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# LOGICAL CONTINUITY

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In discussing the principles to be found in our thinking about species and genera in nature, Kant enunciated this law:

> ". . . there are no species or sub-species which (in the view of reason) are the nearest possible to each other; intermediate species or sub-species being always possible, the difference of which from each of the former is always smaller than the difference existing between these."[1]

Hamilton called Kant's law the law of "Logical Continuity". Here is how Hamilton puts it:

> ". . . no two coördinate species touch so closely on each other, but that we can conceive other or others intermediate."[2]

He cites the pairs, men and orang-utangs, and elephants and rhinoceroses, as classes that conform to the law. But he holds that there are many classes that do not conform. I am going to consider his counter-cases. Hamilton argued:

> ". . . all angles are either acute or right or obtuse. For between these three coördinate species or genera no others can possibly be interjected, though we may always subdivide each of these, in various manners, into a multitude of lower species."[3]

Furthermore, there are classes distinguished from each other by *contradictory* attributes:

> "For example: —in the Cuvierian classification the genus *animal* is divided into the two species of *vertebrata* and *invertebrata*, that is into animals with a

1. Immanuel Kant, *Critique of Pure Reason*, London, J. M. Dent & Sons, 1945, p. 382.
2. Sir William Hamilton, *Lectures on Metaphysics and Logic*, Boston, Gould and Lincoln, 1860, Vol. II, p. 149.
3. *Ibid.*

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backbone—with a spinal marrow; and animals without a backbone—without spinal marrow. Is it possible to conceive the possibility of any intermediate class?"[4]

I believe that Hamilton is right in asserting that there are classes of angles that do not conform to Kant's law; but perhaps Kant would grant this. The law is intended to govern "the *empirical* exercise of reason".[5] Kant is not forced to hold that our examination of kinds of Euclidian angles is such an exercise.

Hamilton's second sort of counter-case can be read as an attack on one or both of these principles:

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*The stronger principle of logical continuity.* Between any two empirical and coördinate classes, **Ks** and **Ls**, a third empirical *and coördinate* class, **Ms**, can be conceived. From this a virtual infinity of empirical classes would follow.[6]

Can we conceive of a class between the classes *vertebrata* and *invertebrata*? If we cannot, it would seem that both the weaker and the stronger forms of the principle of logical continuity are false. Perhaps this is Hamilton's point; but if it is, I suspect him of relying on our old unreliable friend the principle of excluded middle.

"The principle of Excluded Third or Middle—viz., between two contradictories (*principium Exclusi Medii vel Tertis*), enounces that condition of thought which compels us, of two repugnant notions, which cannot both coexist, to think either the one or the other as existing. Hence arises the general axiom—Of contradictory attributions, we can only affirm one of a thing; and if one be explicitly affirmed, the other is implicitly denied. . . *A either is or is not B.*"[7]

Two classes, **Ks** and **Ls**, of some genus, **Gs**, are "distinguished from each other by contradictory attributes" if and only if we distinguish **Ks** from **Ls** by means of some property, $P$, such that if any $G$ has property $P$ it is a $K$, and if any $G$ does not have property $P$ it is an $L$. This is how the two classes *vertebrata* and *invertebrata* are related in the Cuvierian system. It is having a backbone that makes an animal a vertebrate, and not having a backbone that makes an animal an invertebrate. Thus from the principle of excluded middle it follows that any particular animal is either a vertebrate or an invertebrate, but not both.

A defender of the Kantian view might now reply that we can conceive of a class of "borderline" cases between these two classes of animals.

4. *Ibid.*

5. *Op. Cit.*, p. 384. My italics.

6. I mention this simply because Kant holds that the virtual infinity of empirical classes follows from his principle of logical continuity. Cf. Kant, p. 383.

7. Hamilton, *Op. Cit.*, p. 59.

Something, $x$, is a "borderline" case of a $K$ if and only if it is intrinsically indeterminate whether $x$ is a $K$, or $x$ is not a $K$.[8]

We can conceive of borderline cases of vertebrates and invertebrates. The foetus of any vertebrate goes through phases such that if it existed as a kind of animal at that phase it would be intrinsically indeterminate whether it was a vertebrate or an invertebrate. It seems clear, then, that Hamilton has not given us a counter-case to the *weaker* principle of logical continuity. Has he refuted the stronger version? Here a great deal depends upon the meaning of "coördinate". Hamilton says:

> "Two or more concepts are coördinated, when each excludes the other from its sphere, but when both go immediately to make up the extension of a third concept, to which they are cosubordinate."[9]

And a bit later:

> "As examples of Subordination and Coördination, — *man, dog, horse,* stand, as correlatives, in subordination to the concept *animal,* and, as reciprocal correlatives, in coördination with each other."[10]

Two classes, **Ks** and **Ls,** are coördinate only if, where $x$ is some individual, "$x$ is a $K$" entails "$x$ is not an $L$", and "$x$ is an $L$" entails "$x$ is not a $K$". Thus "Walter is a dog" entails "Walter is not a man", and "Walter is a man" entails "Walter is not a dog". The fact that we can conceive of a class of borderline cases of vertebrates does not help the *stronger* principle of logical continuity. The class of borderline cases *is not coördinate with the class of vertebrates or invertebrates.*

Can we conceive of a third, *and coördinate*, class between the Cuvierian classes *vertebrata* and *invertebrata*? Let's try. Imagine a gradual continuum of animals ranging from a clear case of a vertebrate, say a frog, to a clear case of an invertebrate, say a flat-worm. Obviously this continuum of specimens could be divided into as many classes of animals as we find useful. We could, for example, divide the continuum into three coördinate classes: **As, Bs,** and **Cs.**

Vertebrates Invertebrates
Frog ---------------------------------------------------- Flat-Worm
**As** **Bs** **Cs**

If we did this, *could* **Bs** form a coördinate class between the Cuvierian vertebrates and invertebrates? The suggestion is tempting, but, I believe, mistaken. "This animal is a $B$" certainly would not entail "This animal is not an invertebrate"; it would entail "This animal is not an $A$" and "This animal is not a $C$".

8. See Max Black, *Language and Philosophy,* Cornell University Press, Ithaca, New York, 1949, p. 28.

9. Hamilton, *Op. Cit.,* p. 134.

10. *Ibid.*

If the class of **As** were one and the same as the class of vertebrates, and the class of **Cs** were one and the same as the class of invertebrates, we could hold that **Bs** formed a class *indirectly* coördinate with the classes *vertebrata* and *invertebrata,* i.e. the three classes could be said to be coördinate *via* this pair of identities. But there are gounds for saying that these classes cannot be one and the same.

A class, **Ks,** and a class, **Ls,** are one and the same if and only if 1) everything that is a $K$ is an $L$, 2) everything that is an $L$ is a $K$, 3) every borderline case of a $K$ is a borderline case of an $L$, and 4) every borderline case of an $L$ is a borderline case of a $K$. Conditions (3) and (4) follow from conditions (1) and (2) and thus do not really need to be written in. It would be self-contradictory to claim that all **Ks** were **Ls** and all **Ls** were **Ks** and also claim that there were borderline cases of **Ks** that were not borderline cases of **Ls,** or that there were borderline cases of **Ls** that were not borderline cases of **Ks.** The class of **As** cannot be one and the same as the class of Cuvierian vertebrates, nor can be class of **Cs** be one and the same as the class of invertebrates, because there are borderline cases of vertebrates that are not borderline cases of **As** (e.g. a clear case of a $B$), and there are borderline cases of invertebrates that are not borderline cases of **Cs** (e.g. a clear case of a $B$).

Apparently we cannot conceive of a third, *and coördinate*, class between two classes distinguished from each other by contradictory attributes. Hamilton seems to have given us a decisive counter-case to the stronger principle of logical continuity, and thus shown that principle to be false.

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