

Erratum

Erratum for Christian Espíndola, “Semantic Completeness of First-Order Theories in Constructive Reverse Mathematics,” *Notre Dame Journal of Formal Logic*, vol. 57, no. 2 (2016), pp. 281–86. DOI [10.1215/00294527-3470433](https://doi.org/10.1215/00294527-3470433).

The proof of the implication $2 \implies 1$ of Theorem 3 was incorrect. Here is a correct proof.

($2 \implies 1$) Let \mathcal{B} be a Boolean algebra, and consider the theory Γ over a language which has a constant for every element of \mathcal{B} (we shall identify such elements with the constants themselves), a unary relation F , ($F(a)$ is thought of as the assertion “ a is in the filter”), and whose axioms are the following:

1. $F(1) \wedge \neg F(0)$;
2. $F(a) \rightarrow F(b)$ for each pair $a \leq b$ in \mathcal{B} ;
3. $F(a) \wedge F(b) \rightarrow F(a \wedge b)$ for every pair a, b in \mathcal{B} ;
4. $F(a) \vee F(\neg a)$ for each a in \mathcal{B} .

Since this theory is finitely satisfiable (because every finite subset of \mathcal{B} generates a finite subalgebra where one can construct an ultrafilter), it is consistent, and hence, by hypothesis, there is a model \mathcal{M} with a satisfaction relation \models . Define now $\mathcal{U} = \{a \in \mathcal{B} : \mathcal{M} \models F(a)\}$. It is easy to prove that \mathcal{U} is an ultrafilter of \mathcal{B} (and so it follows that every Boolean algebra has an ultrafilter, which is a well-known equivalent of Boolean prime ideal theorem). Indeed, $1 \in \mathcal{U}$ since $\mathcal{M} \models F(1)$, and $0 \notin \mathcal{U}$ since $\mathcal{M} \not\models F(0)$ by the consistency property. If a and b are in \mathcal{U} , then $\mathcal{M} \models F(a)$ and $\mathcal{M} \models F(b)$, so by soundness we have $\mathcal{M} \models F(a \wedge b)$ and hence, $a \wedge b$ belongs to \mathcal{U} . If $a \leq b$ and a is in \mathcal{U} , then $\mathcal{M} \models F(a) \rightarrow F(b)$, and since $\mathcal{M} \models F(a)$, by soundness we get $\mathcal{M} \models F(b)$ and so b is in \mathcal{U} . Finally, since for every a in \mathcal{B} we have $\mathcal{M} \models F(a) \vee F(\neg a)$, then by the consistency property we have that either $\mathcal{M} \models F(a)$ or $\mathcal{M} \models F(\neg a)$; that is, either a or $\neg a$ is in \mathcal{U} .