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Research Article

New Iterative Method Based on Laplace Decomposition Algorithm

Sabir Widatalla[1,2] and M. Z. Liu[1]

[1] *Department of Mathematics, Harbin Institute of Technology, Harbin 150001, China*
[2] *Department of Physics and Mathematics, College of Education, Sinnar University, Singa 107, Sudan*

Correspondence should be addressed to Sabir Widatalla; sabirtag@yahoo.com

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We introduce a new form of Laplace decomposition algorithm (LDA). By this form a new iterative method was achieved in which there is no need to calculate Adomian polynomials, which require so much computational time for higher-order approximations. We have implemented this method for the solutions of different types of nonlinear pantograph equations to support the proposed analysis.

1. Introduction

Since 2001, Laplace decomposition algorithm (LDA) has been one of the reliable mathematical methods for obtaining exact or numerical approximation solutions for a wide range of nonlinear problems.

The Laplace decomposition algorithm was developed by Khuri in [2] to solve a class of nonlinear differential equations. The basic idea in Laplace decomposition algorithm, which is a combined form of the Laplace transform method with the Adomian decomposition method, was developed to solve nonlinear problems. The disadvantage of the Laplace decomposition algorithm is that the solution procedure for calculation of Adomian polynomials is complex and difficult and takes a lot of computational time for higher-order approximations as pointed out by many researchers [3–5].

The Laplace decomposition algorithm plays an important role in modern scientific research for solving various kinds of nonlinear models; for example, Laplace decomposition algorithm was used in [6] to solve a model for HIV infection of $CD4^{+}T$ cells; LDA was employed in [7] to solve Abel's second kind singular integral equations. In [8] it was used to solve boundary Layer equation.

Even though there has been some developments in the LDA [8–11], the use of Adomian polynomials has not been abandoned.

The main purpose of this paper is to introduce a new iterative method based on Laplace decomposition algorithm procedure without the need to compute Adomian polynomials and thus reduce the size of calculations needed.

The scheme is tested for some classes of pantograph equations, and the results demonstrate reliability and efficiency of the proposed method.

2. Basic Idea of LDA and the New Technique

To illustrate the basic concept of Laplace decomposition algorithm, we consider the following general nonlinear model:

$$L^{(m)}u = Nu + Ru + g(t), \tag{1}$$

where $L^{(m)}$ is the highest order derivative, R and N are linear and nonlinear operators, respectively, and $g(t)$ is an inhomogeneous term.

Applying the Laplace transform (denoted throughout this paper by $\mathscr{L}$) to both sides of (1) and using given conditions, we obtain

$$\mathscr{L}[u(t)] = \mathscr{H}(s) + \mathscr{G}(s) + s^{-m}\mathscr{L}[Nu] + s^{-m}\mathscr{L}[Ru], \tag{2}$$

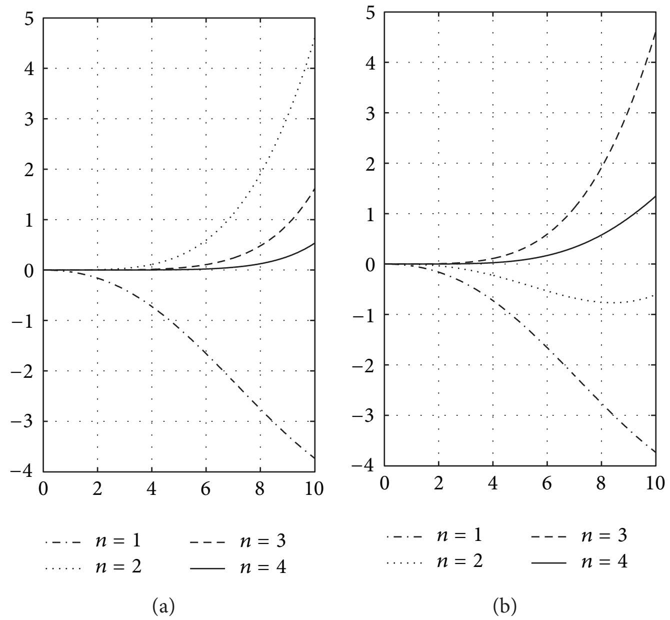

FIGURE 1: The error functions $E_n(t_i)$ for Example 1: (a) present method and (b) standard LDA.

where

$$\mathscr{H}(s) = \sum_{r=0}^{m-1} s^{-(r+1)} u^{(r)}(0), \qquad \mathscr{G}(s) = s^{-m} \mathscr{L}[g(t)]. \tag{3}$$

The Laplace decomposition algorithm defines the unknown function $u(t)$ by an infinite series as

$$u(t) = \sum_{n=0}^{\infty} u_n(t), \tag{4}$$

where the components $u_n(t)$ will be determined recurrently. Substituting this infinite series into (2) and using the linearity of Laplace transform lead to

$$\begin{aligned} \sum_{n=0}^{\infty} \mathscr{L}[u_n(t)] &= \mathscr{H}(s) + \mathscr{G}(s) \\ &\quad + s^{-m} \sum_{n=0}^{\infty} \mathscr{L}[Nu_n] + s^{-m} \sum_{n=0}^{\infty} \mathscr{L}[Ru_n]. \end{aligned} \tag{5}$$

Also the nonlinear functions Nu_n are defined by infinite series as follows:

$$Nu_n = \sum_{n=0}^{\infty} A_n, \tag{6}$$

where A_n are the Adomian polynomials [12], depending only on $u_0, u_1, \ldots, u_n$, and defined by

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[N\left(\sum_{i=0}^{n} \lambda^i u_i \right) \right]_{\lambda=0}, \quad n = 0, 1, 2, \ldots. \tag{7}$$

Substituting (6) into (5), we get

$$\begin{aligned} \sum_{n=0}^{\infty} \mathscr{L}[u_n(t)] &= \mathscr{H}(s) + \mathscr{G}(s) + s^{-m} \sum_{n=0}^{\infty} \mathscr{L}[A_n] \\ &\quad + s^{-m} \sum_{n=0}^{\infty} \mathscr{L}[Ru_n]. \end{aligned} \tag{8}$$

The Laplace decomposition algorithm presents the recurrence relation as

$$\begin{aligned} u_0(t) &= \mathscr{L}^{-1}[\mathscr{H}(s) + \mathscr{G}(s)], \\ u_{n+1}(t) &= \mathscr{L}^{-1} s^{-m} \mathscr{L}[A_n] \\ &\quad + \mathscr{L}^{-1} s^{-m} \mathscr{L}[Ru_n], \quad n = 0, 1, 2, \ldots. \end{aligned} \tag{9}$$

Applying the inverse Laplace transform to (8) leads to

$$\begin{aligned} \sum_{n=0}^{\infty} [u_n(t)] &= \mathscr{L}^{-1}[\mathscr{H}(s) + \mathscr{G}(s)] + \left(\mathscr{L}^{-1} s^{-m} \mathscr{L} \right) \\ &\quad \times \sum_{n=0}^{\infty} A_n + \left(\mathscr{L}^{-1} s^{-m} \mathscr{L} \right) \sum_{n=0}^{\infty} Ru_n. \end{aligned} \tag{10}$$

Equation (10) can be written as

$$\begin{aligned} u_0(t) &+ u_1(t) + u_2(t) + \cdots + u_n(t) + \cdots \\ &= \mathscr{L}^{-1}[\mathscr{H}(s) + \mathscr{G}(s)] \\ &\quad + \left(\mathscr{L}^{-1} s^{-m} \mathscr{L} \right) [A_0 + A_1 + A_2 + \cdots] \\ &\quad + \left(\mathscr{L}^{-1} s^{-m} \mathscr{L} \right) [R(u_0 + u_1 + u_2 + \cdots)]. \end{aligned} \tag{11}$$

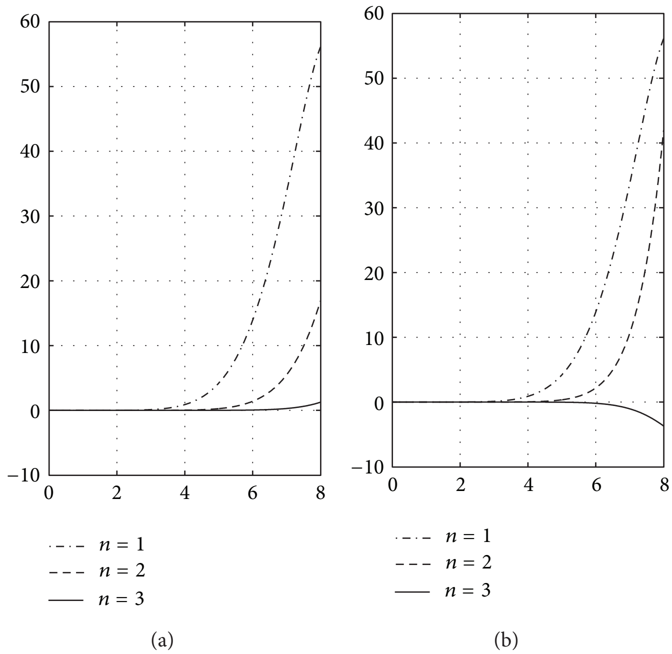

FIGURE 2: The error functions $E_n(t_i)$ for Example 2: (a) present method and (b) standard LDA.

By taking $v_n = \sum_{i=0}^{n} u_i$ and (11), the following procedure can be constructed:

$$\begin{aligned} v_0 &= \mathscr{L}^{-1}\left[\mathscr{H}(s)+\mathscr{G}(s)\right] \\ v_{n+1} &= v_0 + \left(\mathscr{L}^{-1}s^{-m}\mathscr{L}\right)\sum_{i=0}^{n}\left[A_i\right] \\ &\quad + \left(\mathscr{L}^{-1}s^{-m}\mathscr{L}\right)\left(R\left(v_n\right)\right), \quad n = 0,1,2,\ldots. \end{aligned} \tag{12}$$

Consequently, the exact solution may be obtained by

$$u(t) = \lim_{n\to\infty} v_n = \lim_{n\to\infty}\sum_{i=0}^{n} u_i. \tag{13}$$

For the analytic nonlinear operator N, we can write

$$\lim_{n\to\infty} N\left(\sum_{i=0}^{n} u_i\right) = \lim_{n\to\infty}\sum_{i=0}^{n} A_i. \tag{14}$$

By considering, (12) and (14) can be reconstructed as

$$\begin{aligned} \widehat{v}_0 &= \mathscr{L}^{-1}\left[\mathscr{H}(s)+\mathscr{G}(s)\right] \\ \widehat{v}_{n+1} &= \widehat{v}_0 + \left(\mathscr{L}^{-1}s^{-m}\mathscr{L}\right)\left(N\left(\widehat{v}_n\right)\right) \\ &\quad + \left(\mathscr{L}^{-1}s^{-m}\mathscr{L}\right)\left(R\left(\widehat{v}_n\right)\right), \quad n = 0,1,2,\ldots. \end{aligned} \tag{15}$$

Equation (15) is a new iteration method based on LDA. The advantage of this scheme is that there is no need to calculate Adomian polynomials.

3. Test Problems

In this section we will apply our scheme to different types of pantograph equations.

Example 1 (see [13]). Consider the following nonlinear pantograph differential equation:

$$\begin{gathered} u'(t) = \frac{1}{4}u(t) + u\left(\frac{t}{2}\right)\left(1 - u\left(\frac{t}{2}\right)\right), \\ u(0) = 1. \end{gathered} \tag{16}$$

The exact solution of this problem $u(t) = 1/2+1/2\cos(\sqrt{2}t/4) + \sqrt{2}/2\sin(\sqrt{2}t/4)$.

Based on the iteration formula (15), we get

$$\begin{aligned} v_0 &= 1, \\ v_{n+1} &= v_0 + \left(\mathscr{L}^{-1}s^{-1}\mathscr{L}\right)\left(\frac{1}{4v_n(t)}\right) \\ &\quad + \left(\mathscr{L}^{-1}s^{-1}\mathscr{L}\right)\left(v_n\left(\frac{t}{2}\right)\left(1 - v_n\left(\frac{t}{2}\right)\right)\right). \end{aligned} \tag{17}$$

Thus, we get

$$\begin{aligned} v_1 &= 1 + \frac{t}{4}, \\ v_2 &= 1 + \frac{t}{4} - \frac{t^2}{32} - \frac{t^3}{192}, \end{aligned}$$

Table 1: Comparison of the absolute errors for Example 1.

t	Exact solution	Standard LDA $n = 5$	Present method $n = 5$
0.2	1.048708864495296	$8.803E-11$	$1.900E-13$
0.6	1.137669652809217	$6.594E-8$	$4.123E-10$
0.8	1.177476957990361	$3.753E-7$	$3.077E-9$
1.0	1.213898289556670	$1.450E-6$	$1.461E-8$
2.0	1.339484983470599	$9.834E-5$	$1.838E-6$
4.0	1.276427846019296	$6.905E-3$	$2.299E-4$
6.0	$8.410651664398781E-1$	$8.393E-2$	$3.960E-3$
8.0	$2.421580540539403E-1$	$4.911E-1$	$3.122E-2$
10	$2.331110590497953E-1$	1.910	$1.643E-1$

Table 2: Comparison of the absolute errors for Example 2.

t	Exact solution	DTM [1] $N = 9$	Standard LDA $n = 3$	Present method $n = 3$
0.2	$1.986693307950612E-1$	$5.2735E-16$	$7.3960E-18$	$6.3679E-18$
0.6	$5.646424733950354E-1$	$9.0678E-11$	$3.8301E-13$	$2.2537E-13$
0.8	$7.173560908995228E-1$	$2.1431E-9$	$8.2825E-12$	$6.0382E-12$
1.0	$8.414709848078965E-1$	$2.4892E-8$	$8.4781E-11$	$1.0616E-11$
2.0	$9.092974268256817E-1$	$5.0015E-5$	$1.5123E-8$	$2.4971E-7$
4.0	$-7.568024953079282E-1$	$9.5074E-2$	$1.2344E-3$	$5.5720E-4$
6.0	$-2.794154981989259E-1$	7.3079	$1.8828E-1$	$5.2101E-2$
8.0	$9.893582466233818E-1$	148.51	3.6889	1.2615

$$v_3 = 1 + \frac{t}{4} - \frac{t^2}{32} - \frac{t^3}{192} + \frac{t^4}{3072} + \frac{t^5}{49152} - \frac{t^6}{589824} - \frac{t^7}{16515072},$$
$$v_4 = 1 + \frac{t}{4} - \frac{t^2}{32} - \frac{t^3}{192} + \frac{t^4}{3072} + \frac{t^5}{30720} - 1.801E^{-6}t^6 - 9.461E^{-8}t^7 + 3.548E^{-9}t^8 + \text{small term}. \tag{18}$$

Knowing that the exact solution of this exampleis given in [13],

$$u(t) = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{\sqrt{2}}{4}t\right) + \frac{\sqrt{2}}{2}\sin\left(\frac{\sqrt{2}}{4}t\right) = 1 + \frac{t}{4} - \frac{t^2}{32} - \frac{t^3}{192} + \frac{t^4}{3072} + \frac{t^5}{30720} - 1.356E^{-6}t^6 - 9.688E^{-8}t^7 + 3.028E^{-9}t^8 + \cdots. \tag{19}$$

We see that the approximation solutions obtained by the present method have good agreement with the exact solution of this problem.

In Table 1 the absolute errors of the present method and standard LDA for $n = 5$ are compared.

Figure 1 compares the numerical errors $E_n(t_i) = |u(t_i) - u_n(t_i)|$ for $n = 1, 2, 3$, and 4 obtained by (a) the present method and (b) the standard LDA. This plot indicates that the series solution obtained by the present method converges faster than the standard LDA.

Example 2. Consider the following nonlinear pantograph integrodifferential equation (PIDE):

$$u''(t) = \frac{3}{5}u\left(\frac{5}{6}t\right) - 3u\left(\frac{1}{6}t\right) - t + \int_0^t \left[u\left(\frac{1}{2}x\right)u\left(\frac{1}{3}x\right) + 2u^2\left(\frac{1}{2}x\right)\right]dx, \quad u(0) = 0, \quad u'(0) = 1. \tag{20}$$

For this example we write iteration formula (15) as

$$\widehat{v}_0 = t - \frac{t^3}{6}, \widehat{v}_{n+1} = v_0 + \left(\mathscr{L}^{-1}s^{-2}\mathscr{L}\right) \times \left(\frac{3}{5v_n(5t/6)} - 3v_n\left(\frac{t}{6}\right)\right) + \left(\mathscr{L}^{-1}s^{-2}\mathscr{L}\right) \times \int_0^t \left[v_n\left(\frac{x}{2}\right)v_n\left(\frac{x}{3}\right) + 2v_n^2\left(\frac{x}{2}\right)\right]dx, \quad n = 0, 1, 2, \ldots, \tag{21}$$

Table 3: Comparison of the absolute errors for Example 3.

t	Exact solution	DTM [1] $N = 4$	Standard LDA $n = 4$	Present method $n = 4$
0.2	$2.442805516320340E-1$	$1.388E-5$	$2.6127E-6$	$2.6093E-6$
0.4	$5.967298790565082E-1$	$4.632E-4$	$1.5387E-4$	$1.5303E-4$
0.6	1.093271280234305	$3.671E-3$	$1.6166E-3$	$1.5961E-3$
0.8	1.780432742793974	$1.617E-2$	$8.4028E-3$	$8.2072E-3$
1.0	2.718281828459046	$5.162E-2$	$2.9756E-2$	$2.8645E-2$
2.0	14.77811219786130	2.111	1.4168	1.1919
3.0	60.25661076956300	21.25	14.341	9.6958

Table 4: The absolute errors for Example 4.

t	Exact solution	Present method		
	$u_1 = e^{-t}\cos(t)$	Ev_{11}	Ev_{12}	Ev_{13}
0.2	$8.024106E-1$	$1.144E-2$	$4.432E-4$	$1.900E-5$
0.4	$6.174056E-1$	$4.990E-2$	$4.274E-3$	$3.656E-4$
0.6	$4.529538E-1$	$4.185E-1$	$1.643E-2$	$2.119E-3$
0.8	$3.130505E-1$	$2.171E-1$	$4.274E-2$	$7.420E-3$
1.0	$1.987661E-1$	$3.437E-1$	$8.925E-2$	$1.960E-2$
	$u_2 = \sin(t)$	Ev_{21}	Ev_{22}	Ev_{23}
0.2	$1.986693E-1$	$2.273E-2$	$5.174E-4$	$1.670E-5$
0.4	$3.894183E-1$	$1.024E-1$	$5.840E-3$	$1.790E-4$
0.6	$5.646425E-1$	$2.575E-1$	$2.630E-2$	$3.282E-4$
0.8	$7.173561E-1$	$5.082E-1$	$8.022E-2$	$1.276E-3$
1.0	$8.414710E-1$	$8.768E-1$	$1.965E-1$	$1.015E-2$

Table 5: The CPU time analysis of the present method and the standard LDA for obtaining the first three components of Examples 1–4.

Solution method	The required CPU time [in seconds]			
	Example 1	Example 2	Example 3	Example 4
Present method	1.1716	2.0121	1.5608	3.0301
Standard LDA	1.1872	2.4046	1.9201	3.3431

and the first n terms are

$$\begin{aligned} v_1 &= t - \frac{1}{3!}t^3 + \frac{1}{5!}t^5 - \frac{67}{272160}t^7 + \frac{31}{15676416}t^9, \\ v_2 &= t - \frac{1}{3!}t^3 + \frac{1}{5!}t^5 - \frac{1}{7!}t^7 + \frac{67163}{25395793920}t^9 + \cdots, \\ v_3 &= t - \frac{1}{3!}t^3 + \frac{1}{5!}t^5 - \frac{1}{7!}t^7 + \frac{1}{9}t^9 - \frac{230719}{8620058050560}t^{11} + \cdots \\ &\vdots \\ v_n &= t - \frac{1}{3!}t^3 + \frac{1}{5!}t^5 - \frac{1}{7!}t^7 + \frac{1}{9}t^9 - \cdots + \frac{(-1)^n}{(2n+1)!}t^{(2n+1)}, \end{aligned} \tag{22}$$

which gives the exact solution by $u(t) = \lim_{n\to\infty} v_n = \sin(t)$.

In Table 2 we compare the absolute errors of the present method for $n = 3$ and the standard LDA for $n = 3$ and the differential transform method described in [1] with nine terms.

Figure 2 displays the numerical errors obtained by the present method and the standard LDA.

Example 3. Consider the following nonlinear PIDE:

$$\begin{gathered} u'(t) + \left(\frac{1}{2}t - 2\right)u(t) - 2\int_0^t u^2\left(\frac{1}{2}x\right)dx = 1, \\ u(0) = 0, \end{gathered} \tag{23}$$

which has the exact solution $u(t) = te^t$. The iteration form of (15) for this example is

$$\begin{aligned} \hat{v}_0 &= t, \\ \hat{v}_{n+1} &= v_0 - \left(\mathscr{L}^{-1}s^{-1}\mathscr{L}\right)\left(\frac{t}{2} - 2\right)v_n + 2\left(\mathscr{L}^{-1}s^{-1}\mathscr{L}\right) \\ &\quad \times \left(\int_0^t v_n^2\left(\frac{x}{2}\right)dx\right), \quad n = 0, 1, 2, \ldots. \end{aligned} \tag{24}$$

We obtain the following successive approximations:

$$\begin{aligned} v_1 &= t + t^2 - \frac{t^3}{6} + \frac{t^4}{24}, \\ v_2 &= t + t^2 + \frac{t^3}{2} - \frac{t^4}{6} + \frac{7t^5}{120} + O(6), \end{aligned}$$

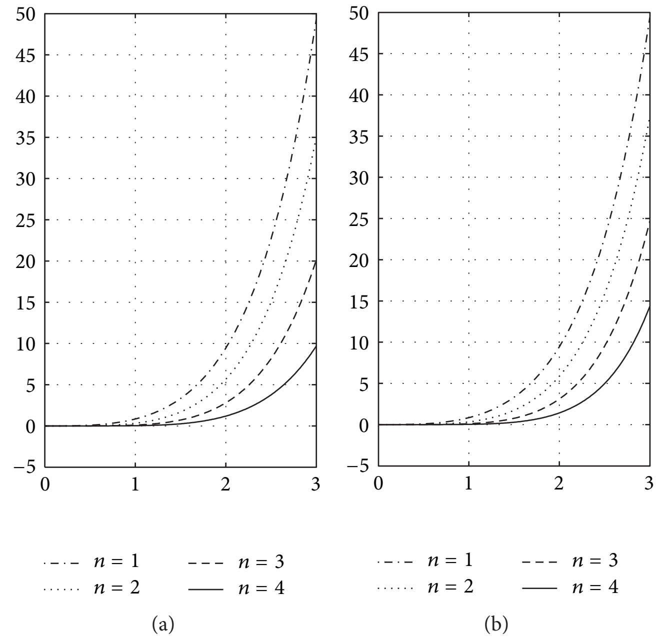

FIGURE 3: The error functions $E_n(t_i)$ for Example 3: (a) present method and (b) standard LDA.

$$v_3 = t + t^2 + \frac{t^3}{2} + \frac{t^4}{6} - \frac{11t^5}{120} + \frac{t^6}{24} + O(7),$$

$$v_4 = t + t^2 + \frac{t^3}{2} + \frac{t^4}{6} + \frac{t^5}{24} - \frac{13t^6}{360} + \frac{103t^7}{5040} + O(8). \tag{25}$$

Note that the exact solution of this example is

$$u(t) = te^t = t + t^2 + \frac{t^3}{2} + \frac{t^4}{6} + \frac{t^5}{24} + \frac{t^6}{120} + \frac{t^7}{720} + \cdots. \tag{26}$$

In Table 3 we compare the absolute errors of the present method for $n = 4$ and the standard LDA for $n = 4$ and the differential transform method described in [1] with four terms.

Figure 3 displays the numerical errors obtained by the present method and the standard LDA.

Example 4. Consider a system of multipantograph equations:

$$\begin{aligned} u_1'(t) &= -u_1(t) - e^{-t}\cos\left(\frac{t}{2}\right)u_2\left(\frac{t}{2}\right) \\ &\quad - 2e^{-(3/4)t}\cos\left(\frac{t}{2}\right)\sin\left(\frac{t}{4}\right)u_1\left(\frac{t}{4}\right), \\ u_2'(t) &= e^t u_1^2\left(\frac{t}{2}\right) - u_2^2\left(\frac{t}{2}\right), \\ u_1(0) &= 1, \qquad u_2(0) = 0. \end{aligned} \tag{27}$$

We can adapt (15) to solve this system as follows:

$$\begin{aligned} v_{10}(t) &= 1, \\ v_{20}(t) &= 0, \\ v_{1(j+1)} &= v_{10} - \left(\mathscr{L}^{-1}s^{-1}\mathscr{L}\right) \\ &\quad \times \left(v_{1j}(t) + e^{-t}\cos\left(\frac{t}{2}\right)v_{2j}\left(\frac{t}{2}\right)\right. \\ &\quad \left. + 2e^{-3t/4}\cos\left(\frac{t}{2}\right)\sin\left(\frac{t}{4}\right)v_{1j}\left(\frac{t}{4}\right)\right), \\ v_{2(j+1)} &= v_{20} + \left(\mathscr{L}^{-1}s^{-1}\mathscr{L}\right) \\ &\quad \times \left(e^t v_{1j}^2\left(\frac{t}{2}\right) - v_{2j}^2\left(\frac{t}{2}\right)\right), \quad j = 0, 1, 2, \ldots. \end{aligned} \tag{28}$$

Table 4 gives the absolute errors $Ev_{ij} = |u_i - v_{ij}|, i = 1, 2, j = 1, 2, 3$ of the present method. The table clearly indicates that when we increase the truncation limit n, we have less error.

Table 5 summarizes the CPU times needed to obtain the first three components of the series solutions pertaining to the four above-mentioned examples by the present method and the standard LDA. The CPU time analysis was conducted on a personal computer with a 3.77 GHz processor and 4 GB of RAM using MATLAB 7.10.

4. Conclusion

In this work, we have presented a new iteration method based on the Laplace decomposition algorithm.

The advantage of the new method is that it does not require Adomian polynomials and thus reduces the calculation size.

The new iterative method has been employed to solve different classes of nonlinear pantograph equations, in which the results obtained are in close agreement with the exact solutions.

The convergence of this method is the subject of ongoing research.

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