

SOME VIEWS OF RUSSELL AND RUSSELL'S LOGIC BY HIS EARLY CONTEMPORARIES

IRVING H. ANELLIS

In the years just prior to World War I, it was not yet definitively certain that the new logic developed and espoused by Gottlob Frege and Bertrand Russell was the “wave of the future” of mathematical logic. To the contrary, the “old” algebraic logic, first articulated by George Boole and Augustus De Morgan in 1847 and 1854 and worked on in the half-century since by Charles Peirce, Ernst Schröder, and a handful of their colleagues remained the focus of much of the research in the field and continued as the source of inspiration and starting point for the research of a new and rising generation of workers. Whereas Frege had eschewed, even belittled, the achievements of the algebraic logicians (see, e.g. [Frege 1880/81, Frege 1882]; see also [Sluga 1987]), Russell was—although he seldom cared too openly or too readily to admit it—reading the work of Peirce and Schröder, along with the work of Giuseppe Peano and his school;¹ the work of Frege himself lay largely forgotten following a brief flurry of generally negative reviews

Based upon an invited talk for the history of logic section of the 12th International Congress of Logic, Methodology, and Philosophy of Science, Oviedo, Spain, August 2003. The talk was undelivered due to illness.

Received 8/30/2004.

© 2005 *The Review of Modern Logic*.

¹[Anellis 1995] includes a discussion of what and when Russell read and wrote of Peirce and Schröder. See [Anellis 1990/91] for details of Russell's notes on Schröder's work.

In a letter of 23 April 1977 to Kenneth Blackwell of the Bertrand Russell Archives, Max Harold Fisch told Blackwell that he had heard from James Kern Feibelman that Russell had once met Peirce. Fisch wrote that he had understood from both Feibelman and Elizabeth Ramsden Eames that Russell met Peirce in 1896 in the United States during a visit to William James [Fisch 1977].

In a letter to Fisch of 4 July 1959, Russell denied that he was influenced by Peirce, or that there was much connection between his work and Peirce's, writing that “there is very little relation between his work in logic and mine” and that Peirce's “treatment of the logic of relations did not seem to me what was appropriate for mathematics and, apart from that, I read very little of his work until my own was finished” [Russell 1959].

by Schröder, John Venn, and a few other interested but critically disapproving parties (see, e.g. [Frege 1972]), until Russell's criticisms of Frege drew renewed attention to it in the *Principles of Mathematics* [Russell 1903a].

In the years before the outbreak of the First World War that witnessed Russell's first endeavors and the publication of his *Principles*, Ernst Schröder's [Schröder 1895–1905] *Vorlesungen über die Algebra der Logik* was still new and viewed as the cutting edge of (algebraic) logic. Peano still proclaimed, in his earliest work on logic, his indebtedness to Boole, Peirce, and Schröder, in the first footnote of his *Arithmetices principia nova methodo exposita* [Peano 1889], while, in a letter to Russell of 19 March 1901 (quoted in [Kennedy 1975, 206], Peano told Russell that his “Sur la logique des relations avec des applications à la théorie des séries” [Russell 1901a] just “fills a gap between the work of Peirce and Schröder on the one hand and” his own “*Formulaire* on the other.” Frege went so far as to label Peano as a follower of Boole [Frege 1897, 370–371] (see also [G. Moore 1988, 109]). Alfred North Whitehead's [Whitehead 1898] *Treatise on Universal Algebra*, also still new, brought together the algebraic logic of Boole with the linear and multilinear algebras of Arthur Cayley, James Joseph Sylvester, William Rowan Hamilton, and Charles Peirce's father Benjamin Peirce, and the calculus of extension of Hermann Günther Grassmann. In his *Treatise*, Whitehead not only named Hamilton and De Morgan as “the first to express quite clearly the general possibilities of algebraic symbolism” [Whitehead 1898, x], but continually expressed his indebtedness to Boole, Grassmann, De Morgan, Schröder, and Venn; and he described his *Treatise* as providing a “thorough investigation of the various systems of Symbolic Reasoning allied to ordinary Algebra,” the “chief examples” of which were Hamilton's Quaternions, Grassmann's Calculus of Extension, and Boole's Symbolic Logic [Whitehead 1898, v]. In his “Memoir on the Algebra of Symbolic Logic” of 1901, Whitehead declared [Whitehead 1901, 139] that “As a matter of history, this algebra [of symbolic logic] has only been continuously studied since the publication of Boole's ‘Laws of Thought’ (1854), and to C. S. Peirce and to Schröder must be assigned the credit of perfecting the laws of its operation. But as a question of logical priority, this algebra must be considered as the first object of mathematical study” by the twofold right of its being “concerned with the fundamental conceptions of classes” and as “the simplest of all algebraic systems.” Cambridge University logician William Ernest Johnson, asked in 1905 to evaluate Whitehead's work, wrote [Johnson 1905] in his “Report to the Subcommittee on Higher Degrees of the Cambridge University Special

Board on Mathematics,” published in the *Cambridge Register*, that Whitehead’s contributions to Boolean algebra and algebraic logic, although yielding “remarkable results and in a manner exhibiting extraordinary power” and “giving new life to the study of symbolic logic,” nevertheless did not receive the attention which they deserved. Edward V. Huntington, Charles Peirce’s one-time correspondent, went further in his “New Sets of Independent Postulates for the Algebra of Logic, with Special Reference to Whitehead and Russell’s *Principia Mathematica*” (see [Huntington 1933, 278]), viewing Whitehead’s work as the culmination and apex of the work in algebraic logic of the last half of the nineteenth century, asserting that algebraic logic, and in particular Boolean algebra and universal algebra was “originated by Boole, extended by Schröder, and perfected by Whitehead.” Russell’s British colleague Arthur Thomas Shearman, writing at the start of the twentieth century on the development of symbolic logic in a work bearing that title [Shearman 1906] of the last half-century, concentrating on the work of the British logicians, saw the work of Frege and Russell as minor natural extensions (and slight refinements) of the work of Boole, De Morgan, and William Stanley Jevons.² Paul Carus, editor of

²Published reactions to Frege’s *Begriffsschrift* [Frege 1879] at the time of its first appearance have been well documented; see, e.g. [Bynum 1972, 15–20]. Reviews of the *Begriffsschrift* were published by Ernst Eduard Reinhold Hoppe [Hoppe 1879], Kurd Lasswitz [Lasswitz 1879], Paul Tannery [Tannery 1879], and Carl Theodore Michaëlis [Michaëlis 1880, Michaëlis 1881]. Of particular significance were the reviews by John Venn [Venn 1880] and Ernst Schröder [Schröder 1880, Schröder 1969]. [Hawkins 1993] relies largely upon circumstantial evidence to speculate upon what Charles Peirce knew of Frege’s work, but is convinced that Peirce knew of the *Begriffsschrift*, as he had received an offprint from Schröder of Schröder’s [Schröder 1880] review, and his students Christine Ladd-Franklin and Allan Marquand each owned a copy of the *Begriffsschrift*, as did the Johns Hopkins University library. A traditional account of the rise of interest in Frege and his work was given by [Stroll 1966]. The view of [Vikko 1998] of the contemporary reception of Frege’s contributions to logic is much more positive than the hitherto standard view, however, in comparison with the reception given to Edmund Husserl’s [Husserl 1891] *Philosophie der Arithmetik*. The paucity of timely reactions, as measured by published reviews, to the remainder of Frege’s major published works, matches, or even falls short of, that evidenced in similar wise, to the unenthusiastic response to the *Begriffsschrift*. Similarly, Frege’s later work received but scant attention; Georg Cantor [Cantor 1885] reviewed Frege’s [Frege 1884] *Grundlegung*; Michaëlis [Michaëlis 1896] noted the first volume of the Frege’s *Grundgesetze*, as did Giuseppe Peano [Peano 1895], and Carl Färber [Färber 1905] noted the second volume of Frege’s [Frege 1903] *Grundgesetze*. It was from Peano that Russell first learned of Frege and his work (see [Kennedy 1973, 368]; see also [Nidditch 1963, esp. p. 109]). Yet, even after obtaining this information from Peano, he looked at, but did not yet for the remainder of 1900 read the

The Monist, shared the view, in discussing “The Nature of Logical and Mathematical Thought” [Carus 1910, 54] that Schröder, Peirce, Peano, Russell and Couturat all belonged to the line of workers who sought to broaden traditional logic by their attempts to “transfer the accomplishments of mathematics upon logic,” and he held [Carus 1910, 54] that Peano “distinguished himself by an application of the algebraic method . . .” Louis Couturat himself asserted, in his review [Couturat 1904] of the *Principles of Mathematics* [Russell 1903a], that Russell’s *Principles* was essentially “*une systématisation et une synthèse*” of the work of Russell’s predecessors, most notably Peano, Whitehead, Schröder, and Russell himself [Couturat 1904, 129–130]. It is clear from Peirce’s comments upon the work of Russell (see, e.g. [Peirce 1903b, MS 459:20]), that he considered Russell to have merely reformulated, in a particularly technical and formal manner, results of logic already established, and that he saw no clear distinction between the new logic and the algebraic mathematical logic of the Boole-Schröder calculus.

If one read the published comments in the decade between publication of Russell’s *Principles* (1903) and Whitehead and Russell’s *Principia Mathematica* (1910–1913), one would in all likelihood be brought to conclude that the work of Peirce and Schröder was still at the forefront of logical research, and that it would, into the foreseeable future, be the point of departure for continued research. Thus, for example, Edwin Bidwell Wilson wrote in the pages of the *Bulletin of the American Mathematical Society* [Wilson 1904, 76] that “Boole had freed us from Aristotelianism and that C. S. Peirce and Schröder had carried the

Grundgesetze. In his *Portraits from Memory* [Russell 1956, 21–22], he wrote about the *Begriffsschrift* that:

Ward gave me Frege’s little book *Begriffsschrift* saying that he did not read the book and did not know whether it had any value. To my shame I have to confess that I did not read it either, until I had independently worked out a great deal of what it contained. This book was published in 1879 and I read it in 1901. I rather suspect that I was its first reader. What first attracted me to Frege was a review of a later book of his by Peano accusing him of unnecessary subtlety. As Peano was the most subtle logician I had at that time come across, I felt that Frege must be remarkable.

Philosopher and psychologist James Ward (1843–1925) was one of Russell’s Cambridge University professors. Russell received the *Begriffsschrift* after he was awarded his fellowship (so that he would have received it from Ward some time during or very soon after the autumn of 1895). The archival evidence shows that Russell in fact first read Frege’s work in mid-June 1902, reading the *Begriffsschrift* [Frege 1879] and the *Grundgesetze* [Frege 1893] between the 17th and 19th, and that he continued to study Frege’s works through the summer.

technique of logic much further.” Maxime Bôcher wrote in the pages of the same journal [Bôcher 1904, 119] that: “Fortunately, the mathematical logicians from Boole down to C. S. Peirce, Schröder, and Peano and his followers,” including, independently, Frege, “have been able to make a rather short list of logical conceptions and principles upon which it would seem that all exact reasoning depends.” Both Josiah Royce and William James, two Harvard philosophers close to Charles Peirce, died in the 1910s; Royce (as quoted in [Ketner 1987, 18]) asserted that “Mr. Charles Peirce has now been for many years the principal representative in this country of a type of investigation in Logic which seems to me, as a student of the subject, to be of very great importance.”³ James (as quoted in [Ketner 1987, 20]) thought that Peirce was “in the very front rank of American thinkers . . . and his Logic when published will unquestionably . . . be recognized all over the world as an epoch-making work.” British mathematician and philosopher of science William Clifford (according to Edward Livingston Youmans, as reported in [Fisch 1986, 129]) in these same years called Peirce “the greatest living logician, and the second man since Aristotle who has added to the subject” of logic “something material, the other man being George Boole . . .” Couturat asserted ([Couturat 1905], [Couturat 1914, 3]) that the “algebra of logic was founded by George Boole (1815–1864) [and] it was developed and perfected by Ernst Schröder (1841–1902),” and in 1914, Philip Jourdain added [Jourdain 1914, iv] that “modern logic is really due to Boole and De Morgan.” Even as late as 1922, Jan Łukasiewicz in his Inaugural lecture at the University of Warsaw, listed Peirce as one of the most prominent representatives of mathematical logic of the day (see [Łukasiewicz 1922, 111]).

An examination of archival documents of Charles Peirce, Christine Ladd-Franklin, and others shows a tendency to reinforce these views, as well as for strong denigration of Russell and his work in logic during the years 1903 to 1913 among many of Russell’s contemporaries, especially, but not exclusively his older contemporaries. Ladd-Franklin, for example, wrote a note in which she complains that Russell (and Whitehead) wrote as if Peirce and Schröder had never existed, clearly insinuating that Russell plagiarized the work of Peirce and Schröder, while Peirce for his part found Russell’s work “nauseating,” and William James, in a letter to Peirce of 24 December 1909 ([James 1909]; quoted in

³The “Logic” (uppercase “L”) to which James referred was a logic textbook on which Peirce had been working. James was seeking—unsuccessfully—to convince a publisher to take on the text; the publisher rejected the text on the ground that it did not offer the latest treatment.

[Perry 1935, 680]) bluntly called Russell “ass,” saying he would prefer being “a-logical, if not illogical” than adopt Russell’s techniques.

Not until after the close of World War I, and thus roughly five years after the third volume of Whitehead and Russell’s *Principia Mathematica* appeared, that is, beginning in 1918, do we clearly notice the adoption of the estimation by the community logicians and historians and philosophers of logic at large that the logistic of Frege and Russell, and particularly the logical system of the *Principia*, is superior to, has surpassed, and ought to replace, the classical Boole-Schröder calculus, and that the latter is limited and in some measure obsolescent, if not obsolete. Thus it is in 1918 that Clarence Irving Lewis who, even in his solid exposition and historical outline of the development of the classical Boole-Schröder calculus, wrote of that calculus as a “classic” in the sense that it was becoming an antique (see [Lewis 1960, 118], in the abridged edition of his *Survey of Symbolic Logic* [Lewis 1918]). But Lewis could not and would not deny the close connection between the classical Boole-Schröder calculus and the emerging logistic of Russell. In his exposition of *Principia* in the second edition [Lewis 1960] of his *Survey*, as Karl Dürr reminded us [Dürr 1968, 112]. Lewis made the reader “aware of the close connection between the older and newer form of logistic,” even though the latter edition excised much of the discussion of Peirce’s contributions.

Contemporaneously, Georg Behrens wrote in *Die Prinzipien der mathematischen Logik bei Schröder, Russell und König* [Behrens 1917, 9–10] that Schröder’s *Vorlesungen über die Algebra der Logik* is a “special mathematical field which leans heavily upon logic,” whereas Russell studied mathematical deduction and, unlike Schröder, “developed a presentation of the logical calculus.” Borrowing a mediæval distinction which was used by Jean van Heijenoort [van Heijenoort 1967a], Schröder, like Boole, De Morgan, Peirce, and the algebraic logicians of the second half of the nineteenth-century, developed a *logica utens*, a specialized logic restricted to a specific universe of discourse, whereas Russell—and Frege—devised a one true logic, a *logica magna*, encompassing the entire universal domain, so that there is only one logic, the mathematical logic of *Principia*. And thus the classical Boole-Schröder calculus is reduced, depending upon one’s perspective, to either a primitive, preparatory stage on the way to mathematical logic, or a specialized case of mathematical logic, the class calculus, or to a mere interesting but minor sidelight, or even a dead-end, in the development of mathematical logic, rather than what it was to its practitioners—the

current state of the art of mathematical logic.⁴ In writing these lines, Behrens cited the “Summary” of *4 (at [Whitehead & Russell 1910–13, I, 114]) in the *Principia Mathematica* that “symbolic logic considered as a calculus had undoubtedly much interest on its own account; but in our opinion this aspect has hitherto been too much emphasized, at the expense of the aspect in which symbolic logic is merely the most elementary part of mathematics, and the logical prerequisite of all the rest.”⁵

In apparent reply to this Russellian claim, but actually *circa* 1897, in an untitled manuscript on Schröder’s logical algebra [Peirce n.d., ca. 1897b, MS 524:4–5], and, we might add, posthumously as well to Behrens’ interpretation of it in contrast with Schröder’s goal in treating the logic of relations in the *Vorlesungen*, Charles Peirce wrote that Schröder developed a calculus which “embraces all ordinary formal logic as nothing but an egregiously simple case.” Peirce immediately continued: “The logic of relations is, therefore, far from being a specialized branch of logic. On the contrary, it greatly enlarges and amplifies all logical conceptions . . .” [Peirce n.d., ca. 1897b, MS 524:2–4]. For Peirce and logicians like him at the end of the nineteenth century, then, the algebraic logic that they developed was, in the sense enumerated, a *logica magna*. Peirce’s remarks, although predating the statement of Whitehead and Russell and the statement of Behrens, as well as all those who thereafter adopted this “Russellian” conception, was typical of attitudes towards the algebraic logic of the nineteenth and early twentieth century of those who were working in the field until that time, and even up until the late 1910s. In that sense, Peirce’s statement may be said to be his anachronistic and posthumous reply to Russell, Behrens, and the followers of Russello-Fregean logic. Peirce’s views of the late 1890s were echoed even after the publication of Russell’s *Principles of Mathematics* in 1903, and if it was not the universal opinion among logicians of that day, it remained prevalent among Peirce’s associates and followers. Thus, for example, in their

⁴How and why this turn in point of view occurred is subject for another discussion, one with which I dealt in part elsewhere—see, e.g., [Anellis & Houser 1991] and [Anellis 1995]—and intend to develop further. Here my purpose is to document the reactions of Russell’s early contemporaries to Russell and his work in logic, basing the illustrations in largest measure upon contemporary, and especially archival, materials.

⁵Compare this with Jean van Heijenoort’s statement, in van Heijenoort’s “Preface” to *From Frege to Gödel* [van Heijenoort 1967b, vi] that although “Boole, De Morgan, and Jevons are regarded as the initiators of modern logic and rightly so,” and the period of their work “would, no doubt leave its mark on the history of logic, but would not count as a great epoch.”

article of 1905 on “Symbolic Logic,” Edward Huntington and Christine Ladd-Franklin wrote [Huntington & Ladd-Franklin 1905, 1] that “Symbolic Logic, or Mathematical Logic, or the Calculus of Logic, — called also the Algebra of Logic (Peirce), Exact Logic (Schröder), and Algorithmic Logic or Logistic (Couturat), — covers exactly the same field as Formal Logic in general . . .” and that these terms are quite synonymous. The implication too is that Russell’s contributions in the *Principles* are a contribution to the same logic to which Boole, De Morgan, Peirce, and Schröder had made contributions.

Notable among Russell’s early contemporaries who, in the last years of the nineteenth and first years of the twentieth century challenged Russell’s view of the primacy of Russell’s work in logic, and even of Russell’s self-appraisal of his contributions to logic, were Louis Couturat, Charles Peirce’s erstwhile student Christine Ladd-Franklin, Charles Peirce himself, and Charles Peirce’s friend and close ally, philosopher William James. Their comments on Russell and his work, both published and private, clearly illustrate that, in the view of many of Russell’s early contemporaries in that period between the publication of Russell’s *Principles of Mathematics* and of Whitehead and Russell’s *Principia Mathematica* [Whitehead & Russell 1910–13], and even for several years thereafter, Russell’s conception of logic not only as was not universally or immediately adopted, but that logicians of the period indeed often enough dealt particularly harshly with Russell and his work in logic.

Some salient illustrations of these negative reactions towards Russell and his work in logic, therefore, follow.

Charles Sanders Peirce and many his friends, students, and colleagues held that Russell held an exaggerated notion of his own importance and in particular of his contributions to logic. Many, including Peirce’s erstwhile Johns Hopkins University student Christine Ladd-Franklin, even went so far as to imply that Russell was plagiarizing the work of Peirce and Schröder. Neither she, nor Peirce, nor any of Peirce’s supporters would have been aware of Russell’s self-appraisal of his contributions, made in a letter to Ottoline Morrell dated 21 August 1912, in which Russell, referring to the *Principles of Mathematics*, wrote (as quoted in [Clark 1975, 189] and [Garciadiego 1991, 132]) that “mathematical philosophers have different thoughts from what they w[oul]d have if I had not existed.” Without of course knowing of this assertion of Russell’s, Harvard philosopher, logician, and Peirce correspondent Josiah Royce, reminiscing with Victor Fritz Lenzen (as reported in [Lenzen 1965, 4]) about the spring of 1914, complained that Russell had “received more attention than any logician since Aristotle.”

Of all the early contemporaries of Russell, Charles Peirce, whose work was broadly slighted and badly treated by Russell, had some of the harshest judgments amongst Russell's elders. While Russell, as we shall see momentarily, sought to take credit for having created the logic of relations, Peirce, in the third of his Lowell lectures of 1903 wrote [Peirce 1903b, MS 459:20], with respect to Whitehead's "On Cardinal Numbers" [Whitehead 1902] to which Russell contributed and which deals with elementary set theory in terms of the logic of relations, presented in Peaneque notation, and to Russell's *Principles of Mathematics*, that:

...quite recently Mr. Whitehead and the Hon[orable] Bertrand Russell have treated of the subject; but they seem merely to have pre[sented] truths already known into a uselessly technical and pedantic form.

Peirce also thought that, in contrast, he did a much better job of the enterprise, writing (as published in the Hartshorne & Weiss edition of his *Collected Papers* [Peirce 1934, 91]) that:

My analyses of reasoning surpasses in thoroughness all that has ever been done in print, whether in words or in symbols—all that De Morgan, Dedekind, Schröder, Peano, Russell, and others have done—to such a degree as to remind one of the differences between a pencil sketch of a scene and a photograph of it.

We know that Russell acquired a copy of Schröder's *Vorlesungen über die Algebra der Logik* in September 1900,⁶ and took notes on both that work and on Schröder's *Der Operationskreis des Logikkalküls* [Schröder 1877] and "Sur une extension de l'idée d'ordre" [Schröder 1901] in 1901 [Russell 1901b],⁷ as well as on Charles Peirce's "On the Algebra of Logic" [Peirce 1880] and "On the Algebra of Logic: A Contribution to the Philosophy of Notation" [Peirce 1885] in 1900–1901 [Russell ca. 1900–1901]. In the *Principles* [Russell 1903a, 10], Russell called Schröder's *Algebra* "the most complete account of non-Peanesque methods," thus seeming to agree with Peirce that the Boole-Schröder calculus, if not Peirce's own treatment and presentation of it, is by far the best and most advanced logic up to that time, excepting perhaps Peano's notational rendition; he would certainly *seem* to also thus have agreed with Peirce's assessment *circa* 1897 of the *Algebra* in the manuscript titled "Schröder's Logic of Relations"

⁶The date is written on the inside board of Russell's copy, to be found in the Russell Archives; see [Anellis 1990/91].

⁷See [Anellis 1990/91] for a sketch of Russell's notes on Schröder.

[Peirce n.d., ca. 1897a, MS 521:12] that “Prof. Schröder’s work is, and must for many years remain, the standard treatise upon exact logic” We know, of course, however, that Russell would *not* have agreed with Peirce on this latter score.

In any event, Russell had little else, and still less explicitly, to say about the contributions either of Peirce or of Schröder. Indeed, he wrote (in [Russell 1946, xv]) that he “read nothing of him [Peirce] until 1900, when I became interested in extending symbolic logic to relations, and learnt from Schröder’s *Algebra der Logik* that Peirce had treated of the subject.” This fully accords with the dates of Russell’s reading notes of Peirce and Schröder; yet it seems a remarkable confession, in light of Huntington’s retrospective remark in “New Sets of Independent Postulates for the Algebra of Logic, with Special Reference to Whitehead and Russell’s *Principia Mathematica*” [Huntington 1933, 278] that algebraic logic and universal algebra were “. . . extended by Schröder, and perfected by Whitehead,” and given that Russell (according to evidence from his own reading log for the years 1891–1902 [Russell 1891–1902]) read Whitehead’s *Treatise* almost as soon as it appeared, in March 1898, and should, had he read it, have found Whitehead’s expressions [Whitehead 1898, x] of indebtedness “in regard to Symbolic Logic to Boole, Schröder, and Venn.”

Russell, however, dismissed the work of Peirce and Schröder as negligible for the development of his own treatment of the logic of relations, saying that he had already completed his work in that field before he read theirs, telling French philosopher and logician Louis Couturat, for example, in a letter of 2 June 1903 [Russell 1903b, R57 = R58] that he read Schröder’s work only after learning of Peano, and that “it is not therefore essential to go through him.” This accords less well with Russell’s earlier recommendation to Couturat, in a letter dated 11 February 1899, that Couturat might find Peirce’s [Peirce 1883a] *Studies in Logic* of some interest. Russell went so far as to claim, in a letter to Philip Jourdain of April 1910 (as quoted in [Grattan-Guinness 1977, 133]) that it was he who “invented” “his” Logic of Relations during September 1900. He also asserted in a letter to Helen Thomas dated 31 December 1900 (as quoted in [Griffin 1992, 207, letter 91]) that in October of that year, preparing what became the *Principles*, that in writing it he “invented a new subject, which turned out to be all mathematics for the first time treated in its essence.”

How much, how carefully, and with what understanding, Russell read the works of Peirce and Schröder, have been matters of speculation.⁸ Christine Ladd-Franklin could, of course, not know what Russell read, or when he read it. But being familiar with Peirce's and Schröder's work as well as with Russell's *Principles*, was in a good position to compare them. In undated notes probably prepared for a Columbia University lecture class, probably *circa* 1913, she wrote [Ladd-Franklin n.d., ca. 1913]:

It should now be clear how the logic of *Principia* is related to the logic we have presented, following the materials of Peirce and Schröder But Whitehead and Russell plainly 'imply' that P[eirce] and S[chröder] were absolutely non-existent!

Charles Peirce held Russell's treatment of relations to be incomplete; in a marginal notation for p. 24, lines 25–28 in his copy of the *Principles*, for example, Peirce wrote: "He considers only dyadic relations." Most of Peirce's marginal notations and comments on Russell concern Russell's published criticisms of his [Peirce's] ostensible failure to distinguish types of collections and between class inclusion and material implication. His marginal annotations in his copy of Russell's *Principles* are littered with comments such as "not so" (p. 12, ll. 12–13), "utterly false" (p. 13, end ¶13), and "Ridiculous modes of formulation" (p. 16, end ¶18).⁹ It was in connection with a discussion of Cantor's [Cantor 1895–97] "Beiträge zur Begründung der transfiniten Mengenlehre" and Cantor's distinction between complete and incomplete totalities and between implication and class inclusion that Peirce wrote [Peirce 1903a, MS 459:19–20] for his third Lowell lecture in 1903 that "... quite recently Mr. Whitehead and the Hon. Bertrand Russell treated of the subject..." In the fifth of his Lowell Lectures, Peirce wrote [Peirce 1903c, MS 469:20] that what "puzzles the Hon. Bertrand

⁸Benjamin Hawkins thinks ([Hawkins 1992, 43–44], [Hawkins 1997, 137]) that Russell actually read very little of Peirce's work on logic, totaling the equivalent of approximately fourteen pages in the third volume of Peirce's *Collected Papers* [Peirce 1933a], merely skimming Peirce's [Peirce 1880] "On the Algebra of Logic" and [Peirce 1885] "On the Algebra of Logic: A Contribution to the Philosophy of Notation." Hawkins says that these "few pages . . . represent Russell's reading of Peirce" on logic, and supposes that that may, at least partially, explain Russell's inaccuracies regarding Peirce.

It should be remarked in comparing my discussion of Russell's reading of Peirce with Hawkins' that I had considerably fuller access to the Russell Archives than did Hawkins.

⁹[Lenzen 1965, 7–8] reported "a few critical remarks by Peirce in the margins" of Peirce's copy of Russell's [Russell 1903a] *Principles*.

Russell in his ‘Principles of Mathematics’ is whether a collection which has but a single individual member is identical with that individual,” and he attributes Russell’s puzzlement to his failure to make appropriate terminological and conceptual distinctions between these. Peirce himself emphasized to Cantor, in letters dated 21 December 1900 and 23 December 1900 (and published in [Peirce 1976, III/2], pp. 767–771, 772–779 respectively), the need to distinguish between sets and classes. It would seem that Russell’s criticisms of Peirce’s failure to distinguish between class inclusion and set membership was, therefore, if not entirely unfair, then at least disingenuous; for Peirce used the same symbol for both, as well as for other relational connectives, but made clear from the context in which he was working which relation he had in mind. The gist of Peirce’s marginalia to his copy of the *Principles* was that Russell’s difficulties with Peirce’s and Schröder’s ostensible lack of proper distinctions was rooted in Russell’s own failure to distinguish material implication and truth-functional implication (conditionality), and in Russell’s erroneous attempt to treat classes, in function-theoretic terms, as individual entities.

Russell’s criticisms of Peirce as failing to distinguish set membership from class inclusion, first found in Russell’s [Russell 1901a] “Sur la logique des relations avec des applications à la théorie des séries,” is revived, in slightly different guise, in the *Principles*, where Russell asserts at once [Russell 1903a, 24] that, while Peirce and Schröder admittedly “realized the great importance of the subject” of the logic of relations, also nonetheless “unfortunately their methods, being based, not on Peano, but on the older Symbolic Logic derived (with modifications) from Boole, are so cumbrous and difficult that most of the applications which ought to be made are practically not feasible,” and he then levels the criticism [Russell 1903a, 24] that: “In addition to the defects of the old Symbolic Logic, their method suffers technically . . . from the fact that they regard a relation essentially as a class of couples, thus requiring elaborate formulæ of summation for dealing with single relations.” In a letter to Philip Jourdain of 15 April 1910 [Russell 1910], Russell (as quoted in [Grattan-Guinness 1977, 134]) went so far as to call Schröder’s methods “hopeless.” In reply to Russell’s assertion [Russell 1903a, 26] that

Peirce and Schröder consider what they call the relative sum of two relations R and S , which holds between x and z , when, if y be any other term whatever, either x has y to the relation R , or y has to z the relation S . This is a complicated notion, which I have found no occasion to

employ, and which is introduced only in order to preserve the duality of addition and multiplication,

Peirce in a letter to Victoria Welby of 12 October 1904 (see p. 30 in [Hardwick 1977, 25–35]), stated that:

As to my algebra of dyadic relations, Russell in his book which is superficial to nauseating to me, has some silly remarks about my “relative addition” etc., which are mere nonsense. He says, or Whitehead says, that the need for it *never* occurs if you bring in the same mode of connection in any other way. It is part of a system which does not bring in the mode of connection in any other way. In that system it is indispensable. But let us leave Russell and Whitehead to work out their own salvation.¹⁰

Shearman [Shearman 1906, 199ff.] followed Russell on this point. Lewis ([Lewis 1918], [Lewis 1960, 102]), however, showed how, in “Upon the Logic of Mathematics” [Peirce 1867],

in respect both to addition and to multiplication, Peirce has here hit upon the same fundamental idea by means of which arithmetical operations are defined in *Principia Mathematica* [vol. II, §A]. The “second intention” of a class term is, in *Principia* $Nc'a; a + b$, in Peirce's discussion, corresponds to what is there called the “arithmetical sum of two logical classes, and $a \times b$ to what is called the “arithmetical product”.

Russell was made aware of Peirce's remark by Welby. Welby, who sought to promote a dialogue between Russell and Peirce, must have passed Peirce's letter along to Russell, since Russell mentions it on the

¹⁰Peirce was aware of the fact that Russell contributed to [Whitehead 1902] and probably suspected Whitehead's hand in [Russell 1903a]. He may have been led to implicate Whitehead in the writing of the *Principles* by the strength of Russell's acknowledgement of Whitehead in the “Preface” [Russell 1903a, xviii], according to which: “At every stage of my work, I have been assisted more than I can express by the suggestions, the criticisms, and the generous encouragement of Mr. A. N. Whitehead,” by the knowledge that the collaboration between Whitehead and Russell went as far back at least to 1902, and by a presumption (evidenced by various unflattering remarks on Russell's lack of mathematical sophistication) that Russell would not have been mathematically capable of engaging on his own in such an enterprise. Another likely factor may have been Whitehead's remarks [Whitehead 1898, 3, 10, 37, 42, 115–116] and [Whitehead 1901, 139], echoed by Russell in the *Principles*, that Peirce's relations are “obscure,” and his expression [Whitehead 1902, 367–368, 378–382] of preference for Russell's notation over Peirce's.

first page of his letter to Welby of November 14, 1904 [Russell 1904b]. He responds, in a letter to Welby of 27 December 1904 [Russell 1904c] that he “does not know where Whitehead or I have said that the need of Dr. Peirce’s Algebra of dyadic relations seldom occurs,” and adds that he thinks that “a symbolism based on Peano’s is practically more convenient, but I hold it quite essential to have a method of expressing relations, & I have always thought very highly of Dr. Peirce for having introduced such a method.”

Given numerous opportunities to respond directly to Russell in print, Peirce proved reluctant to do so. In private, his remarks concerning Russell and Russell’s work in logic were negative. Asked to review the *Principles* for the journal *Science*, Peirce delayed for over year; he told Christine Ladd-Franklin in a letter of 27 July 1904 [Peirce & Ladd-Franklin 1891–1908, MS L237:27] that: “. . . a year has passed since I agreed to notice Russell’s vol. I, and I feel its pretentiousness so strongly that I cannot fail to express it in a notice.” In a similar vein, he wrote (as quoted in [Peirce 1976, III/2, 78]) to Australian philosopher Frederick William Frankland on May 8, 1906 that: “In my opinion Russell and Whitehead are blunderers constantly confusing different questions.” In a letter to Welby of 1 December 1903 [Peirce 1903d], Peirce concluded, from a cursory reading of the *Principles*, that “whatever merit it may have as a digest of what others have done, it is pretentious and pedantic — attributing to its author merit that cannot be accorded him.”

Here is the first suggestion that the *Principles* was not to be esteemed an original work, the first suggestion of possible plagiarism. The theme of Russell’s lack of originality will be found elsewhere as well. It is, indeed, a clear example of damning by faint praise.

Christine Ladd-Franklin also raised the question of Russell’s lack of originality, more forcefully than did Peirce himself; she wrote, as already cited, that Russell and Whitehead “plainly ‘imply’ that P[eirce] and S[chröder] were absolutely non-existent!” Josiah Royce and Fergus Kernan [Royce & Kernan 1916, 706–707] have likewise asserted, albeit somewhat less blatantly suggesting plagiarism, concerning “. . . Peirce’s researches on the algebra of logic, and in particular the logic of relatives,” that “many of the most recent researches, including those of Bertrand Russell, are still due to his,” Peirce’s, “influence, although Russell, . . . has a some-what inadequate sense of his own generally indirect indebtedness to Peirce’s work in this field.”

It would not be difficult to find examples of Russell’s failure to assign proper credit. The so-called “principle of reduction” given as axiom 10 in the *Principles* [Russell 1903a, §18, p. 17], for example, according to

which $((p \supset q) \supset p) \supset q$ (when $p \supset p$ and $q \supset q$), is exactly the “fifth icon” which is to be found in Peirce’s “On the Algebra of Logic: A Contribution to the Philosophy of Notation” [Peirce 1885, p. 189], and which we today call Peirce’s Law.

Ladd-Franklin was among those who sought to prod Peirce into replying to Russell; thus she wrote in a letter to Peirce of 24 July 1904 [Ladd-Franklin 1904]:

Do tell me how it strikes you—all this recent work of Bertrand Russell, Peano, Couturat & their school, which they make so much of. Don’t you think that they exaggerate both its originality & its importance? Are you not going to write something on the subject?

Similar requests were to be had from Eliakim Hastings Moore, editor of the American Mathematical Society’s *Transactions*, who on 14 October 1902 wrote to Peirce [E. Moore 1902], asking him to respond to Whitehead’s [Whitehead 1901] article “On Cardinal Numbers,” and in particular to compare his own work with the work of Whitehead and Russell, noting that Whitehead and Russell wrote in Peano’s notation and used “Russell’s additions on the algebra of relations in general. . . . It would give me much pleasure if you would let me know what you think concerning Russell’s work, especially in comparison with your own” When Peirce failed to respond to Moore, Moore again asked Peirce, in a letter of 31 December 1903 [E. Moore 1903] what he thought of Russell’s *Principles of Mathematics*.

When, finally Peirce did write on the *Principles* [Peirce 1903a], it was in conjunction with Welby’s [Welby 1903] *What is Meaning?*, in which Peirce lightly dismissed the *Principles*, merely acknowledging that “the severe and scholastic labors” that went into it “bespeaks a grit and industry, as well as a high intelligence,” while adding that: “Whoever wishes a convenient introduction to the remarkable researches into the logic of mathematics that have been made during the last sixty years . . . will do well to take up this book.” Peirce’s remarks, he confessed to Welby (as quoted in [Hardwick 1977, 9]), were intended to serve as a contrast between the two books, and to strongly hint that the *Principles* was quite unoriginal. Not surprisingly, in a letter to William James of 25 December 1909, Peirce (as quoted in [Peirce 1976, III/2, 867–877]; see esp. pp. 873–874) wrote of his own work on the logic of relatives that it “simply revolutionizes Logic” and that it “ought to be the Logic of the Future.”

Louis Couturat, a younger contemporary of Russell’s in the sense that he was more nearly Russell’s age, who was early on one of Russell’s

staunchest supporters and best interpreters for French philosophers, and one of those not associated personally with Peirce, in his *L'algebra de la logique* ([Couturat 1905]; see English translation: [Couturat 1914, 92]), wrote that the algebra of logic “ought . . . to develop into a logic of relations, which LEIBNIZ foresaw, which PEIRCE and SCHRÖDER founded, and which PEANO and RUSSELL seem to have established on definite foundations,” thus evidently sharing with Arthur Shearman, Giuseppe Peano, and Paul Carus, among many others, the view that the work of Russell, Peano, and Frege belonged within, and indeed was inseparable from, the work of the older algebraic logicians, that the logic that was to be found in works such as Russell’s *Principles of Mathematics* was a refinement and continuation of the work that had led to, and was presented as the most contemporary state of, the classical Boole-Schröder calculus. I interpret Couturat’s remark here to assert that Peano and Russell (along with Frege, we might add), set a foundation underneath the classical Boole-Schröder calculus, rather than replaced it with an altogether new symbolic logic. Elsewhere, Couturat felt compelled to explicitly warn Russell against the denigration of the Boole-Schröder calculus to the profit of Peano’s mathematical logic, writing, in a letter to Russell dated 27 January 1901 [Couturat 1901], that:

...one must not deprecate the Boole-Schröder system and sacrifice it to Peano: it has its goal and its use in pure logic just as Peano’s has its in Math. And the theory of logical equations has great importance since all logical problems amount to a system of equations and inequations.

Russell nevertheless dismissed Schröder’s work by telling Couturat, in a letter of 9 June 1903 [Russell 1903c], that Schröder speaks “prose” without knowing it.

Norbert Wiener’s assertions, in his doctoral dissertation for Harvard University in 1913, offering a comparison between the logical systems of Schröder’s *Vorlesungen über die Algebra der Logik* and Whitehead and Russell’s *Principia Mathematica*, in which Wiener asserts that essentially the two systems are equal in expressive power, and the discussions which Wiener had held with Russell about his claims [Wiener 1913], has already been treated by Ivor Grattan-Guinness [Grattan-Guinness 1975]; moreover, Geraldine Brady [Brady 2000] has reproduced salient and extensive excerpts, including the introductory and concluding chapters, of Wiener’s thesis, and I need not here repeat any but the main point: namely, that Wiener regarded the two systems

as essentially equivalent, and Russell's retort: namely that Wiener considered only "the more conventional parts of *Principia Mathematica*" (see [Grattan-Guinness 1975, 130]). It is instructive, however, to consider the pasigraphy discussion at the end of the nineteenth century between Peano and Schröder in light of the same "discussion," a few years later, between Russell and Wiener.

Of all the criticisms leveled against Frege's *Begriffsschrift*, the one that continually recurred was that cumbrousness of the notation (this charge is repeated in every review of Frege's [Frege 1879] *Begriffsschrift* booklet; see [Frege 1972]). The core of the issue for Frege and Schröder, and later for Schröder and Peano, was not the notation so much as the nature and purpose of the enterprise at hand—in van Heijenoort's terminology, of logic as calculus and logic as language [van Heijenoort 1967a] (see [Frege 1880/81], [Frege 1882], [Frege 1895], [Frege 1897], [Schröder 1898a], [Schröder 1898b] for the original discussions and [Peckhaus 1990/91] for an analysis). While Russell was asserting that the logic of relations of Peirce and Schröder was "cumbrous," ineffectual, and even "hopeless," as compared with that of Peano, Schröder and Peirce defended their notation. We have already heard Peirce call Russell's treatment "uselessly technical and pedantic" [Peirce 1903b, MS 459:20] and compare his own treatment to a photograph while others', Russell's included, could best be compared to a mere "pencil sketch" [Peirce 1934, 91], while Russell reciprocated (as quoted in [Grattan-Guinness 1977, 134]) by calling Schröder's methods "hopeless." We have also heard Russell say of the algebraic logicians that their methods are "cumbrous and difficult" and that much that is required of it is "practically not feasible" [Russell 1903a, 24]; he wrote, in his letter to Welby of 11 November 1904 [Russell 1904a, 2–3] that Peirce and Schröder's logical methods are inferior to such a degree that they cannot answer "many fundamental questions (fundamental, I mean, to the foundations of mathematics & the principles of symbolism)" Whitehead became convinced by Russell that Peano's notation was indeed superior, and soon adopted it to his own use. In the *Treatise on Universal Algebra* [Whitehead 1898, 3, 10, 37, 42, 115–116], and the "Memoir on the Algebra of Symbolic Logic" [Whitehead 1901, 139] he called Peirce's relations "obscure," and in "On Cardinal Numbers" [Whitehead 1902, 367–368, 378–382] expressed his preference for Russell's Peanesque notation for relations over Peirce's. E. H. Moore, in his efforts to elicit a response from Peirce to Whitehead's [Whitehead 1902] article, wrote on 14 October 1902 [E. Moore 1902] that it was written in Peano's notation and uses Russell's "additions on the algebra of relations in general. The Italian

school believe in the new symbolism as a calculus in terms of which it is highly convenient to work, and not merely as an algebra of mathematical logic.” Schröder’s paper on pasigraphy, its present state and the pasigraphic movement in Italy” [Schröder 1898a, Schröder 1898b] was intended as a response to the challenge from Peano’s notation, arguing that Peano’s symbolism did not appear to be capable of expressing relatives.¹¹ Peirce, possibly in response to Russell’s condescending remark in the *Principles* that Schröder’s *Algebra* is “the most complete account of non-Peanesque methods” and to comments in Whitehead’s [Whitehead 1902] “On Cardinal Numbers” on the Peano notation, asserted that: “Such ridiculously exaggerated claims have been made for Peano’s system, though not, as far as I am aware, by its author, that I shall prefer to refrain from expressing my opinion of its value” (see [Peirce 1933b, 514]).

Cumulatively, the views of these logicians and others suggest that there was nothing particularly inevitable about the predominance which Russell or his conception of logic, or, for that matter, his conception of his ostensibly singular contributions to the development of symbolic logic, would come to enjoy in later years. For Peirce, *circa* 1897, for example, “Prof. Schröder’s work [*Algebra*] is, and must for many years remain, the standard treatise upon exact logic ...” [Peirce n.d., ca. 1897a, MS 521:12].

That Russell was aware of the criticisms of him by Peirce and Peirce’s supporters is attested to by the fact that he expressed the hope, in his letter to Welby of 11 November 1904 [Russell 1904a, 2–3], that the second volume of the *Principles* which he planned “will do much to persuade such opponents as Mr. C. S. Peirce” of the correctness of his own views and position.

One can quote (as we already have), a number of logicians, mathematicians, and philosophers in the last years of the nineteenth century and into the second decade of the twentieth who found the work of Peirce and/or Schröder to be still at the forefront of logical research; among them William James, Edwin Bidwell Wilson, Josiah Royce, William Clifford, while Peirce and Schröder formed something of a mutual admiration society in the late nineteenth century.¹² Whether their *fin de siècle* assessments were correct or not, it is clear that there

¹¹See [Peckhaus 1989, 1] and [Peckhaus 1990/91] on Schröder’s purpose in writing about pasigraphy as a rejection of Peano’s notation. [Peckhaus 1989] shows that Schröder’s reading of Peirce’s [Peirce 1883b] “Logic of Relatives” convinced him of the efficacy of Peirce’s treatment.

¹²See, e.g. [Houser 1990/91] on Peirce and Schröder, as seen through their correspondence.

was little certainty or unanimity regarding the accomplishments of Russell, either in *The Principles of Mathematics*, nor yet even in his and Whitehead's *Principia Mathematica*, at the time they appeared, and that Peirce and Schröder were not yet easily dismissed as having fallen into oblivion or obsolescence.

It should not be supposed, however, that Russell was entirely without supporters, some quite influential, in the years under consideration. We know, for example, that David Hilbert felt frustrated that the hostilities of World War I thwarted his plans to have Russell visit Göttingen (see [Reid 1986]). The anonymous reviewer of the first volume of Whitehead and Russell's *Principia*—since identified to be none other than the authors' colleague Cambridge University mathematician Godfrey Harold Hardy—was enthusiastic [Hardy 1910].

By the middle of the 1920s, however, the views expressed by Behrens in 1917 and 1918 and others came to predominate, and even those who held that the *Principia Mathematica* did not, yet, provide a formal logic sufficient for the complete development of mathematics, thought that Russell's conception of logic and mathematics and their relationship would be the basis for any future work. Thus, for example, Oswald Veblen, in his "Remarks on the Foundations of Geometry," based on his American Mathematical Society presidential address of 1924, prognosticated, from the standpoint of the foundations of mathematics [Veblen 1925, 141], that: "In the process of constructing [such a foundation] we are likely to adopt the Russell point of view that mathematics is coextensive with logic."

As we contemplate the marked shift in attitude that we detect in the work and minds of logicians between the years that saw publication of Schröder's *Vorlesungen über die Algebra der Logik* at one extreme and of Whitehead and Russell's *Principia Mathematica* at the other, and

Writing to Paul Carus and to Peirce himself, Schröder compared Peirce with Aristotle and Leibniz (see Schröder's letter to Carus of 6 March 1893, Robin catalog #L392 [Schröder 1893] and letter of 16 February 1896 to Peirce, Robin catalog MS L421 [Schröder 1896], quoted in [Fisch 1972, 487, 488] and [Fisch 1986, 251] and [Houser 1990/91] and [Houser 1997, 4], and a letter to Paul Carus, MS L392, 6 March 1893, quoted by [Fisch 1972, 487, 488] and [Fisch 1986, 251] and [Houser 1997, 4]) that "however ungrateful [Peirce's] countrymen and contemporaries might prove, [Peirce's] fame would shine like that of Leibniz and Aristoteles into all the thousands of years to come . . .," while Peirce wrote in 1903 (published in [Peirce 1976, III/1, 347]) that: "I am not so in love with my own system as the late Professor Schröder was," and telling Victoria Welby, in a letter of 12 October 1904 (as quoted in [Hardwick 1977, 29]) that "my friend Schröder fell in love with my algebra of dyadic relations. The few pages I gave to it in my [1883]," referring to his "Logic of Relatives" [Peirce 1883b], "were proportionate to its importance."

the attitudes that we detect in the years following the close of World War I, we might consider Lewis Mumford's closing thoughts on the values, achievements, and significance of what he called the 'brown decades' of 1865 to 1895 as he considered the cultural and intellectual climate of those decades: he wrote [Mumford 1931, p. 113]: "A definite change in our life took place around 1895, and there is something in back of it that is lost in a mere account of things, forces, machinery, institutions, events: something that eludes us and yet seems to hold a clue. Perhaps it was only a colour. But what was valid in the art and thought of the Brown Decades did not cease to exist, even though it was temporarily forgotten." He then asks and answers his own two salient questions: Does their work—the work of those who produced and created in the 'brown decades'—lead towards our own? In part, at least, it certainly does. Does it lead to work even greater than our own? We can only hope so!

REFERENCES

ARCHIVES

- [RA] Bertrand Russell Archives, Ready Archives, Mills Memorial Library, McMaster University, Hamilton, Ontario.
- [CPA] Charles Peirce Archives, Houghton Library, Harvard University; microfilm copy at Peirce Edition Project, Indiana University-Purdue University at Indianapolis.
- [LFP] Ladd-Franklin Papers, Rare Book and Manuscript Library, Columbia University.
- [WJC] The William James Collection, Houghton Library, Harvard University.
- [NWP] Norbert Wiener Papers, Harvard University and Massachusetts Institute of Technology.

BIBLIOGRAPHY

- [Anellis 1990/91] Anellis, Irving H. 1990/91. "Schröder Material at the Russell Archives," *Modern Logic* **1**, 237–247.
- [Anellis 1995] ———. 1995. "Peirce Rustled, Russell Pierced: How Charles Peirce and Bertrand Russell Viewed Each Other's Work in Logic, and an Assessment of Russell's Accuracy and Rôle in the Historiography of Logic," *Modern Logic* **5**, 270–328.
- [Anellis & Houser 1991] Anellis, Irving H. & Nathan R. Houser. 1991. "Nineteenth Century Roots of Algebraic Logic and Universal Algebra," in Hajnal Andréka, J. Donald Monk, and István

- Németi (editors), *Algebraic Logic* (Proceedings of a Conference in Budapest, 1988), *Colloquia Mathematica Societatis Janós Bolyai* **54** (Amsterdam/Oxford/New York: North-Holland), 1–36.
- [Behrens 1917] Behrens, Georg. 1917. “Die Prinzipien der mathematischen Logik bei Schröder, Russell und König.” Ph.D. thesis, Universität Hamburg, 1917; Hamburg: Berngruber und Henning; Kiel.
- [Bôcher 1904] Bôcher, Maxime. 1904. “The Fundamental Conceptions and Methods of Mathematics,” *Bulletin of the American Mathematical Society* (1) **11**, 115–135.
- [Brady 2000] Brady, Geraldine. 2000. “From Peirce to Skolem: A Neglected Chapter in the History of Mathematical Logic.” Amsterdam/New York/London: Elsevier Science, North-Holland.
- [Bynum 1972] Bynum, Terrell Ward. 1972. “On the Life and Work of Gottlob Frege,” in [Frege 1972, 1–54].
- [Cantor 1885] Cantor, Georg. 1885. Review of [Frege 1884], *Deutsche Literaturzeitung* **6**, nr. 20, Spalte 728–729; reprinted: Georg Cantor, (Ernst Zermelo, ed.), *Gesammelte Abhandlungen mathematischen und philosophischen Inhalts*. Berlin: Julius Springer, 1932; reprinted: Hildesheim: Olms, 1966, 440–441.
- [Cantor 1895–97] ———. 1895–97. “Beiträge zur Begründung der transfiniten Mengenlehre,” *Mathematische Annalen* **46** (1895), 481–512, **49** (1897), 207–246.
- [Carus 1910] Carus, Paul. 1910. “The Nature of Logical and Mathematical Thought,” *The Monist* **20**, 33–75.
- [Clark 1975] Clark, Ronald W. 1975. *The Life of Bertrand Russell*. London: Jonathan Cape & Weidenfeld and Nicolson.
- [Couturat 1901] Couturat, Louis. 1901. Letter to Bertrand Russell, 27 January 1901; RA: MS C3.
- [Couturat 1904] ———. 1904. Comptes rendu de [Russell 1903a], *Bulletin des Sciences mathématiques* (2) **28**, 129–147.
- [Couturat 1905] ———. 1905. *L’algèbre de la logique*. Paris: Gauthier-Villars.
- [Couturat 1914] ———. 1914. *The Algebra of Logic* (Lydia G. Robinson, translator). Chicago/London:

- Open Court. (English translation of the previous.)
- [Dürr 1968] Dürr, Karl. 1968. "Lewis and the History of Logic," in Paul Arthur Schilpp (editor), *The Philosophy of C. I. Lewis* (Lasalle: Open Court Press), 89–114.
- [Färber 1905] Färber, Carl. 1905. Review of [Frege 1903], *Jahrbuch über die Fortschritte Mathematik* **34**, 71–72.
- [Fisch 1972] Fisch, Max Harold. 1972. "Peirce and Leibniz," *Journal of the History of Ideas* **33**, 485–496; reprinted: [Fisch 1986].
- [Fisch 1977] ———. 1977. Letter to Kenneth Blackwell, 23 April 1977; RA; copy CPA.
- [Fisch 1986] ———. 1986. (Kenneth Lane Ketner & Christian J. W. Kloesel, editors), *Peirce, Semiotics, and Pragmatism*. Bloomington: Indiana University Press.
- [Frege 1879] Frege, Gottlob. 1879. *Begriffsschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens*. Halle: L. Nebert; English translation by Stefan Bauer-Mengelberg; [van Heijenoort 1967b], 1–82.
- [Frege 1880/81] ———. 1880/81. "Booles rechnende Logik und die Begriffsschrift," in [Frege 1969], 9–52; English translation in [Frege 1979], 9–46.
- [Frege 1882] ———. 1882. "Booles logische Formelsprache und meine Begriffsschrift," in [Frege 1969], 53–59; English translation: [Frege 1979], 47–52.
- [Frege 1884] ———. 1884. *Die Grundlagen der Arithmetik*. Breslau: Verlag von Wilhelm Koebner.
- [Frege 1893] ———. 1893. *Grundgesetze der Arithmetik. Begriffsschriftlich abgeleitet*, Bd. I. Jena: H. Pohle.
- [Frege 1895] ———. 1895. "Kritische Beleuchtung einiger Punkte in E. Schröders Vorlesungen über die Algebra der Logik," *Archiv für systematische Philosophie* **1**, 433–465; reprinted: [Frege 1966], 92–112; English translation in [Frege 1984], 210–228.
- [Frege 1897] ———. 1897. "Über die Begriffsschrift des Herrn Peano und meine einige," *Verhandlungen der Königl. Sächsischen Gesellschaft der Wissenschaften zu Leipzig. Math.-Phys. Klasse* **48**, 362–378; reprinted: English translation; [Frege 1984], 234–248.

- [Frege 1903] ———. 1903. *Grundgesetze der Arithmetik. Begriffsschriftlich abgeleitet*, Bd. II. Jena: H. Pohle.
- [Frege 1966] ———. 1966. (Günther Patzig, hsg.), *Logische Untersuchungen*. Göttingen: Vandenhoeck & Ruprecht.
- [Frege 1969] ———. 1969. (Hans Hermes, Friedrich Kambartel, Friedrich Kaulbach, hsg.), *Nachgelassene Schriften*, Hamburg: Felix Meiner Verlag.
- [Frege 1972] ———. 1972. (Terrell Ward Bynum, editor & translator), *Conceptual Notation, and Related Articles*. Oxford: Clarendon Press.
- [Frege 1979] ———. 1979. (Peter Long, Robert White, translators), *Posthumous Writings*. Chicago: University of Chicago Press.
- [Frege 1984] ———. 1984. (Brian F. McGuinness, translator; Peter T. Geach, editor), *Collected Papers on Mathematics, Logic, and Philosophy*. Oxford: Basil Blackwell.
- [Garciadiego 1991] Garciadiego, Alejandro. 1991. "Bertrand Russell's Emotional State circa 1901–1902," in Hardy Grant, Israel Kleiner, & Abe Shenitzer (editors), *Proceedings of the Canadian Society for History and Philosophy of Mathematics/Société Canadienne d'Histoire et Philosophie des Mathématiques*, vol. 4, Seventeenth Annual Meeting, Queen's University, Kingston, Ontario, May 27–29, 1991, 131–143.
- [Grattan-Guinness 1975] Grattan-Guinness, Ivor. 1975. "Wiener on the Logics of Russell and Schröder: An Account of His Doctoral Thesis, and of His Discussion of It with Russell," *Annals of Science* **32**, 103–132.
- [Grattan-Guinness 1977] ———. 1977. *Dear Russell — Dear Jourdain: A Commentary on Russell's Logic, based on His Correspondence with Philip Jourdain*. New York: Columbia University Press; London: Duckworth.
- [Griffin 1992] Griffin, Nicholas. 1992. (Editor.) *The Selected Letters of Bertrand Russell*, vol. I: *The Private Years, 1884–1914*. Boston/New York/London: Houghton-Mifflin.
- [Hardwick 1977] Hardwick, Charles S. 1977. (Editor.) *Semiotic and Significs: The Correspondence between Charles S. Peirce and Victoria Lady Welby*. Bloomington/ London: Indiana University Press.

- [Hawkins 1992] Hawkins, Benjamin Sanford, Jr. 1992. "Peirce and Russell: The History of a Neglected 'Controversy'"; preprint, ts., 74pp.
- [Hawkins 1993] ———. 1993. "Peirce and Frege, A Question Unanswered," *Modern Logic* **3**, 376–383.
- [Hawkins 1997] ———. 1997. "Peirce and Russell: The History of a Neglected Controversy," in Nathan Houser, Don D. Roberts & James Van Evra (editors), *Studies in the Logic of Charles Sanders Peirce* (Indianapolis/Bloomington: Indiana University Press), 111–146.
- [Hoppe 1879] Hoppe, Ernst Eduard Reinhold. 1879. Review of [Frege 1879], *Archiv der Mathematik und Physik* (ser. 1) **63**, *Literarischer Bericht CCLII*, 44–45. English translation by Terrell Ward Bynum: [Frege 1972], 209–210.
- [Houser 1990/91] Houser, Nathan. 1990/91. "The Peirce-Schröder Correspondence," *Modern Logic* **1**, 206–236.
- [Houser 1997] ———. 1997. "Introduction: Peirce as Logician," in Nathan Houser, Don D. Roberts, & James W. Van Evra (eds.), *Studies in the Logic of Charles Sanders Peirce*. Indianapolis/Bloomington: Indiana University Press), 1–22.
- [Huntington 1933] Huntington, Edward V. 1933. "New Sets of Independent Postulates for the Algebra of Logic, with Special Reference to Whitehead and Russell's Principia Mathematica," *Transactions of the American Mathematical Society* **35**, 274–304.
- [Huntington & Ladd-Franklin 1905] Huntington, Edward V. & Christine Ladd-Franklin. 1905. "Symbolic Logic," *Encyclopedia Americana*, vol. 9 (New York: Americana Company); reprint, 6pp.
- [Husserl 1891] Husserl, Edmund G. A. 1891. *Philosophie der Arithmetik. Psychologisches und logische Untersuchungen*. Halle: Pfeiffer.
- [James 1909] James, William. 1909. Letter to C. S. Peirce, 24 December 1909; (WJC) letter 3418.
- [Johnson 1905] Johnson, William Ernest. 1905. "Report to the Sub-Committee on Higher Degrees of the Cambridge University Special Board on Mathematics," *Cambridge Register* **28**.
- [Jourdain 1914] Jourdain, Philip Edward Bertrand. 1914. "Preface" to [Couturat 1914], iii–x.

- [Kennedy 1973] Kennedy, Hubert C. 1973. "What Russell Learned from Peano," *Notre Dame Journal of Formal Logic* **14**, 367–372.
- [Kennedy 1975] ———. 1975. "Nine Letters from Giuseppe Peano to Bertrand Russell," *History and Philosophy of Logic* **13**, 205–220.
- [Ketner 1987] Ketner, Kenneth Lane. 1987. Introductory essay, "Charles Sanders Peirce," in J. J. Stuhr (editor), *Classical American Philosophy: Essential Readings and Interpretive Essays* (New York/Oxford: Oxford University Press), 13–31.
- [Ladd-Franklin 1904] Ladd-Franklin, Christine. 1904. Letter to Charles Peirce, 24 July 1904 (Robin catalog L237:201–202); (CPA).
- [Ladd-Franklin n.d., ca. 1913] ———. N.d., ca. 1913. "Methods of ..."; unidentified ms., 1p., n.d., ca. 1913; re: Whitehead & Russell in *Principia* on Peirce & Schröder; (LFP).
- [Lasswitz 1879] Lasswitz, Kurd. 1879. Rezension von G. Frege, "Begriffsschrift," *Jenaer Literaturzeitung* **16**, 248–249. English translation by Terrell Ward Bynum: [Frege 1972], 210–212.
- [Lenzen 1965] Lenzen, Victor Fritz. 1965. "Reminiscences of a Mission to Milford, Pennsylvania," *Transactions of the Charles S. Peirce Society* **1**, 3–11.
- [Lewis 1918] Lewis, Clarence Irving. 1918. *A Survey of Symbolic Logic, the Classic Algebra of Logic*. Berkeley: University of California Press.
- [Lewis 1960] ———. 1960. *A Survey of Symbolic Logic, the Classic Algebra of Logic*. New York: Dover Publications, abridged and corrected edition of [Lewis 1918].
- [Łukasiewicz 1922] Łukasiewicz, Jan. 1922. "On Determinism," in Jan Łukasiewicz (Ludwik Borkowski, ed.), *Selected Works* (Amsterdam/London: North-Holland, 1970), 110–128.
- [Michaëlis 1880] Michaëlis, Carl Theodor. 1880. Review of [Frege 1879], *Zeitschrift für Volkerpsychologie und Sprachwissenschaft* **12**, 232–240; English translation by Terrell Ward Bynum: [Frege 1972], 212–218.
- [Michaëlis 1881] ———. 1881. Review of [Frege 1879], *Jahrbuch über die Fortschritte der Mathematik* **11**, 48–49.

- [Michaëlis 1896] ———. 1896. Review of [Frege 1893], *Jahrbuch über die Fortschritte der Mathematik* **25**, 101–102.
- [E. Moore 1902] Moore, Eliakim Hastings. 1902. Letter to Charles Peirce, 14 October 1902; (Robin catalog MSL299:15); (CPA).
- [E. Moore 1903] ———. 1903. Letter to Charles Peirce, 31 December 1903; (Robin catalog MSL299:43–46); (CPA).
- [G. Moore 1988] Moore, Gregory H. 1988. “The Emergence of First-Order Logic,” William Aspray & Philip Kitcher (editors), *History and Philosophy of Modern Mathematics* (Minneapolis: University of Minnesota Press), 95–135.
- [Mumford 1931] Mumford, Lewis. 1931. *The Brown Decades: A Study of the Arts in America, 1865–1895*. New York: Harcourt, Brace & Co.; New York: Dover Publications, 2nd rev. ed., 1955.
- [Nidditch 1963] Nidditch, Peter Harold. 1963. “Peano and the Recognition of Frege,” *Mind* **72**, 103–110.
- [Peano 1889] Peano, Giuseppe. 1889. *Arithmetices principia nova methodo exposita*. Torino: Bocca; English translation by Jean van Heijenoort in Jean van Heijenoort (editor), *From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931* (Cambridge, Mass.: Harvard University Press, 1967), 84–97; English translation by Hubert C. Kennedy in Hubert C. Kennedy (editor), *Selected Works of Giuseppe Peano* (Toronto/Buffalo: University of Toronto Press, 1973), 101–134.
- [Peano 1895] ———. 1895. Review of [Frege 1893], *Rivista di Matematica* **5**, 122–128.
- [Peckhaus 1989] Peckhaus, Volker. 1989. Letter to Irving H. Anellis, 22 May 1989, ts., 3pp.
- [Peckhaus 1990/91] ———. 1990/91. “Ernst Schröder und die ‘pasigraphischen System’ von Peano und Peirce,” *Modern Logic* **1**, 173–205.
- [Peirce 1867] Peirce, Charles Sanders. 1867. “Upon the Logic of Mathematics,” *Proceedings of the American Academy of Arts and Sciences* **7**, 402–412.
- [Peirce 1880] ———. 1880. “On the Algebra of Logic,” *American Journal of Mathematics* **3**, 15–57.
- [Peirce 1883a] ———. 1883a. (Editor.) *Studies in Logic*. Boston: Little, Brown & Co.; reprinted: Baltimore: Johns Hopkins University Press, 1983.

- [Peirce 1883b] _____ . 1883b. "The Logic of Relatives," in [Peirce 1883a], 187–203; reprinted: [Peirce 1933a], 195–210 and [Peirce 1989], 453–466.
- [Peirce 1885] _____ . 1885. "On the Algebra of Logic: A Contribution to the Philosophy of Notation," *American Journal of Mathematics* **7**, 180–202.
- [Peirce n.d., ca. 1897a] _____ . N.d., ca. 1897a. "Schröder's Logic of Relations," 54p. MS, n.d., ca. 1897 (Robin catalog MS 521); (CPA).
- [Peirce n.d., ca. 1897b] _____ . N.d., ca. 1897b. <Schröder and the Logic of Relations>, 8p. MS., n.d., ca. 1897 (Robin catalog MS 524); (CPA).
- [Peirce 1903a] _____ . 1903a. Review of [Welby 1903] and [Russell 1903a], *The Nation* **77** (15 October), 308–309; reprinted: [Hardwick 1977, 157–159].
- [Peirce 1903b] _____ . 1903b. "Lowell Lectures. 1903, Lecture 3" (Robin catalog MS 459); (CPA). Published as "Lowell Lectures. 1903. Lecture III" in [Peirce 1976], vol. III/1, 365–388.
- [Peirce 1903c] _____ . 1903c. "Lowell Lectures. 1903, Lecture 5" (Robin catalog MS 469); (CPA). Published as "Lowell Lectures. 1903. Lecture V (469–470)" in [Peirce 1976], vol. III/1, 333–354.
- [Peirce 1903d] _____ . 1903d. Letter to Victoria Welby, 1 December 1903; 3p. MS; (CPA).
- [Peirce 1904] _____ . 1904. Letter to Victoria Welby, 12 October 1904; (CPA); published [Hardwick 1977].
- [Peirce 1933a] _____ . 1933a. (Charles Hartshorne & Paul Weiss, editors), *Collected Papers of Charles Sanders Peirce*, vol. III: *Exact Logic*. Cambridge, Mass.: Harvard University Press.
- [Peirce 1933b] _____ . 1933b. (Charles Hartshorne & Paul Weiss, editors), *Collected Papers of Charles Sanders Peirce*, vol. III: *The Simplest Mathematics*. Cambridge, Mass.: Harvard University Press.
- [Peirce 1934] _____ . 1934. (Charles Hartshorne & Paul Weiss, editors), *Collected Papers of Charles Sanders Peirce*, vol. V: *Pragmatism and Pragmaticism*. Cambridge, Mass.: Harvard University Press.

- [Peirce 1976] ———. 1976. (Carolyn Eisele, editor), *The New Elements of Mathematics*, 4 vols. The Hague/Paris: Mouton.
- [Peirce 1989] ———. 1989. (Christian J. W. Kloesel, ed.), *Writings of Charles S. Peirce: A Chronological Edition*, vol. 4: 1879–1884. Bloomington/Indianapolis: Indiana University Press.
- [Peirce & Ladd-Franklin 1891–1908] Peirce, Charles Sanders & Christine Ladd-Franklin. 1891–1908. Correspondence (Robin catalog MS L237); (CPA).
- [Perry 1935] Perry, Ralph Barton. 1935. *The Thought and Character of William James*. New York: Harper & Row, 1935.
- [Reid 1986] Reid, Constance. 1986. *Hilbert — Couturat*. New York/Heidelberg/Berlin/Tokyo: Springer-Verlag.
- [Royce & Kernan 1916] Royce, Josiah, & Fergus Kernan. 1916. “Charles Sanders Peirce,” *Journal of Philosophy* **13**, 701–709.
- [Russell 1891–1902] Russell, Bertrand. 1891–1902. “What Shall I Read?,” MS RA. Published in Kenneth Blackwell, Nicholas Griffin, Richard Rempel, & John Slater (editors), *Collected Papers of Bertrand Russell*, I (London: Allen & Unwin, 1983), 347–365.
- [Russell 1899] ———. 1899. Letter to Louis Couturat, 11 February 1899; RA.
- [Russell ca. 1900–1901] ———. Ca. 1900–1901. Notes on [Peirce 1880] and [Peirce 1885]; 3p. MS; (RA).
- [Russell 1901a] ———. 1901a. “Sur la logique des relations avec des applications à la théorie des séries,” *Revue de Mathématiques/Rivista di Matematica* **7**, 115–148.
- [Russell 1901b] ———. 1901b. Notes on Schröder, *Vorlesungen über die Algebra der Logik*, 6p. MS, RA file #230:030460.
- [Russell 1903a] ———. 1903a. *Principles of Mathematics*. London: Cambridge University Press.
- [Russell 1903b] ———. 1903b. Letter to Louis Couturat, 2 June 1903; MS R57, alternately transcribed as R58; RA.
- [Russell 1903c] ———. 1903c. Letter to Louis Couturat, 9 June 1903; MS R56; RA.
- [Russell 1904a] ———. 1904a. Letter to Victoria Welby, November 11, 1904; 4p. MS; RA.
- [Russell 1904b] ———. 1904b. Letter to Victoria Welby, November 14, 1904; 3p. MS; RA.

- [Russell 1904c] _____ . 1904c. Letter to Victoria Welby, 27 December 1904; 3p. MS; RA.
- [Russell 1910] _____ . 1910. Letter to Philip E. B. Jourdain, 15 April 1910; RA.
- [Russell 1946] _____ . 1946. "Foreword" to James Kern Feibelman, *An Introduction to Peirce's Philosophy* (New York: Harper), i–xx.
- [Russell 1956] _____ . 1956. *Portraits from Memory and Other Essays*. London: Allen & Unwin; New York: Simon & Schuster.
- [Russell 1959] _____ . 1959. Letter to Max Fisch, 4 July 1959; RA; copy: CPA.
- [Schröder 1877] Schröder, Ernst. 1877. *Der Operationskreis des Logikkalküls*. Leipzig: B. G. Teubner.
- [Schröder 1880] _____ . 1880. Review of [Frege 1879], *Zeitschrift für Mathematik und Physik*, Historisch-literaturische Abteilung **25**, 81–93. English translation by Terrell Ward Bynum: [Frege 1972], 218–232; English translation by Victor H. Dudman in [Schröder 1969].
- [Schröder 1893] _____ . 1893. Letter to Paul Carus, 6 March 1893; (Robin catalog MS L421); (CPA).
- [Schröder 1895–1905] _____ . 1895–1905. *Vorlesungen über die Algebra der Logik*. Leipzig: B. G. Teubner; [first two volumes of three, reprinted: New York: Chelsea, 1966].
- [Schröder 1896] _____ . 1896. Letter to Charles Peirce, 16 February 1896; (Robin catalog MS L392); (CPA).
- [Schröder 1898a] _____ . 1898a. "Über Pasigraphie, ihren gegenwärtigen Stand und die pasigraphische Bewegung in Italien," in Ferdinand Rudio (ed.), *Verhandlungen des Ersten Internationalen Mathematiker-Kongressses in Zürich von 9. bis 11. August 1897* (Leipzig, 1898), 147–162.
- [Schröder 1898b] _____ . 1898b. "On Pasigraphy: Its Present State and the Pasigraphic Movement in Italy," *The Monist* **9**, 44–62, 320.
- [Schröder 1901] _____ . 1901. "Sur une extension de l'idée d'ordre," *Logique et histoire des Sciences III, Bibliothèque du Congrès International de Philosophie* (Paris: Armand Colin), 235–240.
- [Schröder 1969] _____ . 1969. (Victor H. Dudman, trans.), "Translation of Schröder's Review of Begriffsschrift," *Southern Journal of Philosophy* **7**, 139–150.

- [Shearman 1906] Shearman, Arthur Thomas. 1906. *The Development of Symbolic Logic: A Critical-Historical Study of the Logical Calculus*. London: Williams and Norgate; reprinted: Dubuque, Iowa: William C. Brown Reprint Library, 1988.
- [Sluga 1987] Sluga, Hans-Dieter. 1987. "Frege Against the Booleans," *Notre Dame Journal of Formal Logic* **28**, 80–98.
- [Stroll 1966] Stroll, Avrum. 1966. "On the First Flowering of Frege's Reputation," *Journal of the History of Philosophy* **4**, 72–81.
- [Tannery 1879] Tannery, Paul. 1879. Review of [Frege 1879], *Revue Philosophique de la France et de l'Étranger* **8**, 108–109. English translation by Terrell Ward Bynum: [Frege 1972], 232–234.
- [van Heijenoort 1967a] van Heijenoort, Jean. 1967a. "Logic as Calculus and Logic as Language," in Robert S. Cohen and Marx W. Wartofsky (editors), *Boston Studies in the Philosophy of Science* **3** (1967), *In Memory of Norwood Russell Hanson, Proceedings of the Boston Colloquium in Philosophy of Science, 1964/1965* (Dordrecht: D. Reidel Publishing Co.), 440–446; reprint: *Synthèse* **17** (1967), 324–330, and in Jean van Heijenoort, *Selected Essays* (Naples: Bibliopolis, 1986, ©1985), 11–16.
- [van Heijenoort 1967b] ———. 1967b. (Editor.) *From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931*. Cambridge, Mass.: Harvard University Press.
- [Veblen 1925] Veblen, Oswald. 1925. "Remarks on the Foundations of Geometry," *Bulletin of the American Mathematical Society* **31**, 121–141.
- [Venn 1880] Venn, John. 1880. Review of "Begriffsschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens." Von Dr. Gottlob Frege . . . , *Mind* (o.s.) **5**, 297; reprinted: [Frege 1972], 234–235.
- [Vikko 1998] Vikko, Risto. 1998. "The Reception of Frege's Begriffsschrift," *Historia Mathematica* **25**, 412–422.
- [Welby 1903] Welby, Victoria. 1903. *What is Meaning?* New York/London: Macmillan.
- [Whitehead 1898] Whitehead, Alfred North. 1898. *A Treatise on Universal Algebra*, I. Cambridge: Cambridge University Press.

- [Whitehead 1901] _____ . 1901. "Memoir on the Algebra of Symbolic Logic," *American Journal of Mathematics* **23**, 139–165, 297–316.
- [Whitehead 1902] _____ . 1902. "On Cardinal Numbers," *American Journal of Mathematics* **24**, 367–394.
- [Whitehead & Russell 1910–13] Whitehead, Alfred North & Bertrand Russell. 1910–13. *Principia Mathematica*, 3 vols. Cambridge: Cambridge University Press.
- [Wiener 1913] Wiener, Norbert. 1913. *A Comparison between the treatment of the Algebra of Relatives by Schröder and that by Whitehead and Russell*; Ph.D. thesis, Harvard University (Harvard transcript and MIT transcript).
- [Wilson 1904] Wilson, Edwin Bidwell. 1904. "The Foundations of Mathematics," *Bulletin of the American Mathematical Society* (1) **11**, 74–93.

PEIRCE PUBLISHING, 629 CENTRAL AVENUE, # 302, FORT DODGE IA 50501-3867, U.S.A.

E-mail address: Irving-Anellis@peircepublishing.com

<http://peircepublishing.com>

