ON THE CONSTRUCTION OF A MATHEMATICAL THEORY OF THE IDENTIFICATION OF SYSTEMS

RICHARD BELLMAN
THE RAND CORPORATION

1. Summary

Let us consider a complex system consisting of a set of interacting subsystems. Let $x_i(t)$ represent the state vector for the i-th subsystem, and suppose that the behavior over time of these vectors is determined by a set of coupled functional equations

$$\frac{dx_i}{dt} = g_i(x_1, x_2, \cdots, x_N, a_i), \qquad i = 1, 2, \cdots, N, \tag{1.1}$$

where the a_i are parameters determining both the structure of the subsystems and the linkage between these subsystems.

Much of classical analysis is devoted to the qualitative and quantitative analysis of the $x_i(t)$ as functions of t and of the structural parameters. Important as this effort is, it represents only a part of the principal objective, which is that of the identifieation of physical systems. By the "identification of systems," we mean the task of determining the structural parameters on the basis of observations over time and position of the inputs and outputs. This is an essential part of the validations of hypotheses and theories.

2. Introduction

Let us consider a complex system consisting of a set of interacting subsystems: Let $x_i(t)$ represent the state vector for the i-th subsystem, and suppose that the behavior over time of these vectors is determined by a set of coupled functional equations

$$\frac{dx_i}{dt} = g_i(x_1, x_2, \cdots, x_N, a_i), \qquad i = 1, 2, \cdots, N, \tag{2.1}$$

where the a_i are parameters determining both the structure of the subsystems and the linkage between these subsystems.

This research is sponsored by the United States Air Force under Project RAND—Contract No. AF 49(638)-700 monitored by the Directorate of Development Plans, Deputy Chief of Staff, Research and Development, Hq USAF. Views or conclusions contained in this Memorandum should not be interpreted as representing the official opinion or policy of the United States Air Force.