Tito A. Mijares, *Percentage Points of the Sum $V_1^{(s)}$ of s Roots* ($s = 1 - 50$) *A Unified Table for Tests of Significance in Various Univariate and Multivariate Hypotheses*. Statistical Center, University of Philippines, Manila, 1964. vii + 241 pp. $8.00.

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The opening sentences of the preface are: "This book consists of two sets of tables that are mostly not available in current literature. Its purpose is to fill an important gap in unifying the treatment of tests of significance in various univariate and multivariate hypotheses. A comprehensive account of distribution theories under various null hypotheses is given in the introduction with numerical examples in each case that statisticians in both academic and non-academic fields will find highly useful." Clearly the author intends the book not merely as a set of tables with limited usefulness but also to have value as an attempt to unify the treatment of tests of significance. Unfortunately the present reviewer finds neither objective achieved adequately and, in fact, doubts the existence of any value at all in the publication.

The first twenty-eight pages deal with introductory material concerning normal theory tests of various standard univariate and multivariate hypotheses, while the succeeding eight pages explain the two sets of tables and include a list of references. The first set of tables occupies 105 pages and the second set 98 pages.

Both tables are associated with a test statistic, proposed by Pillai (1954), (1955), which is the trace (or sum of the latent roots) of a certain matrix that arises in different guises in various classical multivariate tests of significance. The standardized third and fourth moments, β_1 and β_2, of the null distribution of this statistic were obtained by Pillai and Mijares (1959), who suggested also that using these values of β_1 and β_2 one could fit a Pearson type distribution to the null distribution in question. Earlier Pillai (1954), (1955) had suggested a Type I beta distribution as an alternate approximation for the same distributional problem.

The first set of tables gives percentage points based on the Pearson type distributional approximation when the number, s, of observed non-zero latent roots takes on values 1(1)50. When $s = 1$ the distribution in question is exactly a Type I beta distribution and lower and upper 0.5%, 1%, 2.5% and 5% points are tabulated for various combinations of values of the two "degrees-of-freedom" parameters. When $s > 1$, only the lower and upper 1% and 5% points are tabulated.

The second set of tables provides, for $s = 2(1)50$, values of the two shape parameters of the Type I beta distribution fitted by matching the first two moments alone, as suggested by Pillai (1954), (1955).

The introductory material which is an attempt at "unifying the treatment of tests of significance" is no more than a regurgitation of well-known results that null distributions in univariate normal theory are special cases of the "corresponding" multivariate distributions. What is not mentioned is that there is no unique correspondence between univariate and multivariate tests of significance and, even with the constraint of affine invariance, when $s > 1$, one is led to several alternate functions of the latent roots of which one is the statistic considered in the present book. All the tests considered by the author as special cases of the statistic $V_1^{(s)}$ are also special cases of each of the other multivariate test statistics that have been proposed. The unification attempted, is, therefore, neither unique nor novel and, as far as the tables are concerned, the present reviewer feels that the generality of them will not lead, for various obvious reasons, to such special cases as Student's t-tables and F-tables being supplanted by the tables in the present book.

The errors in the introductory material range from minor non-technical ones to major implicit and explicit technical ones. The minor errors include: (i) references mentioned in the text but not included in the list of references (e.g. Roy (1939) on p. 1, K. Pearson (1934) on pp. 7 and 8, Pillai (1956) on p. 29); and (ii) poorly worded and/or imprecise statements (e.g. "The distribution obtained from samples of normal population under various types of null hypotheses can be reduced into that of the generalized beta distribution given by . . .", "As in the univariate case the distributions under null hypotheses in many types of multivariate problems are reducible into the form (1) by using analogous transformations", "In two-tailed tests the confidence probability for the 2α level of significance becomes . . .", "If $\mathbf{Y}$ is obtained from a sample of size n, viz., the mean vector $\bar{\mathbf{Z}} = \bar{\mathbf{X}} - \nu, \cdots$").

In an intermediate category of errors belong imprecise and unproductive statements like the following: (i) On p. 6, while discussing the likelihood-ratio test for homogeneity of several variances the statement is made that the null distribution of the likelihood-ratio "can be adequately represented by the beta-function distribution . . .". It is not clear that the author is aware of the work of Bartlett (1937) and numerous others on this problem, and he does not justify the adequacy of his approximation which is different from the more familiar one. (ii) On p. 33, in describing the second set of tables, which give the two parameters of a fitted beta distribution, the author states, "For $s > 1$, the 'goodness' of the fitting may be gleaned numerically from the differences of the third and fourth moments of the approximate distribution from those of the true distribution". Clearly the author does not here explicitly recognize that fitting moments is not simply related to fitting percentage points or tail areas, which he appears to have recognized in an earlier paper. (Pillai and Mijares (1959).)

As examples of errors of a more major variety, the following may be mentioned: (i) On p. 2, talking about the hypothesis of no bivariate correlation, the author states, "In a sense the hypothesis is likely to be true the nearer r^2 is to unity." (ii) On p. 4, where there is a discussion of the two-sample test for equality of variances, the author says, "For a $2\alpha\%$ level of significance, we use $\alpha\%$ on each tail." It is not clear that the author is aware that such a procedure does not in general yield an unbiased test. (iii) On p. 9 appears the statement, "Given two independent χ_i^2, with respective d.f. ν_i, and variance σ_i^2, $i = 1, 2, \cdots$." (iv) In several places dealing with the multivariate case (e.g. statement following equation (2.3) on p. 14 and statements in Section 3.3 on p. 26), the author does not seem to realize that a Wishart distribution for a $p \times p$ covariance matrix is only defined when its dimension, p, does not exceed its "degrees-of-freedom". In situations when this constraint on p is not met, however, the distribution of the non-null latent roots is a well-defined entity and this distribution is all that one needs. (See Section 2.3 of Roy and Gnanadesikan (1959).) (v) The recurrence relationship, for the moments about the origin, given on pp. 30 and 31, are valid only for the multidimensional integral excluding the normalizing constant, $c(s, m, n)$, and not for quantities defined as in μ_1 (which should moreover be μ_1') on p. 29. (See Pillai and Mijares (1959).)

The above list of errors in the introductory material is unfortunately incomplete and is only indicative of the types of errors and not their numbers. The last sentence of the author's preface is, "There is no guaranty that this book is free from errors of any kind and I would be grateful for any error brought to my attention by the user".

The "highly useful" numerical examples are far from insightful analyses of the data in each case which are merely used to show how a statistic calculated from them could be compared with the tabulated percentage points. In fact, almost all the examples, including the computations of the basic statistics involved in making the test of significance, are taken over from the literature sources cited. The only difference lies in the final comparison of the statistic with the percentage points tabulated here instead of more familiar tables such as F-tables.

Regardless of the introductory material, it is perhaps legitimate to inquire if the tables *per se* are likely to be useful. The present reviewer does not, for a variety of reasons, think that these tables in their present form have sufficient useful content to warrant their bulk or even their publication. They do not have significant additional value over the published tables of Pillai (1957), (1960). The present tables are admittedly more extensive in various ways such as the considerably larger maximum values considered for the number of observed non-zero roots and for the two sets of "degrees of freedom". For such large values, however, the reviewer for one feels that an explicit, simple and unifying "asymptotic" result would be far more useful than the bulkiness of the present tables which may be a concomitant of the approach of simply extending the range of values of a tabulation.

For the special cases, such as the t and F, the existent tables of percentage points for these individual cases are far more versatile than the present tables. For the multivariate case when $s > 2$, not enough is known about the alternate test procedures available to feel that the statistic of concern here is centrally important. As far as the user is concerned, granting even that he is truly interested in a test of the usual extreme null hypothesis against the completely general alternative hypothesis, the approximation of the null distribution here by a familiar distribution like the beta distribution is all that is likely to be of interest. In the present era of computers, therefore, the formulae for computing the parameters of such an approximating distribution and some computable approximation for getting percentage points and/or tail areas of the approximating distribution are all that would be needed. Certainly one does not need bulky tables giving 3 or 4 decimal places (with no discussion of accuracies involved) for getting just four percentage points.

Purely from considerations of numerical analysis, the tables may be criticized for inadequate attention to interpolation and extrapolation questions and insufficient information on accuracies and details of procedures used to obtain the percentage points from those in the Pearson and Hartley tables. Incidentally, the present reviewer for one wonders what intrinsic value obtains from knowing, to 3 decimal places, values like 349.199 and 775.052 for the parameters of an approximating beta distribution. The bulk of these values tabulated in the second set of tables are either in the hundreds or the thousands!

In summary, hence, the present reviewer cannot unfortunately find anything positively good about the book to recommend it even for the limited objectives of usefulness for tests of significance. More importantly, any such massive effort of computation, whose sole use is for dubious tests of hypotheses, is of questionable value in the light of the real needs of data analysis. To illustrate the point in the current context, if one were interested in assessing the homogeneity of a collection of variances as in Example 4 on p. 7, then a more instructive analysis than the one provided by the author would be a chi-squared probability plot of the ordered variances. The present reviewer did in fact make such a probability plot and he found interesting indications of possible peculiarities of the data including distributional ones. Unfortunately he could not pursue the analysis further since the reference in this case is one whose details are not included in the bibliography!

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