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HOMOMORPHISMS OF BUNCE-DEDDENS ALGEBRAS

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The homomorphisms of a Bunce-Deddens algebra A are described. Necessary and sufficient conditions for an automorphism of the canonical UHF-subalgebra of A to have an extension to an automorphism of A are given.

The Bunce-Deddens algebras were introduced in [**5**]. They are interesting particular examples of inductive limits of the form $\varinjlim C(X_i, F_i)$ (where the F_i's are finite dimensional C^*-algebras), whose study was suggested in [**9**]. In this paper we analyse the homomorphisms and the automorphisms of the Bunce-Deddens algebras, since their good knowledge could spread some light in the above general problem raised by E. G. Effros.

A Bunce-Deddens algebra A is a certain C^*-inductive limit $\varinjlim C(\mathbf{T}, M_{n(i)})$ (see [**5**]). It contains a canonical UHF-algebra B, namely the C^*-subalgebra generated by the constant functions in the algebras $C(\mathbf{T}, M_{n(i)})$. Necessary and sufficient conditions for an automorphism of B to have an extension to an automorphism of A are given (Theorem 2). A key fact proved in this paper is that B is dense in A with respect to the norm given by the unique trace of A (see Proposition 2). It is also shown that the centralizer of $\{\Phi \in \operatorname{Aut}(A)\colon \Phi(B) = B\}$ in $\operatorname{Aut}(A)$ is trivial (Proposition 5) and the same thing about the centralizer of $\{\Phi \in \operatorname{Aut}(B)\colon (\exists)\widetilde{\Phi} \in \operatorname{Aut}(A)$ such that $\widetilde{\Phi}_{|B} = \Phi\}$ in $\operatorname{Aut}(B)$ (Proposition 4).

We also describe the endomorphisms of the Bunce-Deddens algebras, showing that they are approximately inner in a weak sense (see Theorem 1 for a more general case), but not necessarily approximately inner, since they don't always induce the identity in K_1 (see Proposition 3).

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1. In this paper we shall consider only unital C^*-algebras.

For a compact space X and C^*-algebra A we shall identify

$C(X, A) = C(X) \otimes A$ in the canonical way and we shall consider the embedding $A \subset C(X, A)$, where, each element in A is seen as a constant function on X.

By a homomorphism of C^*-algebras we shall mean a unital $*$-homomorphism and by an automorphism of a C^*-algebra, a $*$-automorphism. Let $\mathrm{Hom}(A, B)$ be the homomorphisms $A \to B$, and $\mathrm{Aut}(A)$ the automorphisms of A. $\mathrm{ad}\, u \in \mathrm{Aut}(A)$ will denote the map $\mathrm{ad}\, u(x) = uxu^*$, $x \in A$, where u is a unitary in A. By a trace of A we shall mean a central state of A.

A Bunce-Deddens algebra A will be the C^*-inductive limit of a system:

$$C(\mathbf{T}, M_{n(1)}) \xrightarrow{\Phi_1} C(\mathbf{T}, M_{n(2)}) \xrightarrow{\Phi_2} \cdots$$

where $(n(i))_i$ is a strictly increasing sequence of positive integers with $n(k)$ dividing $n(k+1)$ for all $k \geq 1$ and where each homomorphism Φ_k is given by:

$$\Phi_k(a) = \begin{bmatrix} a & & & & \\ & a & & 0 & \\ & & \cdot & & \\ & 0 & & \cdot & \\ & & & & a \end{bmatrix}, \qquad a \in M_{n(k)},$$

$$\Phi_k\left(\begin{bmatrix} 0 & 0 & 0 & & 0 & z \\ 1 & 0 & 0 & & 0 & 0 \\ 0 & 1 & 0 & \ddots & 0 & 0 \\ & & & \cdot & & \\ 0 & 0 & 0 & & 1 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 & 0 & 0 & & 0 & z \\ 1 & 0 & 0 & & 0 & 0 \\ 0 & 1 & 0 & \ddots & 0 & 0 \\ & & & \cdot & & \\ 0 & 0 & 0 & & 1 & 0 \end{bmatrix}$$

(see [5]). Here $z \in C(\mathbf{T})$ is the map given by $\mathbf{T} \ni t \mapsto t \in \mathbf{C}$. We shall simply denote by $S \in A$ the unitary represented in each $C(\mathbf{T}, M_{n(i)})$ by the matrix:

$$\begin{bmatrix} 0 & 0 & 0 & & 0 & z \\ 1 & 0 & 0 & & 0 & 0 \\ 0 & 1 & 0 & \ddots & 0 & 0 \\ & & & \cdot & & \\ 0 & 0 & 0 & & 1 & 0 \end{bmatrix}$$

Note that A is simple [5], has a unique trace ([4], see also [1]) and is the C^*-algebra generated by B and S (see e.g. [5]).

We shall say that (A, B) is a *canonical pair* if

$$A = \varinjlim (C(\mathbf{T}, M_{n(i)}), \Phi_i)$$

is a Bunce-Deddens algebra (as above) and $B \subset A$ is the UHF-algebra given by $B = \varinjlim (M_{n(i)}, \Phi_{i|M_{n(i)}})$.

For a C^*-algebra A, we shall denote by $U(A)$ the unitary group of A. We denote $U(n) := U(M_n)$ (of course, by M_n we mean the $n \times n$ complex matrices). $K_1(A)$ will denote the K_1-group ([**12**], [**2**], [**8**]) and if $\Phi \in \mathrm{Hom}(A, B)$, $K_1(\Phi): K_1(A) \to K_1(B)$ denotes the natural group homomorphism.

For a space X, we shall denote by $\mathrm{Vect}(X)$ the isomorphism classes of complex vector bundles on X. We say, that $\mathrm{Vect}(X)$ is *torsion free* if any $E \in \mathrm{Vect}(X)$ such that $E \oplus E \oplus \cdots \oplus E$ (n-times) is a trivial vector bundle for some n, is (isomorphic to) the trivial bundle.

In this paper we shall consider only C^*-inductive limits with unital injective bonding homomorphisms.

2. We begin with a general result, which will be used in the sequel. It shows that any two homomorphisms from a UHF-algebra to a more general C^*-inductive limit are approximately inner equivalent:

PROPOSITION 1. *Consider two homomorphisms Φ, Ψ: $A \to B = \varinjlim B_i$. Here A is a UHF-algebra and each B_i is a direct sum of C^*-algebras of the form $C(X, M_n)$, where each X is a compact connected space such that* $\mathrm{Vect}(X)$ *is torsion free.*

Then, there is a sequence $(u_n)_{n\geq 1}$ in $U(B)$ such that:

$$\Phi(x) = \lim_n u_n \Psi(x) u_n^*, \qquad x \in A.$$

Proof. Suppose that $A = \varinjlim A_i$, where each A_i is a full matrix algebra. For any fixed i, arguing as in [**3**, Lemma 2.3], we find v_i, w_i in $U(B)$ and $j = j(i)$ such that:

$$v_i \Phi(A_i) v_i^*, \; w_i \Psi(A_i) w_i^* \subset B_j.$$

Using ([**6**]; see also [**7**, Corollary 2.2]) for each component (in B_j) of $v_i\Phi(\cdot)v_i^*$, $w_i\Psi(\cdot)w_i^*: A_i \to B_j$, we obtain finally $u_i \in U(B)$ such that:

$$\Phi(x) = u_i \Psi(x) u_i^*, \qquad x \in A_i.$$

Since for any $p \geq q$ and any $x \in A_q$ we have:

$$\Phi(x) = u_q \Psi(x) u_q^* = u_p \Psi(x) u_p^*$$

one easily obtains:

$$\Phi(x) = \lim_n u_n \Psi(x) u_n^*, \qquad x \in A.$$

NOTATIONS. For a C^*-algebra A with a unique trace τ, we shall denote by $L^2(A)$ the separate completion of A with respect to the seminorm $A \ni a \mapsto \tau(a^*a)^{1/2} \in \mathbf{R}_+$. The induced norm on $L^2(A)$ will be denoted by $\|\cdot\|_\tau$. Note that $(L^2(A), \|\cdot\|_\tau)$ is a Hilbert space. When $(x_n)_{n\geq 1}$ is a sequence in $L^2(A)$ with $\|x_n - x\|_\tau \to 0$ for some $x \in L^2(A)$, we shall write $\tau\text{-}\lim_n x_n = x$.

The following proposition will be important in the sequel:

PROPOSITION 2. *Let (A, B) be a canonical pair (see §1). Then B is dense in $(L^2(A), \|\cdot\|_\tau)$ (where τ is the trace of A).*

Proof. Consider $A = \varinjlim (C(\mathbf{T}, M_{n(i)}), \Phi_i)$ as in §1. Since A is simple and is generated as C^*-algebra by B and S (see §1), it is enough to prove that $S \in \overline{B}^{\|\cdot\|_\tau}$.

For each $m \in \mathbf{N}$, one has:

$$S = b_m + e^{(m)}_{1, n(m)} S^{n(m)}$$

where

$$b_m := \sum_{i=1}^{n(m)-1} e^{(m)}_{i+1, i}$$

and $(e^{(m)}_{i,j})_{i,j=1}^{n(m)}$ is the canonical system of matrix units in $M_{n(m)} \subset C(\mathbf{T}, M_{n(m)})$.

We have:

$$\begin{aligned}\|S - b_m\|_\tau^2 &= \tau((S^{n(m)})^* e^{(m)}_{n(m), 1} e^{(m)}_{1, n(m)} S^{n(m)}) \\ &= \tau(e^{(m)}_{n(m), n(m)}) = \frac{1}{n(m)} \to 0 \quad \text{as } m \to \infty\end{aligned}$$

and each $b_m \in B$. Hence $S \in \overline{B}^{\|\cdot\|_\tau}$, and the proof is completed.

The following corollary was obtained in **[1]** (and in the particular case when the Bunce-Deddens algebra is of type 2^∞ in **[5]**). Our proof is simpler and shorter.

COROLLARY. *Let (A, B) be a canonical pair. Then $B' \cap A = \mathbf{C} \cdot 1_A$.*

Proof. Let τ be the trace of A. Take an element x in $B' \cap A$. By the above proposition we deduce that it belongs to the center of A (the maps $(A, \|\cdot\|_\tau) \ni a \mapsto ax \in (A, \|\cdot\|_\tau)$ and $(A, \|\cdot\|_\tau) \ni a \mapsto xa \in (A, \|\cdot\|_\tau)$ are continuous) which is trivial since A is a simple C^*-algebra.

The following result gives a description of the homomorphisms between two Bunce-Deddens algebras. Observe first that if A and B are Bunce-Deddens algebras such that $\mathrm{Hom}(A, B) \neq \varnothing$, then $A \subset B$ (see [**5**, Theorem 2 and the proof of Theorem 4]).

THEOREM 1. *Let (A, D) be a canonical pair and B a Bunce-Deddens algebra such that $A \subset B$. Let τ be the trace of B.*

If $\Phi \in \mathrm{Hom}(A, B)$ then there is a sequence $(u_n)_{n\geq 1}$ in $U(B)$ such that:

(a) $\Phi(x) = \tau\text{-}\lim_n u_n x u_n^*$, $x \in A$, *and*

(b) $\Phi(x) = \lim_n u_n x u_n^*$, $x \in D$.

Proof. Proposition 1 gives a sequence $(u_n)_{n\geq 1}$ in $U(B)$ such that (b) holds. The fact that (a) is satisfied for the same sequence $(u_n)_{n\geq 1}$ follows using Proposition 2 and also the fact that for any $x \in A$ and $n \in \mathbf{N}$ we have:

$$\|u_n x u_n^*\|_\tau = \|\Phi(x)\|_\tau = \|x\|_\tau$$

by the unicity of the trace on a Bunce-Deddens algebra.

Having the above result, one could suspect that any endomorphism of a Bunce-Deddens algebra is approximately inner. The answer follows from:

PROPOSITION 3. *Let (A, B) be a canonical pair. Then there is a symmetry Φ of A such that $K_1(\Phi) = -\mathrm{id}_{K_1(A)}$ (and hence Φ is not approximately inner).*

The proof follows from the following lemma (see also [**2**, 10.11.5]):

LEMMA 1. *Let (A, B) be a canonical pair. Then, there is a symmetry Φ of A such that $\Phi(S) = S^*$ and $\Phi(B) = B$.*

Proof. Suppose that $A = \varinjlim (C(\mathbf{T}, M_{m(i)}), \Phi_i)$ as in §1. For each n, take:

$$u_n := \begin{bmatrix} & 0 & & 1 \\ & & 1 & \\ & \cdot^{\cdot^{\cdot}} & & \\ & & 0 & \\ 1 & & & \end{bmatrix} \in U(m(n)) \subset U(C(\mathbf{T}, M_{m(n)})).$$

Observe that $u_n = u_n^*$ and that each diagram:

$$\begin{array}{ccc} M_{m(n)} & \xrightarrow{\Phi_n | M_{m(n)}} & M_{m(n+1)} \\ \Big\downarrow \mathrm{ad}\, u_n & & \Big\downarrow \mathrm{ad}\, u_{n+1} \\ M_{m(n)} & \xrightarrow{\Phi_n | M_{m(n)}} & M_{m(n+1)} \end{array}$$

commutes. Hence we obtain an automorphism Φ of B such that:

$$\Phi(x) = \lim_n u_n x u_n, \qquad x \in B.$$

Let τ be the trace of A. We shall prove that:

$$\lim_n \|u_n S u_n - S^*\|_\tau = 0$$

which, by Theorem 2, will imply that Φ extends to an automorphism of A, also denoted by Φ, such that $\Phi(S) = S^*$ (don't forget that $u_n = u_n^*$).

Since for any arbitrary fixed n we have:

$$S = \begin{bmatrix} 0 & 0 & & 0 & 0 & z \\ 1 & 0 & & 0 & 0 & 0 \\ & & \ddots & & & \\ 0 & 0 & & 1 & 0 & 0 \\ 0 & 0 & & 0 & 1 & 0 \end{bmatrix} \in C(\mathbf{T}, M_{m(n)})$$

(see §1) we get:

$$u_n S u_n = \begin{bmatrix} 0 & 1 & 0 & & 0 & 0 \\ 0 & 0 & 1 & & 0 & 0 \\ & & & \ddots & & \\ 0 & 0 & 0 & & 0 & 1 \\ z & 0 & 0 & & 0 & 0 \end{bmatrix}.$$

Then one easily obtains:

$$\|u_n S u_n - S^*\|_\tau^2 = \tau\left(\begin{bmatrix} |z-\overline{z}|^2 & & & & 0 \\ & 0 & & & \\ & & 0 & & \\ & & & \ddots & \\ 0 & & & & 0 \end{bmatrix}\right)$$

$$\leq 4\cdot\tau\left(\begin{bmatrix} 1 & & & & \\ & 0 & & 0 & \\ & & 0 & & \\ & 0 & & \ddots & \\ & & & & 0 \end{bmatrix}\right)$$

$$= \frac{4}{m(n)} \to 0 \quad \text{as } n \to \infty.$$

Since $\Phi^2|_B = \mathrm{id}_B$ ($u_n = u_n^*$ for each n), $\Phi^2(S) = S$ and A is the C^*-algebra generated by B and S, it follows that Φ is a symmetry of A.

Proof of Proposition 3. Let Φ be the symmetry of A given by the above lemma. Suppose that $K_1(\Phi) = \mathrm{id}_{K_1(A)}$. Then $[\Phi(S)] = [S^*] = [S]$ in $K_1(A)$ and hence $2[S] = 0$. But it is known that $K_1(A) = \mathbf{Z}$ and that $[S]$ is a generator (see **[11]** and **[10]**). It follows that $[S] = 0$, a contradiction.

Now we are interested in knowing under which conditions an automorphism of B extends to an automorphism of A; here (A, B) will be a canonical pair. The answer to this natural problem is given by:

THEOREM 2. *Consider $\Phi \in \mathrm{Aut}(B)$ and let $(u_n)_{n\geq 1}$ be a sequence in $U(B)$ such that $\Phi(x) = \lim_n u_n x u_n^*$, $x \in B$. Let τ be the trace of A. Then:*

$$\Phi \text{ extends to an automorphism of } A$$
$$\Leftrightarrow \tau\text{-}\lim_n u_n S u_n^*,\ \tau\text{-}\lim_n u_n^* S u_n \in A$$

and when Φ extends, it has a unique extension $\widetilde{\Phi} \in \mathrm{Aut}(A)$, where:

$$\widetilde{\Phi}(x) = \tau\text{-}\lim_n u_n x u_n^*$$

and

$$\widetilde{\Phi}^{-1}(x) = \tau\text{-}\lim_n u_n^* x u_n$$

for any $x \in A$.

In the proof of this theorem we shall use the following:

LEMMA 2. *Let* (A, B) *be a canonical pair and* D *a* C^**-algebra with a unique trace. If* Φ, $\Psi \in \mathrm{Hom}(A, D)$ *are such that*:

$$\Phi|_B = \Psi|_B$$

then:

$$\Phi = \Psi.$$

Proof. Since A and D have unique traces, denoted by τ respectively σ, one obtains:

$$\|\Phi(x)\|_\sigma = \|\Psi(x)\|_\sigma \le \|x\|_\tau, \qquad x \in A.$$

Hence, using Proposition 2 and the fact that $\Phi|_B = \Psi|_B$, it follows that $\Phi = \Psi$.

Proof of Theorem 2. Observe first that the unicity of the extension (when it exists) follows from the above lemma.

"$\Rightarrow$" Let $\widetilde{\Phi} \in \mathrm{Aut}(A)$ be such that $\widetilde{\Phi}|_B = \Phi$. Then, by the proof of Theorem 1 and the above remark, it follows that:

$$\widetilde{\Phi}(x) = \tau\text{-}\lim_n u_n x u_n^*, \qquad x \in A.$$

Hence $\tau\text{-}\lim_n u_n S u_n^* = \widetilde{\Phi}(S) \in A$.

The other relation is obtained working with Φ^{-1}.

"$\Leftarrow$" If B is seen in its GNS representation in $B(L^2(B))$ associated with the (unique) trace of B, we have:

$$\Phi(x) = UxU^*, \qquad x \in B,$$

where $U \in U(B(L^2(B)))$ is given by $U(b) := \Phi(b)$, $b \in B$. Since $L^2(B) = L^2(A)$ (by Proposition 2), we have $U \in U(B(L^2(A)))$ and we can define $\widetilde{\Phi} \in \mathrm{Hom}(A, B(L^2(A)))$ by:

$$\widetilde{\Phi}(x) = UxU^*, \qquad x \in A.$$

Here A is seen in its GNS representation in $B(L^2(A))$ associated with the (unique) trace of A. Obviously $\widetilde{\Phi}|_B = \Phi$.

By the proof of Proposition 2 there is a sequence $(b_k)_{k\geq 1}$ in B such that $\|S-b_k\|_\tau \to 0$ and $\|b_k\|=1$, $k\geq 1$. Then, since $x_n \overset{\|\cdot\|_\tau}{\to} 0$ in A means $x_n \overset{\text{so}}{\to} 0$ in $B(L^2(A))$ when $\{\|x_n\|\}$ is bounded, we have:

$$\begin{aligned}\widetilde{\Phi}(S) = USU^* &= \text{so-}\lim_k Ub_kU^* \\ &= \text{so-}\lim_k \Phi(b_k) = \text{so-}\lim_k \left(\text{so-}\lim_n u_nb_ku_n^*\right).\end{aligned}$$

On the other hand observe that:

$$\tau\text{-}\lim_n u_nxu_n^* \text{ exists in } L^2(A) \quad \text{for any } x\in A$$

(use the fact that the limit already exists for any $x\in B$, use Proposition 2 and the equality $\|u_nxu_n^*\|_\tau = \|x\|_\tau$, true for $x\in A$ and $n\in \mathbf{N}$). Therefore, we may write:

$$\begin{aligned}&\left\|\tau\text{-}\lim_n u_nb_ku_n^* - \tau\text{-}\lim_n u_nSu_n^*\right\|_\tau \\ &\qquad = \lim_n \|u_n(b_k-S)u_n^*\|_\tau = \|b_k-S\|_\tau\end{aligned}$$

and hence:

$$\tau\text{-}\lim_k \left(\tau\text{-}\lim_n u_nb_ku_n^*\right) = \tau\text{-}\lim_n u_nSu_n^* \quad (\text{in } L^2(A)).$$

But, by hypothesis, $\tau\text{-}\lim_n u_nSu_n^* \in A$. Using again that $x_n \overset{\|\cdot\|_\tau}{\to} 0$ in A means $x_n \overset{\text{so}}{\to} 0$ in $B(L^2(A))$ when $\{\|x_n\|\}$ is bounded, we have:

$$\widetilde{\Phi}(S) = \text{so-}\lim_k \left(\text{so-}\lim_n u_nb_ku_n^*\right) = \text{so-}\lim_n u_nSu_n^* \in A.$$

But A is the C^*-algebra generated by B and S. Hence $\widetilde{\Phi}(A)\subset A$ and, as in "$\Rightarrow$", we deduce:

$$\widetilde{\Phi}(x) = \tau\text{-}\lim_n u_nxu_n^*, \qquad x\in A.$$

The proof ends if we repeat the above arguments for Φ^{-1}, where $\Phi^{-1}(x) = \lim_n u_n^*xu_n$, $x\in B$, in this way we get $\widetilde{\Phi}^{-1}\in \operatorname{Aut}(A)$.

Question. Does any automorphism of B extend to an automorphism of A, whenever (A, B) is a canonical pair?

Our feeling is that the answer is negative.

REMARK. If we replace the above B with a certain C^*-subalgebra of A, it is easy to see that the answer to the corresponding question is negative. Let $A = \varinjlim (C(\mathbf{T}, M_{n(k)}), \Phi_k)$ be a Bunce-Deddens algebra as in §1, where $n(k) = 2^k$, $k \geq 1$. Let D be the C^*-algebra which is the closure in A of the constant diagonal functions in $C(\mathbf{T}, M_{2^k})$, $k \geq 1$. Observe that there are canonical isomorphisms $D \cong C^*_{\text{red}}(G) \cong C(\widehat{G})$, where $G := \{z \in \mathbf{T} | z^{2^k} = 1$ for some integer $k \geq 1\}$ and hence $\widehat{G}$ is the group of the dyadic integers. It is not difficult to see that there are automorphisms of D which do not preserve the trace (induced from A) and, hence, cannot be extended to A.

Let again (A, B) be a canonical pair. Denote

$$H = \{\Phi \in \text{Aut}(B) \colon (\exists)\widetilde{\Phi} \in \text{Aut}(A) \text{ such that } \widetilde{\Phi}|_B = \Phi\}$$

and $G = \text{Aut}(B)$. We shall prove that the centralizer of H in G is trivial:

PROPOSITION 4. $\{\Phi \in G\colon \Phi \circ \Psi = \Psi \circ \Phi$ *for any* $\Psi \in H\} = \{\text{id}_B\}$.

Proof. Fix $\Phi \in G$ which commutes with every element of H. Since for any $u \in U(B)$, $\text{ad}\, u \in G$ belongs also to H, we have:

$$\Phi \circ \text{ad}\, u = \text{ad}\, u \circ \Phi \Leftrightarrow \Phi(u)^* u$$

commutes with

$$\Phi(B) = B \Leftrightarrow \Phi(u)^* u \in \mathbf{T} \cdot 1_B$$

(since B is simple and hence its center is trivial).

Therefore, for any $u \in U(B)$ we have:

$$\Phi(u) = \gamma(u) u$$

where $\gamma\colon U(B) \to \mathbf{T}$ is a continuous map.

Let τ be the (unique) trace of B. Then, we obtain:

$$\tau(u) = \tau(\Phi(u)) = \gamma(u)\tau(u), \qquad u \in U(B).$$

But it is not difficult to see that $\{u \in U(B)\colon \tau(u) \neq 0\}$ is dense in $U(B)$. Therefore:

$$\gamma(u) = 1, \qquad u \in U(B)$$

which implies that:

$$\Phi(u) = u, \qquad u \in U(B)$$

and hence:

$$\Phi = \text{id}_B.$$

Also, we can prove the following:

PROPOSITION 5. *Let (A, B) be a canonical pair. Then, the centralizer of* $\{\Phi \in \mathrm{Aut}(A)\colon \Phi(B) = B\}$ *in* $\mathrm{Aut}(A)$ *is trivial.*

Proof. Fix $\Phi \in \mathrm{Aut}(A)$ which commutes with every element in $\{\Psi \in \mathrm{Aut}(A)\colon \Psi(B) = B\}$. For any $u \in U(B)$, $\mathrm{ad}\, u \in \mathrm{Aut}(A)$ and $\mathrm{ad}\, u(B) = B$. Hence:

$$\Phi \circ \mathrm{ad}\, u = \mathrm{ad}\, u \circ \Phi, \qquad u \in U(B).$$

Since A is simple, we deduce (as in the proof of the above proposition) that:

$$\Phi|_B = \mathrm{id}_B.$$

By Lemma 2, it follows that:

$$\Phi = \mathrm{id}_A.$$

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