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Research Article

An Iterative Algorithm for the Split Equality and Multiple-Sets Split Equality Problem

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The multiple-sets split equality problem (MSSEP) requires finding a point $x \in \cap_{i=1}^{N} C_i$, $y \in \cap_{j=1}^{M} Q_j$ such that $Ax = By$, where N and M are positive integers, $\{C_1, C_2, \ldots, C_N\}$ and $\{Q_1, Q_2, \ldots, Q_M\}$ are closed convex subsets of Hilbert spaces H_1, H_2, respectively, and $A : H_1 \to H_3$, $B : H_2 \to H_3$ are two bounded linear operators. When $N = M = 1$, the MSSEP is called the split equality problem (SEP). If $B = I$, then the MSSEP and SEP reduce to the well-known multiple-sets split feasibility problem (MSSFP) and split feasibility problem (SFP), respectively. One of the purposes of this paper is to introduce an iterative algorithm to solve the SEP and MSSEP in the framework of infinite-dimensional Hilbert spaces under some more mild conditions for the iterative coefficient.

1. Introduction and Preliminaries

1.1. Introduction. Let $\{C_1, C_2, \ldots, C_N\}$ and $\{Q_1, Q_2, \ldots, Q_M\}$ be nonempty closed convex subsets of real Hilbert spaces H_1 and H_2, respectively, and let $A : H_1 \to H_2$ be a bounded linear operator. The multiple-sets split feasibility problem (MSSFP) is to find a point x satisfying the property:

$$x \in \bigcap_{i=1}^{N} C_i, \qquad Ax \in \bigcap_{j=1}^{M} Q_j, \tag{1}$$

if such point exists. If $N = M = 1$, then the MSSFP reduce to the well-known split feasibility problem (SFP).

The SFP and MSSFP were first introduced by Censor and Elfving [1] and Censor et al. [2], respectively, which attract many authors' attention due to its applications in signal processing [1] and intensity-modulated radiation therapy [2]. Various algorithms have been invented to solve it; see [1–8], e.t.

Recently, Moudafi [9] propose a new split equality problem (SEP): let H_1, H_2, and H_3 be real Hilbert spaces; let $C \subseteq H_1$, $Q \subseteq H_2$ be two nonempty closed convex sets; and let $A : H_1 \to H_3$, $B : H_2 \to H_3$ be two bounded linear operators. Find $x \in C$, $y \in Q$ satisfying

$$Ax = By. \tag{2}$$

When $B = I$, SEP reduces to the well-known SFP.

Naturally, we propose the following multiple-sets split equality problem (MSSEP) requiring to find a point $x \in \cap_{i=1}^{N} C_i$, $y \in \cap_{j=1}^{M} Q_j$ such that

$$Ax = By, \tag{3}$$

where N and M are positive integers; $\{C_1, C_2, \ldots, C_N\}$ and $\{Q_1, Q_2, \ldots, Q_M\}$ are closed convex subsets of Hilbert spaces H_1, H_2, respectively, and $A : H_1 \to H_3$, $B : H_2 \to H_3$ are two bounded linear operators.

In the paper [9], Moudafi gave an alternating CQ-algorithm and relaxed alternating CQ-algorithm iterative algorithm for solving the split equality problem.

We use Γ to denote the solution set of SEP, that is,

$$\Gamma = \{(x, y) \in H_1 \times H_2, Ax = By, x \in C, y \in Q\}, \tag{4}$$

and assume consistency of SEP so that Γ is closed, convex, and nonempty.

Let $S = C \times Q$ in $H = H_1 \times H_2$ and define $G : H \rightarrow H_3$ by $G = [A, -B]$; then $G^* G : H \rightarrow H$ has the matrix form

$$G^* G = \begin{bmatrix} A^* A & -A^* B \\ -B^* A & B^* B \end{bmatrix}. \quad (5)$$

The SEP problem can be reformulated as finding $w = (x, y) \in S$ with $Gw = 0$ or solving the following minimization problem:

$$\min_{w \in S} f(w) = \frac{1}{2} \|Gw\|^2. \quad (6)$$

In paper [10], we used the well-known Tychonov regularization that got some algorithms to converge strongly to the minimum-norm solution of the SEP.

Note that the convergence of the above algorithms depends on the exact requirements of the iterative coefficient. Therefore, the aim of this paper is to introduce an iterative algorithm to solve the SEP and MSSEP in the framework of infinite-dimensional Hilbert spaces under some more mild conditions for the iterative coefficient.

Throughout the rest of this paper, I denotes the identity operator on Hilbert space H and $\mathrm{Fix}(T)$ is the set of the fixed points of an operator T. An operator T on a Hilbert space H is *nonexpansive* if, for each x and y in H, $\|Tx - Ty\| \le \|x - y\|$. T is said to be *averaged*, if there exists $0 < \alpha < 1$ and a nonexpansive operator N such that $T = (1 - \alpha)I + \alpha N$.

Let P_S denote the projection from H onto a nonempty closed convex subset S of H; that is,

$$P_S(w) = \min_{x \in S} \|x - w\|. \quad (7)$$

It is well known that $P_S(w)$ is characterized by the following inequality:

$$\langle w - P_S(w), x - P_S(w) \rangle \le 0, \quad \forall x \in S, \quad (8)$$

and P_S is nonexpansive and averaged.

We now collect some elementary facts which will be used in the proofs of our main results.

Lemma 1 (see [11, 12]). *Let X be a Banach space, C a closed convex subset of X, and $T : C \rightarrow C$ a nonexpansive mapping with $\mathrm{Fix}(T) \neq \emptyset$. If $\{x_n\}$ is a sequence in C weakly converging to x and if $\{(I - T)x_n\}$ converges strongly to y, then $(I - T)x = y$.*

Lemma 2 (see [13]). *Let H be a Hilbert space and $\{w_n\}$ a sequence in H such that there exists a nonempty set $S \subseteq H$ satisfying the following.*

(i) *For every $w \in S$, $\lim_{n \to \infty} \|w_n - w\|$ exists.*

(ii) *Any weak-cluster point of the sequence $\{w_n\}$ belongs to S.*

Then, there exists $\widetilde{w} \in s$ such that $\{w_n\}$ weakly converges to $\widetilde{w}$.

Lemma 3 (see [4]). *Let A and B be averaged operators and suppose that $\mathrm{Fix}(A) \cap \mathrm{Fix}(B)$ is nonempty. Then $\mathrm{Fix}(A) \cap \mathrm{Fix}(B) = \mathrm{Fix}(AB) = \mathrm{Fix}(BA)$.*

The following lemma is vital in our main results.

Lemma 4. *Let $T = I - \gamma G^* G$, where $0 < \gamma < \lambda = 2/\rho(G^* G)$ with $\rho(G^* G)$ being the spectral radius of the self-adjoint operator $G^* G$ on H. Then we have the following:*

(1) $\|T\| \le 1$ *(i.e., T is nonexpansive) and averaged;*

(2) $\mathrm{Fix}(T) = \{(x, y) \in H, Ax = By\}$, $\mathrm{Fix}(P_S T) = \mathrm{Fix}(P_S) \cap \mathrm{Fix}(T) = \Gamma$;

(3) $w \in \mathrm{Fix}(P_S T)$ *if and only if w is a solution of the variational inequality $\langle G^* G w, v - w \rangle \ge 0$, for all $v \in S$.*

Proof. (1) It is easily proved that $\|T\| \le 1$; we only prove that $T = I - \gamma G^* G$ is averaged. Indeed, choose $0 < \beta < 1$, such that $\gamma/(1 - \beta) < 2/\rho(G^* G)$; then $T = I - \gamma G^* G = \beta I + (1 - \beta)V$, where $V = I - \gamma/(1 - \beta) G^* G$ is a nonexpansive mapping. That is to say, T is averaged.

(2) If $w \in \{(x, y) \in H, Ax = By\}$, it is obvious that $w \in \mathrm{Fix}(T)$. Conversely, assuming that $w \in \mathrm{Fix}(T)$, we have $w = w - \gamma G^* G w$. Hence $\gamma G^* G w = 0$; then $\|Gw\|^2 = \langle G^* G w, w \rangle = 0$; we get that $w \in \{(x, y) \in H, Ax = By\}$. It follows $\mathrm{Fix}(T) = \{(x, y) \in H, Ax = By\}$.

Now we prove $\mathrm{Fix}(P_S T) = \mathrm{Fix}(P_S) \cap \mathrm{Fix}(T) = \Gamma$. By $\mathrm{Fix}(T) = \{(x, y) \in H, Ax = By\}$, $\mathrm{Fix}(P_S) \cap \mathrm{Fix}(T) = \Gamma$ is obvious. On the other hand, since $\mathrm{Fix}(P_S) \cap \mathrm{Fix}(T) = \Gamma \neq \emptyset$, and both P_S and T are averaged, from Lemma 3, we have $\mathrm{Fix}(P_S T) = \mathrm{Fix}(P_S) \cap \mathrm{Fix}(T)$.

(3) Consider

$$\begin{aligned} &\langle G^* G w, v - w \rangle \ge 0, \quad \forall v \in S \\ &\quad \Longleftrightarrow \langle w - (w - \gamma G^* G w), v - w \rangle \\ &\quad \ge 0, \quad \forall v \in S \\ &\quad \Longleftrightarrow w = P_S(w - \gamma G^* G w) \\ &\quad \Longleftrightarrow w \in \mathrm{Fix}(P_S T). \end{aligned} \quad (9)$$

□

2. Iterative Algorithm for SEP

In this section, we establish an iterative algorithm that converges weakly to a solution of SEP.

Algorithm 5. Choose an arbitrary initial point $w_0 = (x_0, y_0)$, and sequence $\{w_n = (x_n, y_n)\}$ is generated by the following iteration:

$$w_{n+1} = (1 - \alpha_n)(I - \gamma G^* G) w_n + \alpha_n P_S (I - \gamma G^* G) w_n, \quad (10)$$

where $\alpha_n \subseteq (0, 1)$ and $0 < \gamma < \lambda = 2/\rho(G^* G)$ with $\rho(G^* G)$ being the spectral radius of the self-adjoint operator $G^* G$ on H.

To prove its convergence we need the following lemma.

Lemma 6. *The sequence $\{w_n\}$ generated by algorithm (10) is Féjer-monotone with respect to Γ; that is to say, for every $w \in \Gamma$,*

$$\|w_{n+1} - w\| \le \|w_n - w\|, \quad \forall n \ge 1, \quad (11)$$

if $\{\alpha_n\} \subseteq (0, 1)$ and $0 < \gamma < \lambda = 2/\rho(G^* G)$.

Proof. Let $u_n = (I-\gamma G^*G)w_n$ and choose $w \in \Gamma$; by Lemma 4, $w \in \mathrm{Fix}(P_S) \cap \mathrm{Fix}(I-\gamma G^*G)$, $Gw = 0$ and we have

$$\begin{aligned}
&\|w_{n+1}-w\|^2 \\
&\quad = \|(1-\alpha_n)u_n + \alpha_n P_S(u_n) - w\|^2 \\
&\quad \le (1-\alpha_n)\|u_n - w\|^2 + \alpha_n \|P_S(u_n) - w\|^2 \\
&\qquad - \alpha_n(1-\alpha_n)\|u_n - P_S(u_n)\|^2 \\
&\quad \le (1-\alpha_n)\|u_n - w\|^2 + \alpha_n\|u_n - w\|^2 \\
&\qquad - \alpha_n(1-\alpha_n)\|u_n - P_S(u_n)\|^2 \\
&\quad = \|u_n - w\|^2 - \alpha_n(1-\alpha_n)\|u_n - P_S(u_n)\|^2.
\end{aligned} \tag{12}$$

Moreover, we have

$$\begin{aligned}
\|u_n - w\|^2 &= \|(I-\gamma G^*G)w_n - w\|^2 \\
&= \|w_n - w\|^2 + \|\gamma G^*Gw_n\|^2 \\
&\quad - 2\langle w_n - w, \gamma G^*Gw_n\rangle \\
&= \|w_n - w\|^2 + \gamma^2\langle Gw_n, GG^*Gw_n\rangle \\
&\quad - 2\gamma\langle Gw_n - Gw, Gw_n\rangle \\
&\le \|w_n - w\|^2 + \gamma^2\lambda\|Gw_n\|^2 \\
&\quad - 2\gamma\langle Gw_n - 0, Gw_n\rangle \\
&= \|w_n - w\|^2 - \gamma(2-\lambda\gamma)\|Gw_n\|^2.
\end{aligned} \tag{13}$$

Hence, we can get that

$$\begin{aligned}
\|w_{n+1}-w\|^2 \le \|w_n - w\|^2 - \alpha_n(1-\alpha_n)\|u_n - P_S(u_n)\|^2 \\
- \gamma(2-\lambda\gamma)\|Gw_n\|^2.
\end{aligned} \tag{14}$$

It follows that $\|w_{n+1}-w\| \le \|w_n - w\|$, for all $w \in \Gamma$, $n \ge 1$. □

Theorem 7. *If* $0 < \liminf_{n\to\infty}\alpha_n \le \limsup_{n\to\infty}\alpha_n < 1$, *then the sequence* $\{w_n\}$ *generated by algorithm* (10) *converges weakly to a solution of SEP* (2).

Proof. Let w be a solution of SEP; according to Lemma 6, we can get that the sequence $\|w_n - w\|$ is monotonically decreasing and converges to some positive real. Since $0 < \liminf_{n\to\infty}\alpha_n \le \limsup_{n\to\infty}\alpha_n < 1$ and $0 < \gamma < \lambda$, by (14), we have

$$\|u_n - P_S(u_n)\| \longrightarrow 0, \qquad \|Gw_n\| \longrightarrow 0, \quad \text{when } n \longrightarrow \infty. \tag{15}$$

Since $\{w_n\}$ is Féjer-monotonicity, it follows that $\{w_n\}$ is bounded. Let $\widetilde{w}$ be a weak-cluster point of $\{w_n\}$ and let $k = 1, 2, \ldots$ be the sequence of indices, such that w_{n_k} converges weakly to $\widetilde{w}$. By Lemma 1, we can get that $G\widetilde{w} = 0$. It follows that $\widetilde{w} \in \mathrm{Fix}(I-\gamma G^*G)$.

Since $u_n = (I-\gamma G^*G)w_n$, it follows that u_{n_k} converges weakly to $\widetilde{w}$. On the other hand, $\|u_n - P_S(u_n)\| \to 0$. Using Lemma 1 again, we obtain that $P_S(\widetilde{w}) = \widetilde{w}$. That is to say, $\widetilde{w} \in \mathrm{Fix}(P_S)$.

Hence $\widetilde{w} \in \mathrm{Fix}(P_S) \cap \mathrm{Fix}(I-\gamma G^*G)$. By Lemma 4, we get that $\widetilde{w}$ is a solution of SEP (2).

The weak convergence of the whole sequence $\{w_n\}$ holds true since all conditions of the well-known Opial's lemma (Lemma 2) are fulfilled with $S = \Gamma$. □

3. Iterative Algorithm for MSSEP

In this section, we establish an iterative algorithm that converges weakly to a solution of MSSEP.

We use $\overline{\Gamma}$ to denote the solution set of MSSEP, that is,

$$\overline{\Gamma} = \left\{(x,y) \in H_1 \times H_2, Ax = By, x \in \bigcap_{i=1}^{N} C_i, y \in \bigcap_{j=1}^{M} Q_j\right\}, \tag{16}$$

and assume consistency of MSSEP so that $\overline{\Gamma}$ is closed, convex, and nonempty.

Without loss of generality, we assume that $N = M$. In fact, if $N > M$, let $Q_j = H_2$, for $j > M$.

Let $S_i = C_i \times Q_i$ in $H = H_1 \times H_2$ and define $G : H \to H_3$ by $G = [A, -B]$; then $G^*G : H \to H$ has the following matrix form:

$$G^*G = \begin{bmatrix} A^*A & -A^*B \\ -B^*A & B^*B \end{bmatrix}. \tag{17}$$

The original problem now can be reformulated as finding $w = (x, y) \in \cap_{i=1}^{N} S_i$ with $Gw = 0$, or, more generally, minimizing the function $\|Gw\|$ over $w \in \cap_{i=1}^{N} S_i$.

Algorithm 8. For an arbitrary initial point $w_0 = (x_0, y_0)$, sequence $\{w_n = (x_n, y_n)\}$ is generated by the following iteration:

$$w_{n+1} = (1-\alpha_n)(I-\gamma G^*G)w_n + \alpha_n P_{S_{i(n)}}(I-\gamma G^*G)w_n, \tag{18}$$

where $i(n) = n(\mathrm{mod}\, N) + 1$, $\alpha_n > 0$ is a sequence in $(0,1)$, and $0 < \gamma < \lambda = 2/\rho(G^*G)$ with $\rho(G^*G)$ being the spectral radius of the self-adjoint operator G^*G on H.

The proof of the following lemma is similar to Lemma 4, and we omit its proof.

Lemma 9. *Let* $T = I - \gamma G^*G$, *where* $0 < \gamma < \lambda = 2/\rho(G^*G)$ *with* $\rho(G^*G)$ *being the spectral radius of the self-adjoint operator* G^*G *on* H. *Then we have* $\mathrm{Fix}(T) = \{(x,y) \in H,\ Ax = By\}$, $\mathrm{Fix}(P_{\cap S_i}T) = \mathrm{Fix}(P_{\cap S_i}) \cap \mathrm{Fix}(T) = \overline{\Gamma}$, *and* $\cap \mathrm{Fix}(P_{S_i}T) = \cap[\mathrm{Fix}(P_{S_i}) \cap \mathrm{Fix}(T)] = \overline{\Gamma}$.

To prove its convergence we also need the following lemma.

Lemma 10. *Any sequence $\{w_n\}$ generated by algorithm (18) is the Féjer-monotone with respect to $\overline{\Gamma}$; namely, for every $w \in \overline{\Gamma}$,*

$$\|w_{n+1} - w\| \le \|w_n - w\|, \quad \forall n \ge 1, \tag{19}$$

provided that $\alpha_n > 0$ is a sequence in $(0,1)$ and $0 < \gamma < \lambda = 2/\rho(G^*G)$.

Proof. Let $u_n = (I - \gamma G^*G)w_n$ and take $w \in \overline{\Gamma}$; by Lemma 9, $w \in \mathrm{Fix}(P_{S_i}) \cap \mathrm{Fix}(I - \gamma G^*G)$, for all $N \ge i \ge 1$, $Gw = 0$ and we have

$$\begin{aligned}\|w_{n+1} - w\|^2 &= \left\|(1-\alpha_n)u_n + \alpha_n P_{S_{i(n)}}(u_n) - w\right\|^2 \\ &\le (1-\alpha_n)\|u_n - w\|^2 + \alpha_n \left\|P_{S_{i(n)}}(u_n) - w\right\|^2 \\ &\quad - \alpha_n(1-\alpha_n)\left\|u_n - P_{S_{i(n)}}(u_n)\right\|^2 \\ &\le (1-\alpha_n)\|u_n - w\|^2 + \alpha_n\|u_n - w\|^2 \\ &\quad - \alpha_n(1-\alpha_n)\left\|u_n - P_{S_{i(n)}}(u_n)\right\|^2 \\ &= \|u_n - w\|^2 - \alpha_n(1-\alpha_n)\left\|u_n - P_{S_{i(n)}}(u_n)\right\|^2.\end{aligned} \tag{20}$$

Moreover, all the same to the proof of Lemma 6, we have

$$\|u_n - w\|^2 \le \|w_n - w\|^2 - \gamma(2 - \lambda\gamma)\|Gw_n\|^2. \tag{21}$$

Hence, we have

$$\begin{aligned}\|w_{n+1} - w\|^2 &\le \|w_n - w\|^2 - \alpha_n(1-\alpha_n)\left\|u_n - P_{S_{i(n)}}(u_n)\right\|^2 \\ &\quad - \gamma(2-\lambda\gamma)\|Gw_n\|^2.\end{aligned} \tag{22}$$

It follows that $\|w_{n+1} - w\| \le \|w_n - w\|$, for all $w \in \overline{\Gamma}$, $n \ge 1$. □

Theorem 11. *If $0 < \liminf_{n\to\infty}\alpha_n \le \limsup_{n\to\infty}\alpha_n < 1$, then the sequence $\{w_n\}$ generated by algorithm (18) converges weakly to a solution of MSSEP (3).*

Proof. From (22) and the fact that $0 < \liminf_{n\to\infty}\alpha_n \le \limsup_{n\to\infty}\alpha_n < 1$ and $0 < \gamma < \lambda = 2/\rho(G^*G)$, we obtain that

$$\sum_{n=0}^{\infty}\left\|u_n - P_{S_{i(n)}}(u_n)\right\|^2 < \infty, \qquad \sum_{n=0}^{\infty}\|Gw_n\|^2 < \infty. \tag{23}$$

Therefore,

$$\lim_{n\to\infty}\left\|u_n - P_{S_{i(n)}}(u_n)\right\| = 0, \qquad \lim_{n\to\infty}\|Gw_n\| = 0. \tag{24}$$

Since $\{w_n\}$ is Féjer-monotone, it follows that $\{w_n\}$ is bounded. Let $\widetilde{w}$ be a weak-cluster point of $\{w_n\}$. Taking a subsequence $\{w_{n_k}\}$ of $\{w_n\}$ such that w_{n_k} converges weakly to $\widetilde{w}$, then, by Lemma 1, we can get that $G\widetilde{w} = 0$; it follows that $\widetilde{w} \in \mathrm{Fix}(I - \gamma G^*G)$.

Let $u_n = (I - \gamma G^*G)w_n$; it follows that u_{n_k} converges weakly to $\widetilde{w}$.

Since

$$\begin{aligned}&\|w_{n+1} - w_n\|^2 \\ &\quad = \left\|(1-\alpha_n)u_n + \alpha_n P_{S_{i(n)}}(u_n) - w_n\right\|^2 \\ &\quad = \left\|\alpha_n\left(P_{S_{i(n)}}u_n - u_n\right) + u_n - w_n\right\|^2 \\ &\quad \le 2\alpha_n^2\left\|\left(P_{S_{i(n)}}u_n - u_n\right)\right\|^2 + 2\|\gamma G^*Gw_n\|^2 \\ &\quad = 2\alpha_n^2\left\|\left(P_{S_{i(n)}}u_n - u_n\right)\right\|^2 + 2\gamma^2\langle Gw_n, GG^*Gw_n\rangle \\ &\quad \le 2\alpha_n^2\left\|\left(P_{S_{i(n)}}u_n - u_n\right)\right\|^2 + 2\gamma^2\lambda\|Gw_n\|^2,\end{aligned} \tag{25}$$

it follows that

$$\sum_{n=0}^{\infty}\|w_{n+1} - w_n\|^2 < \infty. \tag{26}$$

On the other hand

$$\begin{aligned}&\|u_{n+1} - u_n\|^2 \\ &\quad = \|w_{n+1} - w_n + \gamma G^*G(w_{n+1} - w_n)\|^2 \\ &\quad \le 2\left(\|w_{n+1} - w_n\|^2 + \|\gamma G^*G(w_{n+1} - w_n)\|^2\right) \\ &\quad \le 2\left(\|w_{n+1} - w_n\|^2 + \gamma^2\lambda\|(w_{n+1} - w_n)\|^2\right).\end{aligned} \tag{27}$$

Hence

$$\sum_{n=0}^{\infty}\|u_{n+1} - u_n\|^2 < \infty. \tag{28}$$

We can get that $\lim_{n\to\infty}\|u_{n+1} - u_n\| = 0$ and $\lim_{n\to\infty}\|u_{n+j} - u_n\| = 0$ for all $j = 1, 2, \ldots, N$.

Moreover, for any $i = 1, 2, \ldots, N$,

$$\begin{aligned}\left\|u_n - P_{S_{n+i}}u_n\right\| &\le \|u_n - u_{n+i}\| + \left\|u_{n+i} - P_{S_{n+i}}u_{n+i}\right\| \\ &\quad + \left\|P_{S_{n+i}}u_{n+i} - P_{S_{n+i}}u_n\right\| \\ &\le 2\|u_n - u_{n+i}\| + \left\|u_{n+i} - P_{S_{n+i}}u_{n+i}\right\| \\ &\longrightarrow 0.\end{aligned} \tag{29}$$

Thus, $\lim_{n\to\infty}\|u_n - P_{S_i}u_n\| = 0$ for all $i = 1, 2, \ldots, N$. Using Lemma 1 again, we obtain that $P_{S_i}(\widetilde{w}) = \widetilde{w}$. That is to say, $\widetilde{w} \in \mathrm{Fix}(P_{S_i})$ for all $i = 1, 2, \ldots, N$.

Hence $\widetilde{w} \in \cap\,\mathrm{Fix}(P_{S_i}) \cap \mathrm{Fix}(I - \gamma G^*G)$. By Lemma 9, we obtain that $\widetilde{w}$ is a solution of MSSEP (3).

The weak convergence of the whole sequence $\{w_n\}$ holds true since all conditions of the well-known Opial's lemma (Lemma 2) are fulfilled with $S = \overline{\Gamma}$. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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