ON KNOTGROUPS OF PARALLELKNOTS

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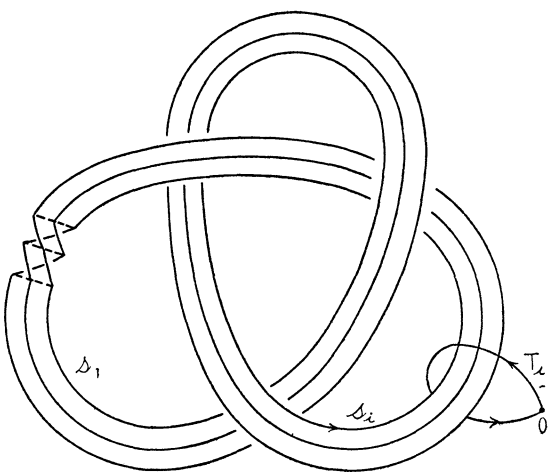
In this short note I will give a geometrical meaning of the knotgroup of a parallelknot, whith is obtained free-group-theoretically in Reidemeister's book[1] by using the addition theorem of fundamental groups, just as proved in case of torusknots in Seifert-Threlfall's.[2]

Let $\mathfrak{K}_{qr}$ be a parallelknot. Let the group of $\mathfrak{K}$ be defined with the generators $S_1, S_2, \cdots, S_n$ and the relations



$\left(\begin{array}{l}\mathfrak{K}\text{: Cloverknot,} \\ \quad q = 3,\ r = 2\end{array}\right)$

$$R_i(S) = S_{i+1}^{-1} S_{\lambda(i)}^{\varepsilon_i} S_i S_{\lambda(i)}^{-\varepsilon_i} \qquad (i = 1, 2, \cdots, n).$$

By eliminating $S_2, S_3, \cdots, S_n$ successively, from these equations, we get

$$(1) \qquad S_1 = L_{n+1} S_1 L_{n+1}^{-1},$$

where

$$L_{n+1} = S_{\lambda(n)}^{\varepsilon_n} S_{\lambda(n-1)}^{\varepsilon_{n-1}} \cdots S_{\lambda(1)}^{\varepsilon_1}.$$

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1) K. Reidemeister: Knotentheorie. Berlin, Springer (1932), p. 57, § 11.

2) Seifert-Threlfall: Lehrbuch der Topologie. Leipzig, Teubner (1934), p. 117, Satz I and p. 180.

Consider a path T_i corresponding to each S_i as in the Figure. As in the case of torusknots (Seifert–Threlfall, p. 180, where $A^m = B^n$), we can take as generators, $T_1, T_2, \cdots, T_n$ and one more Q, which corresponds to the center line (Seele) of the tube (Schlauch). Then, by the addition theorem in Seifert–Threlfall § 52, the defining relations between them are

$$R_i(T_k) = 1 \qquad (i = 1, 2, \cdots, n)$$

and one more relation of the type

$$Q^q = X,$$

that is obtained by representing $\mathfrak{K}_{qr}$ itself in two ways, as in case of torusknots: $A^m = B^n$. The one is the q–times of the center line, namely Q^q, and the other is $L_{n+1}^{-q} S_1^r$ under a suitable direction sense, because L_{n+1} represents the knot $\mathfrak{K}$ and S_1^r represents r–times of turnings around the tube and moreover, by (1), S_1 and L_{n+1} are commutative. Thus the defining relations are

$$\begin{cases} R_i(T_k) = 1 & (i = 1, 2, \cdots, n), \\ Q^q L_{n+1}^q T_1^{-r} = 1, & \end{cases}$$

where

$$L_{n+1} = T_{\lambda(n)}^{\varepsilon_n} T_{\lambda(n-1)}^{\varepsilon_{n-1}} \cdots T_{\lambda(1)}^{\varepsilon_1}.$$

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