Institute of Mathematical Statistics
LECTURE NOTES–MONOGRAPH SERIES

Small Sample Asymptotics

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Hayward, California

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The production of the *IMS Lecture Notes–Monograph Series* is managed by the IMS Business Office: Jessica Utts, IMS Treasurer, and Jose L. Gonzalez, IMS Business Manager.

Library of Congress Catalog Card Number: 90-84215

International Standard Book Number 0-940600-18-8

Printed in the United States of America

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PREFACE

The aim of this monograph is to provide a coherent development of the ideas and techniques of small sample asymptotics. This term was coined by Frank Hampel and reflects our aims of obtaining asymptotic expansions which give accurate results for small sample sizes n, even down to $n = 1$. The central question is that of finding good approximations to the density of statistics in situations where the computation of the exact density is intractable. The techniques presented here have a broad applicability which we have tried to demonstrate in several sections of the monograph.

This monograph brings together results from a number of papers over the past 35 years. The beginning point for the authors were the fundamental papers by Daniels (1954) and Hampel (1973) in which the basic ideas of the approximation for the mean were developed. Both authors are indebted to both Professors Daniels and Hampel for their support and encouragement in putting together this monograph.

In the development and presentation of the approximations, a basic principle was that the approximations had to be computable. To that end, we have included many numerical examples by means of which the reader can judge the quality of the approximations. Our techniques provide alternatives to Edgeworth expansions on the one hand and the bootstrap on the other. The small sample approximations are simpler than Edgeworth expansion and can be thought of as a series of low order Edgeworth expansions. We obtain much better numerical accuracy than the Edgeworth and our density approximations are always positive unlike the Edgeworth. The direct approximation of the cumulative distribution function allows us to by pass the resampling of the bootstrap in the construction of confidence intervals, for instance.

Our presentation begins with results for the mean and then uses these techniques and results to obtain approximations for more interesting statistics. In all the results, there is the key idea of recentering the underlying density at the point where we want to approximate the density of the statistics and then using a normal approximation at that point. The ideas can be developed in the complex plane by using saddlepoint and steepest descent techniques and in the reals by using conjugate densities. Where possible we give a parallel development to contrast the two approaches.

The material in the book is reasonably self-contained and should be accessible to graduate students in statistics. Our hope is that the readers will be stimulated to use and develop small sample approximations in problems of interest to them.

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April 1990