RANDOM OPERATOR EQUATIONS

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1. Introduction and summary

In recent years a number of papers have appeared in which random variables with values in spaces more general than the space of real or complex numbers were dealt with. These papers differ mainly in their approach to the generalization of notions known from classical probability theory. Thus measurability, integrability, convergence, and so on, are treated differently in different papers.

Many of these papers studied some problems connected with the theory of random operator equations. It should be mentioned that Czechoslovak probabilists have systematically attacked this part of probabilistic functional analysis since the year 1955, when Špaček [22] published his paper on random equations. Dealing with contraction mappings, Špaček proved the measurability of the random fixed point in the case of the space of real numbers and, under some further restrictive assumptions, the measurability of the fixed point for arbitrary complete metric spaces. All Špaček's restrictive assumptions were removed in [12], provided the complete metric space is question is separable. The proof of this result is based on the limit theorem for arbitrary metric spaces (see [13]) and on the substitution theorem for separable metric spaces (see [13]).

Under essentially the same restrictive assumptions, Špaček studied in [23] the measurability of invertible random transformations, being partially motivated by the necessary and sufficient condition for almost sure regularity of "function-space type" measurable transformations as developed in [21].

Further papers closely connected with random operator equations are concentrated mainly in the Transactions of the First and Second Prague Conferences. Of these [9], [13], [16], and [20] deal with general properties of generalized random variables; [7], [8], [10], [11], [14], and [17] deal with stochastic approximation methods and experience theory; and [15] deals with inverse and adjoint transformations. Some other papers published in the Transactions, though not directly related to random operator equations, have played an important part in the process of forming the ideas of experience theory and their connection with general stochastic approximation methods.

So much for the contributions of the Czechoslovak school of probabilistic functional analysis. Probabilists of other countries have, of course, contributed to a great extent to the development of this area of probability theory, and a number of references can be found in [9].