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A NOTE ON NATURAL DEDUCTION

ALEX BLUM

Theorem: *If S is a system of natural deduction which has an effective procedure for transforming its formulae into conjunctive normal form* (from now 'cnf') *and counts among its rules of inference Conjunction and*

$$\text{Tautology-Introduction } (\mathbf{T\text{-}I}): \quad \frac{\ldots p \vee \ldots \vee \sim p \ldots}{\ldots \sim p \vee \ldots \vee p \ldots}$$

then S has a proof procedure for validity which is (transparently) *effective.*[1]

Proof: Let u be any valid formula in S. Let

$$u': a_1 \cdot a_2 \cdot a_3 \cdot \ldots \cdot a_n$$

be the expansion of u in cnf. We shall now prove u' valid.

1.	a_1	**T-I**
2.	a_2	**T-I**
3.	a_3	**T-I**
⋮		
n.	a_n	**T-I**
$n+1$.	$a_1 \cdot a_2 \cdot a_3 \cdot \ldots \cdot a_n$	1, 2, 3, Conjunction

QED

1. The theorem also holds if in place of **T-I**, S contains: Conditional Proof, Addition, Tautology, Implication and Commutation as rules of inference.

Proof:

1′.	p	assumption of limited scope
2′.	$p \vee p$	1′, Addition
3′.	p	2′, Tautology
4′.	$p \supset p$	1′-3′, Conditional Proof
5′.	$\sim p \vee p$	4′, Implication
6′.	$\sim p \vee p \vee \ldots$	5′, Addition

Clearly, any inference sanctioned by **T-I** will be either 5′ or a Commutation variant of either 5′ or 6′.

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Should either of the steps 1, 2, 3, . . . or n, fail to be instances of **T-I**, then clearly, contrary to our hypothesis, either u' is not in cnf or it is not valid.

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