DECISION THEORY FOR PÓLYA TYPE DISTRIBUTIONS. CASE OF TWO ACTIONS, I

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1. Introduction

In a recent paper by the author and H. Rubin [1] a thorough analysis was made of the form of essentially complete classes when the densities $p(x \mid \omega)$ have monotone likelihood ratios. The form of the Bayes strategies was determined and complete classes of strategies were characterized. In order to carry out this analysis, very stringent conditions were imposed on the loss functions. For example, in the case of two actions it was assumed that $L_1(\omega) - L_2(\omega)$ possesses at most one sign change. This condition of one sign change is intrinsically connected to the fact that the density $p(x \mid \omega)$ has a monotone likelihood ratio. In this paper we deal with the general case of two actions where no assumptions are made concerning the number of sign changes of $L_1(\omega) - L_2(\omega)$. However, the class of densities $p(x \mid \omega)$ is specialized to that of the exponential family in the first half of this paper and then the results are extended to a class of distributions which we call Pólya type distributions. Although the exponential distributions constitute a subfamily of the Pólya type distributions, we have chosen to treat all the decision theory associated with this class of distributions separately in order to best illustrate the scope of the ideas and methods developed below. In section 8 the results for the exponential family of distributions are then extended to the case when the underlying distributions are Pólya type.

A thorough study is made of form of complete classes, Bayes strategies and admissibility. Further results are obtained which relate Liapounoff's theorem on the range of a vector measure to the family of distributions under consideration.

2. Preliminaries and definitions

Let the observed random variable be denoted by x ranging over X and the unknown state of nature by ω in Ω. The sets X and Ω are one-dimensional and will be identified with subsets of the real line. To expedite the discussion we take Ω to be an interval.

The cumulative distribution of x, when the state of nature is ω, is assumed to belong to the exponential family, that is,

$$P(x, \omega) = \beta(\omega) \int_{-\infty}^{x} e^{t\omega} d\mu(t), \tag{1}$$

where $\beta(\omega) > 0$ for ω in Ω and μ is a σ-finite measure defined on the real line with the spectrum of μ equal to X. (The spectrum of μ is defined to be the set of all x such that for

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