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Erratum: Optimal linear drift for the speed of convergence of an hypoelliptic diffusion*†

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Abstract

Erratum for *Optimal linear drift for the speed of convergence of an hypoelliptic diffusion*, A. Guillin, and P. Monmarché, Electron. Commun. Probab. 21 (2016), paper no. 74, 14 pp. doi:10.1214/16-ECP25.

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The authors correct the two following mistakes:

1. At page 5, line -20, it is proved in [13, Corollary 12] that the entropy converges at rate $2\rho(A)$

$$\mathrm{Ent}_{\psi_\infty}\left(e^{tL^*_{A,D}}h\right) \quad \leq \quad ce^{-2\rho(A)t}\mathrm{Ent}_{\psi_\infty}(h)\,,$$

   and not simply $\rho(A)$ as it has been written.

2. At page 9, line 8, C should be replaced by C^T:

$$\partial_t\left(\alpha''(h_t)\left(\nabla h_t\right)^T M\nabla h_t\right) \quad \leq \quad 2\alpha''(h_t)\left(\nabla h_t\right)^T MC^T\nabla h_t$$

   Indeed, the Jacobian Matrix of the function $b(x) = Cx$ is C^T and not C. This initial mistake has the following chain of consequences:

   - At page 9, from line 9 to 15, $S^{\frac{1}{2}}$ should be systematically replaced by $S^{-\frac{1}{2}}$. For the computations to hold, the matrix $\widetilde{J}$ should be taken equal to its opposite, meaning that at page 8, the line -5 should be

$$\left(\widetilde{J}\right)_{k,l} = \frac{\nu_k + \nu_l}{\nu_k - \nu_l}.$$

   - At page 9, the computation from line -6 to line -3 should be replaced by

$$\mathrm{Ent}_{\psi_\infty}(h_t) \quad \leq \quad \frac{1}{2}\int \frac{(\nabla h_t)^T S^{-1}\nabla h_t}{h_t}\mathrm{d}\psi_\infty$$

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$$
\begin{aligned}
&\le \frac{1}{2\nu_1}\int \frac{\left|Q^{\frac{1}{2}}S^{-\frac{1}{2}}\nabla h_t\right|^2}{h_t}\mathrm{d}\psi_\infty \\
&\le \frac{e^{-2\lambda(t-s)}\nu_N}{2\nu_1}\int \frac{\left|S^{-\frac{1}{2}}\nabla h_s\right|^2}{h_s}\mathrm{d}\psi_\infty, \\
&\le \frac{\nu_N}{2\nu_1 \min\sigma(S)}\, e^{-2\lambda(t-s)}\int \frac{|\nabla h_s|^2}{h_s}\mathrm{d}\psi_\infty.
\end{aligned}
$$

Note that an annoying factor $\frac{\max\sigma(S)}{\min\sigma(S)}$ has disappeared.

As a consequence of both these corrections, the main result is improved to the following correct statement:

Theorem 2. For any $C > 1$ we can construct $(A, D) \in \mathcal{I}(S)$ such that for all $h > 0$, with finite entropy, and for all $t, t_0 > 0$ with $t \ge t_0$,

$$
Ent_{\psi_\infty}\left(e^{(t-t_0)L^*_{A,D}}e^{t_0 L_{-S,I_N}}h\right) \le C\frac{1}{2t_0\min\sigma(S)}e^{-2(\max\sigma(S))(t-t_0)}Ent_{\psi_\infty}(h).
$$

Moreover it is possible to construct $(A, D) \in \mathcal{I}(S)$ with $\|A\|_F \le 4N^2\sqrt{\frac{(\max\sigma(S))^3}{\min\sigma(S)}}$ (where $\|A\|_F = \sqrt{\mathrm{Tr}\,(A^T A)}$ is the Frobenius norm) such that for all $h > 0$, with finite entropy, and for all $t \ge t_0 > 0$

$$
Ent_{\psi_\infty}\left(e^{(t-t_0)L^*_{A,D}}e^{t_0 L_{-S,I_N}}h\right) \le \frac{1}{t_0\min\sigma(S)}e^{-2(\max\sigma(S))(t-t_0)}Ent_{\psi_\infty}(h).
$$