pothesis; that is to say, it is sufficient that the hypothesis be true for the thesis to be true; while it is necessary that the thesis be true for the hypothesis to be true also. When a theorem and its reciprocal are true we say that its hypothesis is the necessary and sufficient condition of the thesis; that is to say, that it is at the same time both cause and consequence.

5. Principle of Identity.—The first principle or axiom of the algebra of logic is the *principle of identity*, which is formulated thus:

(Ax. I) $$a < a,$$

whatever the term a may be.

C. I.: "All a's are a's", i. e., any class whatsoever is contained in itself.

P. I.: "a implies a", i. e., any proposition whatsoever implies itself.

This is the primitive formula of the principle of identity. By means of the definition of equality, we may deduce from it another formula which is often wrongly taken as the expression of this principle:

$$a = a,$$

whatever a may be; for when we have

$$a < a, \quad a < a,$$

we have as a direct result,

$$a = a.$$

C. I.: The class a is identical with itself.

P. I.: The proposition a is equivalent to itself.

6. Principle of the Syllogism.—Another principle of the algebra of logic is the principle of the *syllogism*, which may be formulated as follows:

(Ax. II) $$(a < b) \quad (b < c) < (a < c).$$

C. I.: "If all a's are b's, and if all b's are c's, then all a's are c's". This is the principle of the *categorical syllogism*.

P. I.: "If a implies b, and if b implies c, a implies c." This is the principle of the *hypothetical syllogism*.

We see that in this formula the principal copula has always the sense of implication because the proposition is a secondary one.

By the definition of equality the consequences of the principle of the syllogism may be stated in the following formulas[1]:

$$(a < b) \quad (b = c) < (a < c),$$
$$(a = b) \quad (b < c) < (a < c),$$
$$(a = b) \quad (b = c) < (a = c).$$

The conclusion is an equality only when both premises are equalities.

The preceding formulas can be generalized as follows:

$$(a < b) \quad (b < c) \quad (c < d) < (a < d),$$
$$(a = b) \quad (b = c) \quad (c = d) < (a = d).$$

Here we have the two chief formulas of the *sorites*. Many other combinations may be easily imagined, but we can have an equality for a conclusion only when all the premises are equalities. This statement is of great practical value. In a succession of deductions we must pay close attention to see if the transition from one proposition to the other takes place by means of an equivalence or only of an implication. There is no equivalence between two extreme propositions unless all intermediate deductions are equivalences; in other words, if there is one single implication in the chain, the relation of the two extreme propositions is only that of implication.

7. Multiplication and Addition.—The algebra of logic admits of three operations, *logical multiplication*, *logical addition*, and *negation*. The two former are binary operations, that is to say, combinations of two terms having as a consequent a third term which may or may not be different from each of them. The existence of the logical product and logical sum of two terms must necessarily answer the purpose of a

---

1 Strictly speaking, these formulas presuppose the laws of multiplication which will be established further on; but it is fitting to cite them here in order to compare them with the principle of the syllogism from which they are derived.