Uniform Interpolation and Layered Bisimulation *

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Summary. In this paper we give perspicuous proofs of Uniform Interpolation for the theories IPC, K, GL and S4Grz, using bounded bisimulations. We show that the uniform interpolants can be interpreted as propositionally quantified formulas, where the propositional quantifiers get a semantics with bisimulation extension or bisimulation reset as the appropriate accessibility relation. Thus, reversing the conceptual order, the uniform interpolation results can be viewed as quantifier elimination for bisimulation extension quantifiers.

1. Introduction

Bisimulation and bounded bisimulation can be used to 'visualize' proofs. The aim of this paper is to present proofs for uniform interpolation results as clearly and perspicuously as possible using bounded bisimulation. Ordinary interpolation for a given theory T says that if $\mathsf{T} \vdash A \to B$, then there is a formula $I(A, B)$ in the language containing only the shared propositional variables, say $\mathbf{q}$, such that $\mathsf{T} \vdash A \to I$ and $\mathsf{T} \vdash I \to B$. Uniform interpolation is a strengthening of ordinary interpolation in which the data in terms of which the interpolant is to be specified are weaker: the interpolant can be found from either A and $\mathbf{q}$ or from $\mathbf{q}$ and B. Thus, if uniform interpolation holds, there is, for every A and $\mathbf{q}$, a 'post-interpolant' $I(A, \mathbf{q})$ such that $\mathsf{T} \vdash A \to I(A, \mathbf{q})$ and, for all B such that $\mathsf{T} \vdash A \to B$ and such that the shared propositional variables of A and B are among $\mathbf{q}$, we have $\mathsf{T} \vdash I(A, \mathbf{q}) \to B$. Similarly for the 'pre-interpolant'. As we will see, uniform interpolation can be viewed as quantifier elimination for certain propositional quantifiers in T: the quantifiers that correspond to the trans-model accessibility relation *bisimulation extension* (or: *bisimulation reset*).

In this paper we prove Uniform Interpolation for IPC (Intuitionistic Propositional Calculus), for K, for GL (Löb's Logic) and for S4Grz.[1] Uni-

* The present paper is in its final form and no similar paper has been or is being submitted elsewhere.

[1] Uniform Interpolation for IPC was first proved by Pitts using proof theoretical methods. It was proved by the present method by Ghilardi and Zawadowski and, independently but later, by the author. Uniform Interpolation for K is due to Ghilardi. Uniform Interpolation for GL was first proved by Shavrukov. It was proved by the present method by the author. To give the due credit it should be pointed out that the method here is similar to the one used by Ghilardi and Zawadowski and, independently, the author, to prove the result for IPC. The result for S4Grz is, as far as I know, new in this paper.