Part C

Infinitary Languages

This part of the book is devoted to languages with infinitely long formulas and their applications. Again the structures are of the sort studied in first-order model theory. Languages with richer structures and infinitely long formulas are studied in Part E. The study of infinitely long formulas is more developed than some of the other parts of extended model theory. In particular, there are several books treating various aspects of the subject, notably Keisler [1971a] and Dickmann [1975]. This part of the present book was planned with the existence of these references in mind, containing chapters that give an introduction to the subject leading into these books as well as chapters that discuss more recent advances.

Chapter VIII presents a wealth of material on $\mathscr{L}_{\omega_1\omega}$ and some of its sublogics. Starting with the original motivations for studying languages with infinitely long formulas, the chapter provides both a basic introduction and an explanation of many of the developments that have taken place since Keisler's [1971a] publication. In addition, it discusses extensions of $\mathscr{L}_{\omega_1\omega}$ by new propositional connectives. The importance of these extensions is not for their intrinsic interest so much, as for the fact that they seem to have all the nice properties of $\mathscr{L}_{\omega_1\omega}$, and so make it difficult to find a characterization of $\mathscr{L}_{\omega_1\omega}$ by its model-theoretic properties.

Chapter IX presents an introduction to the stronger logics $\mathscr{L}_{\kappa\lambda}$, one that leads into Dickmann's book [1975] on this topic but also goes beyond it with the presentation of some more recent results. Special emphasis is given to partial isomorphisms and their applications, and to Hanf number computations.

One of the more recent developments in infinitary logic is that dealing with game quantification which has grown out of the work of Svenonius [1965], Moschovakis [1972] and Vaught [1973b]. The logic $\mathscr{L}_{\omega_1\omega}$ and $\mathscr{L}_{\infty\omega}$ allow only finite strings of quantifiers at any stage in the transfinite process of building formulas. $\mathscr{L}_{\omega_1\omega_1}$ and $\mathscr{L}_{\infty\omega_1}$ permit infinitely long strings of the forms

$$\forall x_1\, \forall x_2 \ldots \phi(x_1, x_2, \ldots)$$

and

$$\exists x_1\, \exists x_2 \ldots \phi(x_1, x_2, \ldots).$$