\Pi &\triangleq J+\tilde{x}^T\tilde{x}-\gamma^2 w^T w \\
&= E\left\{\begin{array}{c}\left[\left(A+\alpha BG+\tilde{\alpha}_{i,j}BG\right)\tilde{x}+B_1 w(i,j)\right]^T P \\ \left[\left(A+\alpha BG+\tilde{\alpha}_{i,j}BG\right)\tilde{x}+B_1 w(i,j)\right]^T \mid \tilde{x}\end{array}\right\} \\
&\quad - \tilde{x}^T P\tilde{x}+\tilde{x}^T\tilde{x}-\gamma^2 w^T w \\
&= \zeta^T \Xi \zeta,
\end{aligned} \tag{29}$$

where $\zeta = \left[\tilde{x}^T \quad \tilde{w}^T\right]^T$.

From condition (11), we have $\Pi < 0$ for any $\zeta \neq 0$; that is,

$$\begin{aligned}
&E\left\{\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1)\end{bmatrix}^T P \begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1)\end{bmatrix} \mid \tilde{x}\right\} \\
&\quad < \tilde{x}^T P\tilde{x} - \tilde{x}^T\tilde{x}+\gamma^2\tilde{w}^T\tilde{w}.
\end{aligned} \tag{30}$$

Taking the expectation of both sides yields

$$\begin{aligned}
&E\left\{\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1)\end{bmatrix}^T P \begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1)\end{bmatrix} \mid \tilde{x}\right\} \\
&\quad < E\left\{\tilde{x}^T P\tilde{x} - \tilde{x}^T\tilde{x}\right\}+\gamma^2\tilde{w}^T\tilde{w}.
\end{aligned} \tag{31}$$

Due to the relationship (31), it can be established that

$$\begin{aligned}
&E\left\{x^v(k+1,0)^T P_2 x^v(k+1,0)\right\} \\
&\quad = E\left\{x^v(k+1,0)^T P_2 x^v(k+1,0)\right\} \\
&E\left\{x^h(k+1,0)^T P_1 x^h(k+1,0)+x^v(k,1)^T P_2 x^v(k,1)\right\} \\
&\quad \leq E\left\{x^h(k,0)^T P_1 x^h(k,0)+x^v(k,0)^T P_2 x^v(k,0)\right\} \\
&\qquad - E\left\{\tilde{x}(k,0)^T\tilde{x}(k,0)\right\}+\gamma^2 w(k,0)^T w(k,0) \\
&E\left\{x^h(k,1)^T P_1 x^h(k,1)+x^v(k-1,2)^T P_2 x^v(k-1,2)\right\} \\
&\quad \leq E\left\{x^h(k-1,1)^T P_1 x^h(k-1,1)\right. \\
&\qquad\quad \left.+x^v(k-1,1)^T P_2 x^v(k-1,1)\right\} \\
&\qquad - E\left\{\tilde{x}(k-1,1)^T\tilde{x}(k-1,1)\right\} \\
&\qquad + \gamma^2 w(k-1,1)^T w(k-1,1) \\
&E\left\{x^h(k-1,2)^T P_1 x^h(k-1,2)\right. \\
&\quad \left.+x^v(k-2,3)^T P_2 x^v(k-2,3)\right\} \\
&\quad \leq E\left\{x^h(k-2,2)^T P_1 x^h(k-2,2)\right. \\
&\qquad\quad \left.+x^v(k-2,2)^T P_2 x^v(k-2,2)\right\} \\
&\qquad - E\left\{\tilde{x}(k-2,2)^T\tilde{x}(k-2,2)\right\} \\
&\qquad + \gamma^2 w(k-2,2)^T w(k-2,2) \\
&\quad \vdots \\
&E\left\{x^h(1,k)^T P_1 x^h(1,k)+x^v(0,k+1)^T P_2 x^v(0,k+1)\right\} \\
&\quad \leq E\left\{x^h(0,k)^T P_1 x^h(0,k)+x^v(0,k)^T P_2 x^v(0,k)\right\} \\
&\qquad - E\left\{\tilde{x}(0,k)^T\tilde{x}(0,k)\right\}+\gamma^2 w(0,k)^T w(0,k) \\
&E\left\{x^h(0,k+1)^T P_1 x^h(0,k+1)\right\} \\
&\quad = \left\{x^h(0,k+1)^T P_1 x^h(0,k+1)\right\}.
\end{aligned} \tag{32}$$

Adding both sides of the inequality system, we have

$$\begin{aligned}
&E\left\{\sum_{j=0}^{k+1}\left[x^h(k+1-j,j)^T P_1 x^h(k+1-j,j)\right.\right. \\
&\qquad\quad \left.\left.+x^v(k+1-j,j)^T P_2 x^v(k+1-j,j)\right]\right\} \\
&\quad < \left\{\sum_{j=0}^{k}\left[x^h(k-j,j)^T P_1 x^h(k-j,j)\right.\right. \\
&\qquad\quad \left.\left.+x^v(k-j,j)^T P_2 x^v(k-j,j)\right]\right\}
\end{aligned}$$

$$+ E\left\{x^v(k+1,0)^T P_2 x^v(k+1,0)\right\}$$
$$+ E\left\{x^h(0,k+1)^T P_1 x^h(0,k+1)^T\right\}$$
$$- E\left\{\sum_{j=0}^{k}\left[\tilde{x}(k-j,j)^T \tilde{x}(k-j,j)\right]\right\}$$
$$+ \gamma^2 \sum_{j=0}^{i} w(k-j,j)^T w(k-j,j). \tag{33}$$

Summing up both sides of this inequality from $k = 0$ to $k = N$, we have

$$E\left\{\sum_{k=0}^{N}\sum_{j=0}^{k}\tilde{x}(k-j,j)^T\tilde{x}(k-j,j)\right\}$$
$$< \gamma^2 \sum_{k=0}^{N}\sum_{j=0}^{k} w(k-j,j)^T w(k-j,j)$$
$$+ \sum_{k=0}^{N}\left(E\left\{x^v(k+1,0)^T P_2 x^v(k+1,0)\right\}\right.$$
$$\left. + E\left\{x^h(0,k+1)^T P_1 x^h(0,k+1)^T\right\}\right)$$
$$- E\left\{\sum_{j=0}^{N}\left[x^h(N+1-j,j)^T P_1 x^h(N+1-j,j)\right.\right.$$
$$\left.\left. + x^v(N+1-j,j)^T P_2 x^v(N+1-j,j)\right]\right\}$$
$$+ E\left\{x^v(0,0)^T P_2 x^v(0,0)\right\} + E\left\{x^h(0,0)^T P_1 x^h(0,0)^T\right\}; \tag{34}$$

that is,

$$E\left\{\sum_{k=0}^{\infty}\sum_{j=0}^{k}\tilde{x}(k-j,j)^T\tilde{x}(k-j,j)\right\}$$
$$< \gamma^2 \sum_{k=0}^{\infty}\sum_{j=0}^{k} w(k-j,j)^T w(k-j,j)$$
$$+ \sum_{k=0}^{\infty}\left(E\left\{x^v(k,0)^T P_2 x^v(k,0)\right\}\right.$$
$$\left. + E\left\{x^h(0,k)^T P_1 x^h(0,k)^T\right\}\right). \tag{35}$$

Considering the zero initial boundary conditions, (35) means

$$\|\tilde{x}\|_E < \gamma^2 \|w\|_2. \tag{36}$$

This completes the proof. □

Remark 5. Theorem 4 provides a sufficient condition of the mean-square asymptotic stability and H_∞ disturbance attenuation level γ for 2D systems with missing measurements. If the communication links existing between the plant and the controller are perfect, that is, there is no packet dropout during their transmission, then $\alpha = 1$ and $\theta = 0$. In this case, the condition in Theorem 4 becomes the condition obtained in [6] for 2D deterministic system. From this point of view, Theorem 4 can be seen as an extension of [6] to 2D systems with missing measurement.

Theorem 4 gives a mean-square asymptotic stability condition with H_∞ disturbance attenuation level γ for system (10) where the controller gain matrix G is known. However, our eventual purpose is to determine a suitable G by system matrices A, B, B_1 and parameter α.

The following well-known lemma is needed in the proof of our main result.

Lemma 6 (Schur complement). *Assume W, L, V are given matrices with appropriate dimensions, where W and V are positive definite symmetric matrices. Then*

$$L^T V L - W < 0, \tag{37}$$

if and only if

$$\begin{bmatrix} -W & L^T \\ L & -V^{-1} \end{bmatrix} < 0, \tag{38}$$

or

$$\begin{bmatrix} -V^{-1} & L \\ L^T & -W \end{bmatrix} < 0. \tag{39}$$

Based on the above lemma, we can give our main result.

Theorem 7. *For the 2D closed-loop system* (10)*, if there exist a positive define symmetric matrix Q and a matrix M satisfying*

$$\Omega = \begin{bmatrix} -Q & 0 & QA^T + \alpha M^T B^T & \theta M^T B^T & Q \\ & -\gamma^2 I & B_1^T & 0 & 0 \\ & & -Q & 0 & 0 \\ & * & & -Q & 0 \\ & & & & -I \end{bmatrix} < 0, \tag{40}$$

then the 2D closed-loop system (10) *is mean-square asymptotically stability and has an H_∞ disturbance attenuation level γ. In this case, a suitable state feedback control law can be given as $G = MQ^{-1}$.*

Proof. The condition in Theorem 4 can be rewritten as

$$\Theta^T \begin{bmatrix} P & & \\ & P & \\ & & I \end{bmatrix} \Theta + \begin{bmatrix} -P & \\ & -\gamma^2 I \end{bmatrix} < 0, \tag{41}$$

where

$$\Theta^T = \begin{bmatrix} (A+\alpha BG)^T & \theta(BG)^T & I \\ B_1^T & 0 & 0 \end{bmatrix}. \tag{42}$$

By applying Lemma 6, condition (41) is equivalent to the following LMI condition:

$$\begin{bmatrix} -P & 0 & (A+\alpha BG)^T & \theta(BG)^T & I \\ & -\gamma^2 I & B_1^T & 0 & 0 \\ & & -P^{-1} & 0 & 0 \\ & * & & -P^{-1} & 0 \\ & & & & -I \end{bmatrix} < 0. \tag{43}$$

Define $Q = P^{-1}$, and pro- and postmultiplying $\mathrm{diag}(Q, I, I, I, I)$ for the above condition give

$$\begin{bmatrix} -Q & 0 & QA^T + \alpha(GQ)^T B^T & \theta(GQ)^T B^T & Q \\ & -\gamma^2 I & B_1^T & 0 & 0 \\ & & -Q & 0 & 0 \\ & * & & -Q & 0 \\ & & & & -I \end{bmatrix} < 0. \tag{44}$$

Set $GQ = M$ to obtain the LMI of (40) and the proof is complete. □

Remark 8. Theorem 7 provides an LMI condition for the mean-square asymptotic stability and H_∞ disturbance attenuation level γ of 2D stochastic system, which can be solved by LMI Toolbox. Then by (40), we also can give a suitable state feedback control law. It is noted that LMI (40) only contains few elements; hence, the computation is not big. However, when the system matrices get bigger, the solution of LMI (40) may be time-consuming due to complicated computation. In practical system, the system dimension is not very big (often less than 5). Hence, the computation complexity is acceptable.

Remark 9. For a fixed γ, the feasibility of (40) is a suboptimal H_∞ controller. When $\gamma$is not fixed, the minimization of γ that satisfies (40) can be searched. Hence, an optimal H_∞ controller can be obtained by solving the following optimization problem:

$$\begin{aligned} &\min_{Q,M} \quad \gamma^2 \\ &\text{s.t} \quad (40). \end{aligned} \tag{45}$$

4. Robust H_∞ Control for Uncertain 2D Systems

In this section, we extend the above design to the case of robust H_∞ control. Consider the following 2D system with uncertain parameter perturbations

$$\begin{aligned} \begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} &= (A+\Delta A)\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + (B+\Delta B)\,u(i,j) \\ &\quad + (B_1 + \Delta B_1)\,w(i,j), \end{aligned} \tag{46}$$

where $\Delta A, \Delta B, \Delta B_1$ denote admissible uncertain perturbations of matrices A, B, and B_1, which can be represented as

$$\Delta A = E\Sigma F_1, \qquad \Delta B = E\Sigma F_2, \qquad \Delta B_1 = E\Sigma F_3, \tag{47}$$

where E, F_1, F_2, F_3 are known real constant matrices characterizing the structures of uncertain perturbations and Σ is an uncertain perturbation of the system that satisfies $\Sigma^T\Sigma \le I$.

Lemma 10. *Assume X, Y are matrices or vectors with appropriate dimensions. For any scalar $\varepsilon > 0$ and all matrices Δ with appropriate dimensions satisfying $\Delta\Delta^T \le I$, the following inequality holds:*

$$X\Delta Y + Y^T\Delta^T X^T \le \varepsilon XX^T + \varepsilon^{-1}Y^T Y. \tag{48}$$

The main result of this section is given in the following theorem.

Theorem 11. *For the uncertain 2D system (46) used the state feedback control law (3) with missing measurement, if there exist a positive define symmetric matrix Q, a matrix M , and scalars $\varepsilon > 0$ satisfying*

$$\begin{bmatrix} -Q & 0 & \Omega_1 & \theta M^T B^T & Q & \Omega_2 & \theta M^T {F_2}^T \\ & -\gamma^2 I & B_1^T & 0 & 0 & F_3^T & 0 \\ & & \varepsilon EE^T - Q & 0 & 0 & 0 & 0 \\ & & & \varepsilon EE^T - Q & 0 & 0 & 0 \\ & & & & -I & 0 & 0 \\ & * & & & & -\varepsilon I & 0 \\ & & & & & & -\varepsilon I \end{bmatrix} < 0, \tag{49}$$

where

$$\Omega_1 = QA^T + \alpha M^T B^T, \qquad \Omega_2 = Q{F_1}^T + \alpha M^T {F_2}^T, \tag{50}$$

then the uncertain 2D system is mean-square asymptotically stability and has an H_∞ disturbance attenuation level γ. In this case, a suitable state feedback control law can be given as $G = MQ^{-1}$.

Proof. By applying Theorem 7, the uncertain 2D system with state feedback control law (3) is mean-square asymptotically stable and has an H_∞ disturbance attenuation level γ, and there exist a positive define symmetric matrix Q, a matrix M satisfying

$$\Omega = \begin{bmatrix} -Q & 0 & Q(A+\Delta A)^T + \alpha M^T (B+\Delta B)^T & \theta M^T (B+\Delta B)^T & Q \\ & -\gamma^2 I & (B_1+\Delta B_1)^T & 0 & 0 \\ & & -Q & 0 & 0 \\ & * & & -Q & 0 \\ & & & & -I \end{bmatrix} < 0. \quad (51)$$

That is,

$$\begin{bmatrix} -Q & 0 & \Omega_1 & \theta M^T B^T & Q \\ & -\gamma^2 I & B_1^T & 0 & 0 \\ & & -Q & 0 & 0 \\ & * & & -Q & 0 \\ & & & & -I \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ E \\ 0 \\ 0 \end{bmatrix} \Sigma \begin{bmatrix} \Omega_2 \\ F_3^T \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} \Omega_2 \\ F_3^T \\ 0 \\ 0 \\ 0 \end{bmatrix} \Sigma^T \begin{bmatrix} 0 \\ 0 \\ E \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} 0 \\ 0 \\ 0 \\ E \\ 0 \end{bmatrix} \Sigma \begin{bmatrix} \theta M^T F_2^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} \theta M^T F_2^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Sigma^T \begin{bmatrix} 0 \\ 0 \\ 0 \\ E \\ 0 \end{bmatrix}^T < 0. \quad (52)$$

Therefore, using Lemma 10, there exists a scalar $\varepsilon > 0$ such that

$$\begin{bmatrix} -Q & 0 & \Omega_1 & \theta M^T B^T & Q \\ & -\gamma^2 I & B_1^T & 0 & 0 \\ & & -Q & 0 & 0 \\ & * & & -Q & 0 \\ & & & & -I \end{bmatrix} + \varepsilon \begin{bmatrix} 0 \\ 0 \\ E \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ E \\ 0 \\ 0 \end{bmatrix}^T + \varepsilon^{-1} \begin{bmatrix} \Omega_2 \\ F_3^T \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \begin{bmatrix} \Omega_2 \\ F_3^T \\ 0 \\ 0 \\ 0 \end{bmatrix} + \varepsilon \begin{bmatrix} 0 \\ 0 \\ 0 \\ E \\ 0 \end{bmatrix}^T \begin{bmatrix} 0 \\ 0 \\ 0 \\ E \\ 0 \end{bmatrix} + \varepsilon^{-1} \begin{bmatrix} \theta M^T F_2^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \begin{bmatrix} \theta M^T F_2^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} < 0. \quad (53)$$

Note that the above condition is equivalent to

$$\begin{bmatrix} -Q & 0 & \Omega_1 & \theta M^T B^T & Q \\ & -\gamma^2 I & B_1^T & 0 & 0 \\ & & \varepsilon E E^T - Q & 0 & 0 \\ & * & & \varepsilon E E^T - Q & 0 \\ & & & & -I \end{bmatrix} + \varepsilon^{-1} \begin{bmatrix} \Omega_2 & \theta M^T {F_2}^T \\ F_3^T & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} (\Omega_2)^T & F_3 & 0 & 0 & 0 \\ \theta F_2 M & 0 & 0 & 0 & 0 \end{bmatrix} < 0. \quad (54)$$

By the Schur complement, LMI (54) implies LMI in Theorem 11 holds.

This completes the proof. □

In addition, by solving the following optimization problem

$$\min_{Q,M,\varepsilon} \quad \gamma^2 \qquad \text{s.t} \quad (49). \quad (55)$$

we can obtain a robust optimal H_∞ controller for the uncertain 2D stochastic systems (46).

Remark 12. This paper considers the problem of H_∞ stabilization for a class of 2D systems with missing measurements. Here, we describe the missing measurements as a Bernoulli random binary distribution, which renders the 2D systems to be stochastic ones. Under this framework, we give the definition of stochastic mean-square stability and H_∞ performance, and then a state feedback control design approach is addressed. The proposed design approach is systematic for 2D stochastic system. The results in this paper can be extended to solve other problems such as network-induced delay and H_∞ filter design.

5. Illustrative Examples

In this section, two numerical examples are used to illustrate the effectiveness of the proposed results.

Example 1. Let us consider 2D system (1) with the following parameters:

$$A = \begin{bmatrix} 0.5 & 0.3 \\ 0.1 & 0.7 \end{bmatrix}, \qquad B = \begin{bmatrix} 0.1 & 0.05 \\ 0.2 & 0 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}. \tag{56}$$

It is assumed that measurements transmitted between the plant and the controller are imperfect; that is, the state signal may be lost during their transmission. Suppose $\alpha = 0.8$; that is, in the communication link, the probability of the data packet missing is 20%. By applying Theorem 7 and solving the optimization problem (45), the minimum H_∞ disturbance attenuation level is $\gamma_{\text{opt}} = 0.32$. Meanwhile, we can obtain

$$Q = \begin{bmatrix} 0.8663 & -0.0405 \\ -0.0405 & 0.8554 \end{bmatrix}, \qquad M = \begin{bmatrix} -0.4429 & -3.7218 \\ -3.7218 & 2.1220 \end{bmatrix}. \tag{57}$$

Hence, a feasible state feedback control law can be selected as

$$G = MQ^{-1} = \begin{bmatrix} -0.7162 & -4.3849 \\ -4.1895 & -2.2824 \end{bmatrix}. \tag{58}$$

Assume the disturbance is

$$w(i,j) = \begin{bmatrix} \dfrac{1}{10ij} \\ \dfrac{1}{10ij} \end{bmatrix}. \tag{59}$$

Simulation results are shown in Figures 1 and 2, where the state response of $x^h(i,j)$ is plotted in Figure 1 and $x^v(i,j)$ is plotted in Figure 2. It can be seen from Figures 1 and 2 that the closed-loop 2D system is asymptotically stable. Hence, even though the 2D system is affected by external disturbances and has significant data dropout in the output measurements, the proposed design approach can guarantee the stability and disturbance attenuation ability for the 2D system.

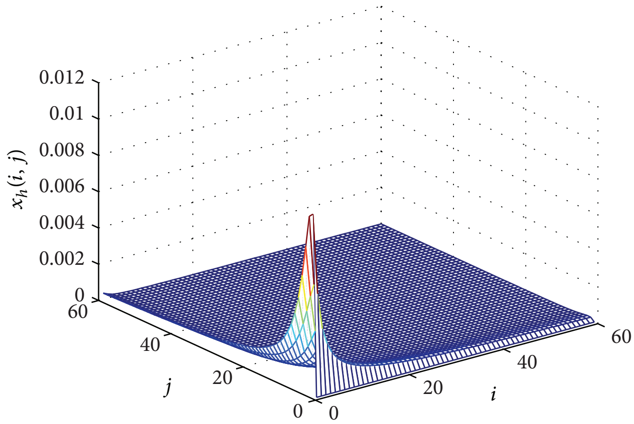

Figure 1: State response of $x^h(i,j)$ for Example 1.

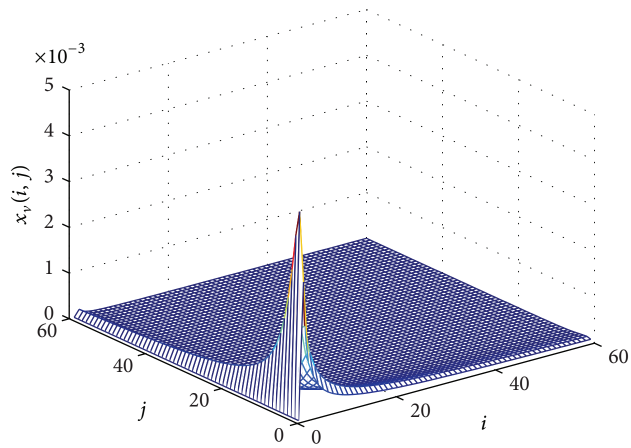

Figure 2: State response of $x^v(i,j)$ for Example 1.

Example 2. Let us consider uncertain 2D system (46) with the following parameters:

$$A = \begin{bmatrix} 0.8 & 0.021 \\ 0.01 & 0.9 \end{bmatrix}, \qquad B = \begin{bmatrix} 0.03 & 0 \\ 0.1 & 0.1 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, \qquad E = \begin{bmatrix} 0.01 & 0.01 \\ 0 & 0.02 \end{bmatrix},$$
$$F_1 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.02 \end{bmatrix}, \qquad F_2 = \begin{bmatrix} 0.01 & 0.01 \\ 0.03 & 0 \end{bmatrix}, \qquad F_3 = \begin{bmatrix} 0.01 & 0.01 \\ 0.01 & 0.02 \end{bmatrix}. \tag{60}$$

It is also assumed that the probability of the data packet missing is 0.2; that is, $\alpha = 0.8$. In this case, by applying Theorem 11 and solving the optimization problem (55), the minimum H_∞ disturbance attenuation level is $\gamma_{\text{opt}} = 0.35$. Meanwhile, we can obtain

$$Q = \begin{bmatrix} 0.5125 & -0.2522 \\ -0.2522 & 0.6073 \end{bmatrix}, \qquad M = \begin{bmatrix} -10.4172 & 10.0783 \\ 10.0783 & -16.9824 \end{bmatrix}, \qquad \varepsilon = 14.4592. \tag{61}$$

Hence, a feasible state feedback control law can be selected as

$$G = MQ^{-1} = \begin{bmatrix} -15.2830 & 10.2485 \\ 7.4205 & -24.8822 \end{bmatrix}. \tag{62}$$

The disturbance $w(i,j)$ is also given as Example 1. Simulation results are shown in Figures 3 and 4, where the state response of $x^h(i,j)$ is plotted in Figure 3 and $x^v(i,j)$ is plotted in Figure 4. It is observed that the closed-loop uncertain 2D system is also asymptotically stable. Hence, the proposed design approach is also effective for uncertain 2D system.

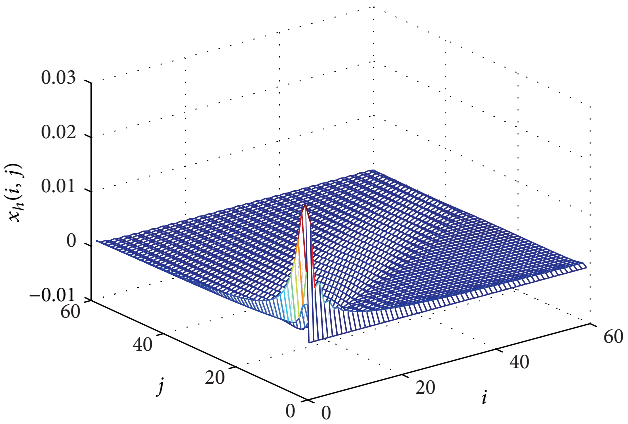

Figure 3: State response of $x^h(i, j)$ for Example 2.

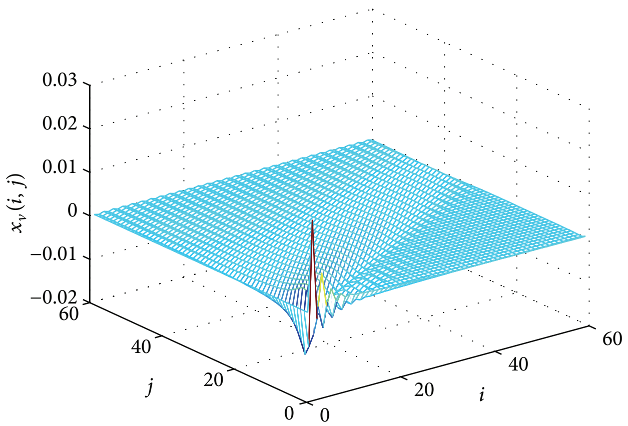

Figure 4: State response of $x^v(i, j)$ for Example 2.

6. Conclusions

In this paper, we have investigated the problem of H_∞ stabilization for a class of 2D systems described with missing measurements. A sufficient condition has been developed in terms of LMIs, which guarantees mean-square asymptotic stability and H_∞ disturbance attenuation level for closed-loop 2D system. The result is also extended to more general cases where the system matrices contain uncertain parameters. Numerical examples have been provided to illustrate the effectiveness of proposed approach. In our future work, the control and filtering problems for networked-based 2D system with packet dropouts and network-induced delay will be discussed.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This work is supported by the Program of NSFC (no. 61203065, no. 61340015, and no. 61104119), the program of Natural Science of Henan Provincial Education Department (12A510013), the program of Open Laboratory Foundation of Control Engineering Key Discipline of Henan Provincial High Education (KG 2011-10), the program of Key Young Teacher of Henan Polytechnic University, and the Doctoral Fund Program of Henan Polytechnic University (B2012-003).

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