$$Q_1 = 2x_1^2 - x_2^2, \qquad Q_2 = x_1^2 - 2x_2^2$$

admit the definite linear combination $Q_1 - Q_2 = x_1^2 + x_2^2$, and the corresponding system (5) admits the indefinite solution $B = x_1x_2$.

EXAMPLE 2. The three forms

$$Q_1 = 2x_1^2 - x_2^2, \qquad Q_2 = x_1^2 - 2x_2^2, \qquad Q_3 = x_1x_2$$

admit the definite linear combination $Q_1 - Q_2 - Q_3 = x_1^2 - x_1x_2 + x_2^2$, but the corresponding system (5) admits no solution form B.

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NOTE ON A CONJECTURE DUE TO EULER

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Euler's conjecture (1772) that

$$x_1^n + \cdots + x_t^n = x^n,$$

where n is an integer greater than 3 and $2 < t < n$, has no solution in rational numbers $x_1, \cdots, x_t, x$ all different from zero, is still unsettled even in its first case, $n = 4$, $t = 3$. It may therefore be of some interest to note a solution of this equation for any $n > 3$ and any $t > 1$ in terms of (irrational) algebraic numbers, which can be made algebraic integers by suitable choice of a homogeneity parameter, all different from zero, all the numbers being polynomials in numbers of degree $2d$, where $4d \leqq 2n - 5 + (-1)^n$. If solutions differing only by a parameter are not considered distinct, there are at least d^{t-1} sets of solutions $x_1, \cdots, x_t, x$.

The solutions described are

$$x_1 = u, \qquad x_2 = r_{t-1}u, \qquad x = (1 + r_1) \cdots (1 + r_{t-1})u;$$

$$x_j = r_{t-j+1}(1 + r_{t-j+2})(1 + r_{t-j+3}) \cdots (1 + r_{t-1})u, \qquad j = 3, \cdots, t,$$

where u is a parameter and the r's are any roots, the same or different, of any factor $F_n(r)$, irreducible in the field of rational numbers, of

$$f(r) \equiv \sum_{s=1}^{n-1} (n, s) r^{n-s-1},$$

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where (n, s) is the binomial coefficient $n!/s!(n-s)!$. For, $(r\neq 0)$, $f(r)=0$ implies $(1+r)^n=1+r^n$; whence the verification is immediate on successive reduction of $x_1^n+x_2^n$, $x_1^n+x_2^n+x_3^n$, $x_1^n+\cdots+x_t^n$. The remarks on d and the number of sets of solutions then follow since $f(r)=0$ is a reciprocal equation, and $F_n(r)$ has no multiple roots.

With $y\equiv r+r^{-1}$, the first seven $F_n(r)$ are

$$\begin{aligned}
n &= 4: \quad 2y+3;\\
n &= 5: \quad y+1;\\
n &= 6: \quad 6y^2+15y+8;\\
n &= 7: \quad y+1;\\
n &= 8: \quad 4y^3+14y^2+16y+7;\\
n &= 9: \quad 3y^3+9y^2+10y+5;\\
n &= 10: \quad 10y^4+45y^3+80y^2+75y+32.
\end{aligned}$$

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