WEIERSTRASS PREPARATION THEOREM

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H. Späth[1] has shown that the Weierstrass preparation theorem is a consequence of the following one:

THEOREM. *Let P and Q be power series in the variables $z_1, z_2, \cdots, z_m$ such that P is regular in z_1 of degree k, that is, $P(z_1, 0, \cdots, 0) = cz_1^k + dz_1^{k+1} + \cdots$, $(c \neq 0)$. Then there exist power series A and B such that*

$$Q - AP = B \tag{1}$$

where B does not contain powers of z_1 higher than the $(k-1)$st. The series A and B are uniquely determined.

It has not been observed that this general form permits of a simple proof by induction when we are concerned only with *formal* series. The solution of (1) is equivalent to the solution of the system

$$(q_n - a_0 p_n - a_1 p_{n-1} - \cdots - a_{n-1} p_1) - a_n p_0 = b_n, \qquad n = 0, 1, 2, \cdots, \tag{2}$$

where $P = \sum p_n z_m^n$, $Q = \sum q_n z_m^n$, $A = \sum a_n z_m^n$, $B = \sum b_n z_m^n$. The theorem is evidently true for the case $m = 1$. Since p_0 is regular in z_1 of degree k, the existence of a formal solution of (1) and its uniqueness is an immediate consequence of an induction from $m-1$ to m variables.

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[1] H. Späth, Journal für die reine und angewandte Mathematik, vol. 161 (1929), pp. 95–100. This contains additional references.