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A bivariate optimal replacement policy with cumulative repair cost limit for a two-unit system under shock damage interaction

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Abstract. In this paper, a bivariate (n, k) replacement policy with cumulative repair cost limit for a two-unit system is studied, in which the system is subjected to shock damage interaction between units. Each unit 1 failure causes random damage to unit 2 and these damages are additive. Unit 2 will fail when the total damage of unit 2 exceed a failure level K, and such a failure makes unit 1 fail simultaneously, resulting in a total failure. When unit 1 failure occurs, if the cumulative repair cost till to this failure is less than a predetermined limit L, then unit 1 is corrected by minimal repair, otherwise, the system is preventively replaced. The system is also replaced at the nth unit 1 failure, or at damage level k $(<K)$ of unit 2, or at total failure. The explicit expression of the long-term expected cost per unit time is derived and the corresponding optimal bivariate replacement policy can be determined analytically or numerically. Finally, a numerical example is given to illustrate the theoretical results for the proposed model.

1 Introduction

The maintenance for multi-unit systems has became more and more complex in the last few decades, because the systems are becoming more complicated having many interacting or dependent units. For this reason, the modelling and optimization of preventive maintenance actions is also more complicated than before. Several preventive maintenance (abbreviated PM) models for multi-unit systems were reviewed in Nicolai and Dekker (2008) and Nowakowski and Werbińka (2009). The failure times are often stochastically dependent between units in the multi-unit system; these dependencies can be classified into economic, structural and stochastic dependencies in Thomas (1986):

(1) Economic dependence between units shows that performing maintenance on several units at the same time is cheaper than the total cost for individual unit separately. By this fact, economic dependence also offers the opportunity to execute group maintenance of many units which can save costs (Nicolai and Dekker

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(2008)). Focused on the concept of economic dependence, Dekker, Wildeman and van der Duyn Schouten (1997) reviewed many literatures on multi-unit maintenance models and distinguished these models into two categories: stationary models and dynamic models.

(2) Structure dependence means that it must maintain the other units simultaneously when we repair the failed units, because the units in the system are always constructed into a whole unit.

(3) Stochastic dependence means that the state (e.g., the age, the failure rate or the model of failure) of one unit influences the lifetime distribution of other units, or there are some external causes to bring these units into simultaneous failure and hence the lifetimes of these units are correlated. Sun et al. (2006) developed an analytical model to analyze this type of failure interaction quantitatively and verify the model using case studies and experiments.

In category of stochastic dependency, Murthy and Nguyen (1985a) first introduced the concept of failure interaction between units; furthermore, failure interactions can be divided into three groups:

(3-1) Type I failure-interaction: When a unit fails, it can induce simultaneous failure or no effect to the other units, which is according to a fixed probability rule. Murthy and Nguyen (1985a) proposed such type of failure interaction for a two-unit system. Murthy and Nguyen (1985b) extended Type I failure interaction to a multi-unit system. Murthy and Wilson (1994) discussed the estimation problem for Type I failure interaction model under different data structures. Jhang and Sheu (2000) extended the fixed probability rule in Murthy and Nguyen (1985a) to be a probability function of time t. Golmakani and Moakedi (2012a) found the optimal periodic inspection interval over a finite time horizon for a two-unit repairable system with failure interactions. Golmakani and Moakedi (2012b) determined the optimal periodic inspection interval for a multi-unit repairable system with failure interactions.

(3-2) Failure rate interaction: When a unit fails, it acts an interior shock to the other units and then accelerates their failure rates. Such type of failure interaction is also called as Type II failure-interaction which is also introduced by Murthy and Nguyen (1985a). Murthy and Casey (1987) studied such type of failure interaction in a two-unit system and derived the optimal replacement policies. Lai and Chen (2006) proposed a periodic replacement model for a two-unit system subjected to failure rate interaction between units. Subsequently, Lai (2007a) extended the concept of failure rate interactions to a multi-unit system. Lai and Chen (2008) modified the model of Lai and Chen (2006) by adding the effect of external shocks. Sung et al. (2012) extended the model of Lai and Chen (2006) to develop a two-parameter $(n; T)$ replacement policy, in which the system is replaced at age T; upon the nth failure of unit-A, or at the first total failure, whichever occurs first.

(3-3) Shock damage interaction: When a unit fails in a two-unit system, it causes a random amount of damage to the other units. Such damages will be accumulated and the system fails when the total damage exceed a failure level. Nakagawa and Murthy (1993) derived an optimal replacement policy to minimize the expected cost rate per unit time, in which the replacement decision is based on the number of failure. Satow and Osaki (2003) extended the work of Nakagawa and Murthy (1993) and proposed a two-parameter $(T; k)$ replacement model for a two-unit system with shock damage interaction, in which the system is replaced at age T, at the total damage to unit 2 exceeds a damage level k, or at unit 2 failure. Wang and Zhang (2009) proposed an optimal replacement policy n^* for a two-unit system with shock damage interaction, in which the repair times follow a geometric process. Sheu et al. (2015) presented a two-parameter (T, N) replacement model for a two-unit system with shock damage interaction. It allows the two generalized situation, one is unit B can have a certain initial damage and the other is that unit B with cumulative damage level z may enter a minor failure state with probability $\pi(z)$ at each unit A minor failure.

Concerning PM models for a multi-unit system, minimal repair is corrective maintenance action that restores the system to an operational state without affecting the level of deterioration. Repair-cost limit policies with minimal repair, which prescribe repairing or replacing decision depending on a single repair cost, have been discussed in several articles. When a failure occurs, the necessary repair cost is evaluated. If the cost exceeds a certain level, the system is replaced; otherwise, it is repaired. Such a repair-cost limit policy was first studied by Drinkwater and Hastings (1967). Modified Single repair-cost limit policy, Lai (2007b) incorporated the concept of cumulative repair cost limit into a periodic replacement model, in which the information of all repair costs is collected for the decision for repairing or replacing the system. This paper overcame the shortcomings of the traditional single repair-cost limit policy. Followed the work of Lai (2007b), Chang, Sheu and Chen (2010) presented a preventive maintenance model for determining the optimal number of minimal repairs before replacement. Chien, Sheu and Chang (2009) extended the work of Lai (2007b) by introducing the random lead time for replacement delivery. Chien, Chang and Sheu (2010) modified the work of Chien, Sheu and Chang (2009) by adding an age-dependent type of failure. Sheu et al. (2010) presented a generalized model for determining the optimal replacement policy based on multiple factors (or more information) such as the number of minimal repairs before replacement and the cumulative repair cost limit. Lai (2012) modified the work of Lai and Chen (2006) by incorporating with the concept of the cumulative repair cost limit to a two-unit system. Chang, Sheu and Chen (2013) modified the work of Chang, Sheu and Chen (2010) by allowing an age-dependent failure type.

Under shock damage interaction between units, we want to incorporate the concept of cumulative repair cost limit to preventive maintenance model for a two-unit system. In real life application, we can have an illustrative example in chemical industry. A system consists of a metal container (unit 2) and pneumatic pump (unit 1) in which chemical reactions take place in metal container and and the temperature of the container is controlled by cold water pump. Whenever pneumatic pump fails, it causes a random amount of damage to metal container. The damage will be accumulated; and when the cumulative damage exceeds a threshold, the system fails. If pneumatic pump can be repaired after failure and then, the cumulative repair cost will be the indicator for the decision of replacing the system.

By adding the concept of cumulative repair cost limit, we extended the work of Satow and Osaki (2003) to present a bivariate (n, k) replacement policy for a two-unit system subject to shock damage interaction, in which the system is replaced preventively at the nth unit 1 failure, (2) when the total damage of unit 2 exceeds k and is less or equal to K, or (3) when the accumulated repair cost of unit 1 exceeds a pre-determined limit L. Therefore, we consider a flexible replacement policy under which the system is replaced at age, or at the occurrence time of some one unit 1 failure when the total damage to unit 2 exceeds a specified level or the accumulated repair cost to unit 1 exceeds a pre-determined limit L, whichever occurs first. Such a policy is general enough and appropriate for maintaining a complex system with multiple units.

The outline of this paper is as follows. Section 2 presents the model analysis, the long-term expected costs per unit time and the optimization. In Section 3, special cases of our model are introduced. A computational example is provided to demonstrate the above results in Section 4. Section 5 gives some conclusions.

2 Model analysis

Notation:

- $\{N_1(t), t \geq 0\}$ NHPP for unit 1 failures.
- $N_1(t)$ the number of unit 1 failures occurred in $[0, t)$.
- $r_1(t)$ the intensity rate of unit 1 failure.
- $R_1(t)$ mean number of unit 1 failures during $[0, t)$.
- S_j the occurrence time of the jth unit 1 failure, $j = 0, 1, 2, 3, \ldots$ with $S_0 = 0$.
- $P_i(t)$ the probability that there is just j unit 1 failures occurred during $[0, t)$.
- D_i random amount of damage to unit 2 by the ith unit 1 failure.
- $H_i(d)$ probability distribution of random variable D_i.
- μ_d mean of random variable D_i.
- W_j total damage to unit 2 up to the jth unit 1 failure with $W_0 = 0$.
- $H^{(j)}(w)$ CDF of W_j.
- Z the occurrence time of the first total failure.
- $\overline{F_Z}(t)$ survival function of random variable Z.
- X_i random repair cost due to the ith unit 1 failure.

$G(x)$	probability distribution of random variable X_i.
μ_x	mean of random variable X_i.
$G^{(j)}(x)$	the j-fold Stieltjes convolution of $G(x)$.
C_0	cost of preventive replacement.
C_1	cost of failure replacement ($C_1 > C_0$).
L	pre-determined limit for the accumulated repair cost.
T	scheduled time for preventive replacement.
$\alpha(n)$	probability that the system is replaced at the nth unit 1 failure.
$\beta(L)$	probability that the system is replaced at when the accumulated repair cost exceeds a pre-determined limit L.
$\delta(k)$	probability that the system is replaced at when the total damage of unit 2 exceeds a level k.
$\eta(K)$	probability that the system is replaced at the first total failure.
$Z(n,k)$	mean length of a replacement cycle.
$R(n,k)$	expected total cost incurred during a replacement cycle.
$C(n,k,L)$	long-term expected cost per unit time.
n^*	n which minimizes $C(n,k,L)$.
k^*	k which minimizes $C(n,k,L)$.

A system composed of two units (denoted as units 1 and 2), in which shock damage interaction exists between units, is considered. Unit 1 fails according to the intensity rate $r_1(t)$ and each unit 1 failure is assumed to be corrected by minimal repair. So, unit 1 failures occur according to a non-homogeneous Poisson process $\{N1(t), t \geq 0\}$ with intensity rate $r_1(t)$ and mean-value function $R_1(t) = \int_0^t r_1(x)\,dx$. Let S_j ($j = 0, 1, 2, 3, \dots$) be the occurrence time of jth unit 1 failure with $S_0 = 0$. The probability that there are at least j unit 1 failures occurred in $[0, t]$ is given by

$$\Pr\big(N_1(t) \geq j\big) = P(S_j \leq t) = \sum_{i=j}^{\infty} \frac{(R_1(t))^i e^{-R_1(t)}}{i!} = \sum_{i=j}^{\infty} P_i(t). \tag{1}$$

In equation (1), $N_1(t)$ denotes the number of unit 1 failures occurred in $[0, t]$. Then, the distribution function of random variable S_j is given by

$$\begin{aligned} f_{s_j}(t) &= \frac{d}{dt} P(S_j \leq t) = \frac{d}{dt} F_j(t) \\ &= \frac{d}{dt} \sum_{i=j}^{\infty} \frac{(R_1(t))^i \exp(-R_1(t))}{i!} = r_1(t) P_{j-1}(t). \end{aligned} \tag{2}$$

Whenever unit 1 fails, it causes a random amount of damage to unit 2. The amount of damage D_i due to the ith unit 1 failure is a sequence of identical and independent random variables with a probability distribution $H_i(d) = P(D_i \leq d)$

and a finite mean μ_d, $i = 1, 2, 3, \ldots$. These damages to unit 2 are additive and let $W_j = \sum_{i=1}^{j} D_i$ be the total damage after jth unit 1 failure with $W_0 = 0$. Then, random variable W_j has the distribution function:

$$P(W_j \leq w) = H^{(j)}(w) = \begin{cases} 1, & j = 0, \\ H_1 * H_2 * \cdots * H_j(w), & j = 1, 2, 3, \ldots, \end{cases} \tag{3}$$

where $H^{(j)}(w)$ is the j-fold Stieltjes convolution of $H(w)$ with itself.

Unit 2 fails whenever the total damage exceeds a failure level K. Each unit 2 failure causes unit 1 into failure at the same time and leads to a total failure of two-unit system. Because unit 2 is not repairable and as a result, a failed two-unit system needs to replaced by a new one. Let random variable Z denote the occurrence time of the first total failure, so the survival function of Z is given by

$$\begin{aligned} \overline{F_Z}(t) &= P(Z > t) = P(W_{N_1(t)} < K) \\ &= \sum_{i=0}^{\infty} P\big(N_1(t) = i, W_i < K\big) = \sum_{i=0}^{\infty} P_i(t) H^{(i)}(K). \end{aligned} \tag{4}$$

Finally, we consider a preventive maintenance model in which minimal repair or replacement takes place according to the following scheme. The cost for minimal repair is evaluated when a unit 1 failure occurs. We assume that the repair cost X_i due to the ith unit 1 failure is a sequence of identical and independent random variables with a probability distribution $G(x) = P(X_i \leq x)$ and a finite mean μ_x, $i = 1, 2, 3, \ldots$. Then, the total repair cost $Y_j = \sum_{i=1}^{j} X_i$ till to the jth unit 1 failure after the installation has a distribution function

$$P(Y_j \leq y) = G^{(j)}(y) = \begin{cases} 1, & j = 0, \\ G_1 * G_2 * \cdots * G_j(y), & j = 1, 2, 3, \ldots, \end{cases} \tag{5}$$

where $G^{(j)}(x)$ is the j-fold Stieltjes convolution of $G(x)$ with itself.

The system can be replaced at four different cases:

1. When the nth unit 1 failure occurs.
2. When the total damage of unit 2 exceeds k and is less or equal to K.
3. When the accumulated repair cost of unit 1 exceeds a pre-determined limit L.
4. When the first total failure occurs.

The first three cases are preventive replacement and the last case is a failure replacement. Let $C(n, k, L)$ be the long-term expected cost per unit time. The cost of preventive replacement is C_0, while the cost of failure replacement is C_1 ($C_1 > C_0$). This problem is just to find an optimal pair (n^*, k^*) to minimize $C(n, k, L)$ in the steady state case.

2.1 Different replacement cases

According to the above description, the replacement of the system can occur at four different cases and the probabilities of four cases are derived as follows.

1. The probability $\alpha(n)$, that the system is preventively replaced at the nth unit 1 failure, is given by

$$\alpha(n) = P(Y_n < L, W_n < k) = G^{(n)}(L)H^{(n)}(k). \tag{6}$$

2. The probability $\beta(L)$, that the system is preventively replaced at the jth ($j < n$) unit 1 failure when the accumulated repair cost exceeds a pre-determined limit L, is given by

$$\begin{aligned}\beta(L) &= \sum_{j=1}^{n} P(Y_{j-1} < L < Y_j, W_j < k) \\ &= \sum_{j=1}^{n} H^{(j)}(k)\big(G^{(j-1)}(L) - G^{(j)}(L)\big).\end{aligned} \tag{7}$$

3. The probability $\delta(k)$, that the system is preventively replaced at some one unit 1 failure when the total damage of unit 2 exceeds a level k, is given by

$$\begin{aligned}\delta(k) &= \sum_{j=1}^{n} P(Y_j < L, W_{j-1} < k < W_j < K) \\ &\quad + \sum_{j=1}^{n} P(Y_{j-1} < L < Y_j, W_{j-1} < k < W_j < K) \\ &= \sum_{j=1}^{n} G^{(j)}(L) \int_0^k \big(H(K-x) - H(k-x)\big)\,dH^{(j-1)}(x) \\ &\quad + \sum_{j=1}^{n}\big(G^{(j-1)}(L) - G^{(j)}(L)\big) \int_0^k \big(H(K-x) - H(k-x)\big)\,dH^{(j-1)}(x) \\ &= \sum_{j=1}^{n} G^{(j-1)}(L) \int_0^k \big(H(K-x) - H(k-x)\big)\,dH^{(j-1)}(x).\end{aligned} \tag{8}$$

4. The probability $\eta(K)$, that the system is replaced at the first total failure, is given by

$$\begin{aligned}\eta(K) &= \sum_{j=1}^{n} P(Y_j < L, W_{j-1} < k < K < W_j) \\ &\quad + \sum_{j=1}^{n} P(Y_{j-1} < L < Y_j, W_{j-1} < k < K < W_j)\end{aligned}$$

$$
\begin{aligned}
&= \sum_{j=1}^{n} G^{(j)}(L)\big(H^{(j-1)}(K) - H^{(j)}(k)\big) \\
&\quad + \sum_{j=1}^{n}\big(G^{(j-1)}(L) - G^{(j)}(L)\big)\big(H^{(j-1)}(K) - H^{(j)}(k)\big) \qquad (9) \\
&= \sum_{j=1}^{n} G^{(j-1)}(L)\int_0^k \overline{H}(K-x)\,dH^{(j-1)}(x) \\
&= \sum_{j=1}^{n-1} G^{(j)}(L)\int_0^k \overline{H}(K-x)\,dH^{(j)}(x).
\end{aligned}
$$

It is easily seen that the equations (6) + (7) + (8) + (9) = 1.

More specifically, we need the following assumptions:

1. The system is monitored continuously so that failures of two units can be detected instanteously.
2. The times taken for minimal repair or replacement are very smaller than the mean time between failures. As a consequence, we can ignore those and treat those as being zero.

2.2 Long-term expected cost per unit time

A replacement cycle is exchanged if a replacement is completed. Here, a replacement cycle is actually a time interval between the installation of the system and the first replacement or a time interval between consecutive replacements. So, the successive replacement cycles will constitute a regenerative process.

Let $Z(n,k)$ and $R(n,k)$ denote the expected length of a replacement cycle and the expected total cost incurred during a replacement cycle, respectively. Using the renewal-reward theorem, we can observe that the long-term expected cost per unit time in the steady-state case is given by Ross (1983):

$$C(n,k,L) = R(n,k)/Z(n,k).$$

The expected length of a replacement cycle $Z(n,k)$ depends on the situation and can be computed as follows:

$$
\begin{aligned}
Z(n,k) = &\left[G^{(n)}(L)H^{(n)}(k)\int_0^\infty t \times P_{n-1}(t)r_1(t)\,dt\right] \\
&+ \left[\sum_{j=1}^{n}\big(G^{(j-1)}(L) - G^{(j)}(L)\big)H^{(j)}(k)\int_0^\infty t \times P_{j-1}(t)r_1(t)\,dt\right] \\
&+ \left[\sum_{j=1}^{n} G^{(j-1)}(L)\int_0^k \big(H(K-x) - H(k-x)\big)\,dH^{(j-1)}(x)\right. \qquad (10)
\end{aligned}
$$

$$\times \int_0^\infty t \times P_{j-1}(t) r_1(t)\, dt \Bigg]$$

$$+ \left[\sum_{j=1}^{n} G^{(j-1)}(L) \int_0^k \overline{H}(K-x)\, dH^{(j-1)}(x) \int_0^\infty t \times P_{j-1}(t) r_1(t)\, dt \right]$$

$$= \left[\sum_{j=0}^{n-1} G^{(j)}(L) H^{(j)}(k) \int_0^\infty P_j(t)\, dt \right]$$

and then, the expected total cost $R(n,k)$ during a replacement cycle is

$$\begin{aligned} R(n,k) &= \left[C_0 + (n-1)\mu_x\right] \times G^{(n)}(L) H^{(n)}(K) \\ &\quad + \sum_{j=1}^{n} \left[C_0 + (j-1)\mu_x\right] \times H^{(j)}(k) \big(G^{(j-1)}(L) - G^{(j)}(L)\big) \\ &\quad + \Bigg[\sum_{j=1}^{n} \left[C_0 + (j-1)\mu_x\right] \\ &\quad \times G^{(j-1)}(L) \int_0^k \big(H(K-x) - H(k-x)\big)\, dH^{(j-1)}(x) \Bigg] \\ &\quad + \Bigg[\sum_{j=1}^{n} \left[C_1 + (j-1)\mu_x\right] \\ &\quad \times G^{(j-1)}(L) \int_0^k \overline{H}(K-x)\, dH^{(j-1)}(x) \Bigg] \\ &= C_0 + (C_1 - C_0) \times \sum_{j=1}^{n} G^{(j-1)}(L) \int_0^k \overline{H}(K-x)\, dH^{(j-1)}(x) \\ &\quad + \mu_x \times \sum_{j=1}^{n-1} G^{(j)}(L) H^{(j)}(k) \\ &= C_0 + (C_1 - C_0) \times \sum_{j=0}^{n-1} G^{(j)}(L) \int_0^k \overline{H}(K-x)\, dH^{(j)}(x) \\ &\quad + \mu_x \times \sum_{j=1}^{n-1} G^{(j)}(L) H^{(j)}(k). \end{aligned} \tag{11}$$

Combining (10) and (11), the long-term expected cost per unit time $C(n,k,L)$ is obtained as follows:

$$C(n,k,L) = \Bigg(C_0 + (C_1 - C_0) \times \sum_{j=0}^{n-1} G^{(j)}(L) \int_0^k \overline{H}(K-x)\,dH^{(j)}(x) + \mu_x \times \sum_{j=1}^{n-1} G^{(j)}(L) H^{(j)}(k)\Bigg) \Bigg/ \Bigg(\sum_{j=0}^{n-1} G^{(j)}(L) H^{(j)}(k) \int_0^\infty P_j(t)\,dt\Bigg). \tag{12}$$

2.3 Optimization

In the steady-state case, this problem is to find the values of n and k that minimize the function $C(n,k,L)$ given by equation (12) under the following assumptions:

(a1) $r_1(t)$ is a continuous and increasing function of t with $r_1(t) \to \infty$ as $t \to \infty$.
(a2) $G^{(n)}(y)$ and $H^{(n)}(w)$ are PF2 (a Polya frequency function of order 2).

From Lemma 3.7 in Barlow and Proschan (1975), it is known that $G^{(n)}(L)$ (or $H^{(n)}(w)$) is decreasing in n for all $L > 0$. In addition, we can observe that $G^{(n)}(L)$ (or $H^{(n)}(w)$) is PF2 if and only if $G^{(n)}(L)/G^{(n-1)}(L)$ (or $H^{(n)}(K)/H^{(n-1)}(K)$) is decreasing in n for all L (or K) >0 (Gottlieb (1980, p. 749)).

First, we shall minimize $C(n,k,L)$ with respect to k for given n and L. A necessary condition that a finite optimal k ($<K$) minimizes $C(n,k,L)$ can be obtained by differentiating $C(n,k,L)$ with respect to k and setting it equal to zero. Hence, $dC(n,k,L)/dk = 0$ holds if and only if

$$\begin{aligned} U(n,k,L) &= B(n,k,L) \times \sum_{j=0}^{n-1} G^{(j)}(L) H^{(j)}(k) \int_0^\infty P_j(t)\,dt \\ &\quad - (C_1 - C_0) \times \sum_{j=1}^{n-1} G^{(j)}(L) \int_0^k \overline{H}(K-x)\,dH^{(j)}(x) - \mu_x \\ &\quad \times \sum_{j=1}^{n-1} G^{(j)}(L) H^{(j)}(k) = C_0, \end{aligned} \tag{13}$$

where

$$B(n,k,L) = \frac{[(C_1 - C_0)\overline{H}(K-k) + \mu_x] \times \sum_{j=1}^{n-1} G^{(j)}(L) h^{(j)}(k)}{\sum_{j=0}^{n-1} G^{(j)}(L) h^{(j)}(k) \int_0^\infty P_j(t)\,dt}. \tag{14}$$

Next, we shall minimize $C(n,k,L)$ with respect to n for given k and L. When the inequalities $C(n+1,k,L) \geq C(n,k,L)$ and $C(n,k,L) < C(n-1,k,L)$ are both satisfied for some finite n, there exist an optimal n^*. In the derivation of these inequalities, we can see that the inequalities $C(n+1,k,L) \geq C(n,k,L)$ and $C(n,k,L) < C(n-1,k,L)$ hold if and only if

$$K(n,k,L) \geq C_0 \quad \text{and} \quad K(n-1,k,L) < C_0, \tag{15}$$

where

$$K(n,k,L) = \begin{cases} \big((C_1 - C_0)M_n Q_n + \mu_x Q_n\big) \times \sum_{j=0}^{n-1} G^{(j)}(L) H^{(j)}(k) \int_0^\infty P_j(t)\,dt \\ \quad - (C_1 - C_0) \sum_{j=1}^{n-1} G^{(j)}(L) \int_0^k \overline{H}(K-x)\,dH^{(j)}(x) \\ \quad - \mu_x \sum_{j=1}^{n-1} G^{(j)}(L) H^{(j)}(k), \qquad n = 1,2,3,\ldots, \\ 0, \qquad n = 0, \end{cases} \tag{16}$$

and

$$M_n = \frac{\int_0^k \overline{H}(K-x)\,dH^{(n)}(x)}{H^{(n)}(k)},$$

$$Q_n = \frac{1}{\int_0^\infty P_n(t)\,dt}.$$

If $K(n,k,L)$ is showed to be an increasing function of n and $\lim_{n\to\infty} K(n,k,L) > C_0$, then the optimal n^* is finite and unique for all $k > 0$. In order to show that $K(n,k,L)$ is an increasing function of n, the following Lemma 1 is needed.

Lemma 1. *Under assumptions* (a1) *and* (a2), *the following results are true*:

(1) $M_n = \int_0^k \overline{H}(K-x)\,dH^{(n)}(x)/H^{(n)}(k)$ *is increasing in* n.
(2) $Q_n = 1/\int_0^\infty P_n(t)\,dt$ *is increasing in* n.

The detailed proof of Lemma 1 is presented in Appendix. We want to find the optimal pair n^* and k^* that minimize $C(n,k,L)$ in (12) under certain sufficient conditions.

Theorem 1. *Under assumption* (a1) *and* (a2), *if* $r_1(t)$ *is an increasing and continuous function of* t, *there exists finite* k^* *and* n^* *that satisfy* (13) *and* (16), *respectively.*

Proof. First, it is evident that $U(n,0,L) = \lim_{k\to 0} U(n,k,L) = 0 < C_0$ and if

$$\begin{aligned}
U(n,K,L) &= \lim_{k\to K} U(n,k,L)\\
&= \frac{(C_1 - C_0 + \mu_x) \times \sum_{j=1}^{n-1} G^{(j)}(L)h^{(j)}(K)}{\sum_{j=0}^{n-1} G^{(j)}(L)h^{(j)}(K)\int_0^\infty P_j(t)\,dt}\\
&\quad \times \sum_{j=0}^{n-1} G^{(j)}(L)H^{(j)}(K)\int_0^\infty P_j(t)\,dt\\
&\quad - (C_1 - C_0 + \mu_x) \times \sum_{j=1}^{n-1} G^{(j)}(L)H^{(j)}(K) + (C_1 - C_0)\\
&\quad \times \sum_{j=1}^{n-1} G^{(j)}(L)H^{(j+1)}(K) > C_0.
\end{aligned}$$

Hence, we can know that then there exists at least one finite optimal k^* such that $U(n,k,L) = C_0$, that is, k^* satisfies (13). Furthermore, if $r_1(t)$ is strictly increasing and

$$\frac{d}{dT}U(T,n,L) = \left(\frac{d}{dk}B(n,k,L)\right) \times \sum_{j=0}^{n-1} G^{(j)}(L)H^{(j)}(k)\int_0^\infty P_j(t)\,dt > 0.$$

Then $U(n,k,L)$ is also a strictly increasing function of t, and hence, the optimal k^* is unique.

In equation (16), we know that $U(0,k,L) = 0 < C_0$. If $\lim_{n\to\infty} K(n,k,L) > C_0$, we can observe that there is a finite n such that equation (16) is satisfied. Using the equation (16) and Lemma 1, we have

$$\begin{aligned}
&K(n+1,k,L) - K(n,k,L)\\
&\quad = \big((C_1 - C_0)M_{n+1}Q_{n+1} + \mu_x Q_{n+1}\big) \times \sum_{j=0}^{n} G^{(j)}(L)H^{(j)}(k)\int_0^\infty P_j(t)\,dt\\
&\qquad - (C_1 - C_0)\sum_{j=1}^{n} G^{(j)}(L)\int_0^k \overline{H}(K-x)\,dH^{(j)}(x)\\
&\qquad - \mu_x \sum_{j=1}^{n} G^{(j)}(L)H^{(j)}(k)\\
&\qquad - \big((C_1 - C_0)M_n Q_n + \mu_x Q_n\big) \times \sum_{j=0}^{n-1} G^{(j)}(L)H^{(j)}(k)\int_0^\infty P_j(t)\,dt
\end{aligned}$$

$$+ (C_1 - C_0) \sum_{j=1}^{n-1} G^{(j)}(L) \int_0^k \overline{H}(K - x)\, dH^{(j)}(x)$$

$$+ \mu_x \sum_{j=1}^{n-1} G^{(j)}(L) H^{(j)}(k)$$

$$= \big[(C_1 - C_0)(M_{n+1} Q_{n+1} - M_n Q_n) + \mu_x (Q_{n+1} - Q_n)\big]$$

$$\times \sum_{j=0}^{n} G^{(j)}(L) H^{(j)}(k) \int_0^\infty P_j(t)\, dt.$$

From Lemma 1, we can know that $(M_{n+1} Q_{n+1} - M_n Q_n) > 0$ and $Q_{n+1} - Q_n > 0$ for any n, furthermore, $\sum_{j=0}^{n} G^{(j)}(L) H^{(j)}(k) \int_0^\infty P_j(t)\, dt > 0$, then $K(n+1, k, L) - K(n, k, L) > 0$ for all n. Thus, $K(n, k, L)$ is increasing in n and there exists a finite and unique n^* that satisfies (16) for all $k < K$.

According the above proof, we can know that there exists finite n^* and k^* that satisfy the equations (13) and (16), respectively. □

3 Special cases

The replacement policy (n, k, L) for a given L can be considered as a generalization of several past models, we have the following special cases:

Case 1. If $L \to \infty$ and $n \to \infty$, it means that the two-unit system is replaced preventively at time T or on unit 2 failure. Thus, $C(n, k, L)$ will be reduced to be

$$C(\infty, k, \infty) = \Bigg(C_0 + (C_1 - C_0) \times \sum_{j=0}^{\infty} \int_0^k H(K - x)\, dH^{(j)}(x) + (C_1 - C_0 + \mu_x) \times \sum_{j=1}^{\infty} H^{(j)}(k)\Bigg) \Bigg/ \Bigg(\sum_{j=0}^{\infty} H^{(j)}(k) \int_0^\infty P_j(t)\, dt\Bigg)$$

which is same as equation (15) in Satow and Osaki (2003).

Case 2. If $L \to \infty$ and $k \to K$, it means that the two-unit system is replaced preventively at the nth unit 1 failure or at total failure. So, $C(n, k, L)$ will be reduced to be

$$C(n, K, \infty) = \frac{C_1 - (C_1 - C_0) H^{(n)}(K) + \mu_x \times \sum_{j=1}^{n-1} H^{(j)}(K)}{\sum_{j=0}^{n-1} H^{(j)}(K) \int_0^\infty P_j(t)\, dt},$$

this is same as equation (12) in Nakagawa and Murthy (1993).

4 Numerical example

In this example, we consider the intensity rate $r_1(t)$ of unit 1 failures with shape parameter β and scale parameter λ is given by

$$r_1(t) = \lambda t^{\beta-1}, \qquad \lambda > 0, \beta > 1.$$

The mean-value function is $R_1(t) = \int_0^t \lambda u^{\beta-1}\,du = \lambda t^\beta/\beta$. We assume that the shape parameter is set at $\beta = 2$ and then, $r_1(t) = \lambda t$ is an increasing function of t. If $\lambda > 1$ indicates that the failure rate of unit 1 increases with time, in other words, unit 1 has the characteristic of "aging process" and unit 1 is more likely to fail as time goes on.

Suppose that the random repair cost X_i for unit 1 failure has an exponential distribution $G(x) = P(X_i \le x)$ and finite mean μ_x. The random damage D_i from consecutive unit 1 failures has an exponential distribution $H_i(d) = P(D_i \le d)$ and finite mean μ_d.

We use an algorithm, which is proposed in Chang et al. (2011), to compute the optimal replacement policy (n^*, k^*) under a pre-determined cumulative repair cost limit L as follows.

Step 1. Set $n = 1$ and $C(0, k_0, L) = \infty$.

Step 2. Compute $\sum_{j=0}^{n-1} G^{(j)}(L)H^{(j)}(k)\int_0^\infty P_j(t)\,dt$, $\sum_{j=1}^{n-1} G^{(j)}(L)\int_0^k \overline{H}(K-x)\,dH^{(j)}(x)$, $\sum_{j=1}^{n-1} G^{(j)}(L)H^{(j)}(k)$ and $\frac{[(C_1-C_0)\overline{H}(K-k)+\mu_x]\times\sum_{j=1}^{n-1} G^{(j)}(L)h^{(j)}(k)}{\sum_{j=0}^{n-1} G^{(j)}(L)h^{(j)}(k)\int_0^\infty P_j(t)\,dt}$ as defined by equations (13) and (14).

Step 3. Find the solutions k^*_{n-1}, k^*_n, and k^*_{n+1} which satisfy equation (13) ($U(n-1, k_{n-1}, L) = C_0$, $U(n, k_n, L) = C_0$ and $U(n+1, k_{n+1}, L) = C_0$, respectively).

Step 4. Compute $C(n-1, k_{n-1}, L)$, $C(n, k_n, L)$ and $C(n+1, k_{n+1}, L)$ as defined by equation (12).

Step 5. If $C(n+1, k_{n+1}, L) \ge C(n, k_n, L)$ and $C(n, k_n, n, L) < \mathrm{C}(n-1, k_{n-1}, L)$, then $n^* = n$, $k^* = k$, $C(n^*, k^*, L) = C(n, k_n, L)$ and go to output. Otherwise, make $n = n + 1$ and go to Step 2.

Output: k^*, n^* and $C(n^*, k^*, L)$.

Stop.

The optimal replacement policy turns out to be (n^*, k^*). It is noted that the foregoing algorithm does not ensure a global minimum. Nevertheless, the algorithm can considerably reduce the search of the optimal values.

For the purpose of easy computation, we consider the following cases:

Case 1: $C_1 = 2000$, $L = 500$, $\mu_x = 50$, $\mu_d = 40$, and $\lambda = 1.2$ are fixed values, K and C_0 are varied.

Case 2: $C_0 = 1000$, $C_1 = 2000$, $K = 800$, $\mu_d = 40$ and $L = 600$ are fixed values, λ and μ_x are varied.

Case 3: $C_0 = 1000$, $C_1 = 2000$, $K = 800$, $L = 600$, and $\lambda = 1.2$ are fixed values, L/μ_x and K/μ_d are varied.

The results of three cases are provided in Tables 1 to 3, respectively. From Tables 1 and 3, we have the following conclusions:

(1) When the failure level K increases, we can know that the optimal n^* and k^* increases but the minimum $C(n^*, k^*, L)$ decreases in Table 1. The greater K implies that it allows more damages from unit 1 failures before system failure, then

Table 1 *Optimal n^*, k^* and $C(n^*, k^*, L)$ at different combination of K and C_0*

	K								
	600			800			1000		
C_0	n^*	k^*	$C(n^*, k^*, L)$	n^*	k^*	$C(n^*, k^*, L)$	n^*	k^*	$C(n^*, k^*, L)$
600	25	471.4	106.1924067	29	666.9	100.2498.96	30	865.9	98.95249052
800	27	489.4	125.5786142	30	685.4	118.4514539	32	884.5	116.9329121
1000	27	505.7	144.5921616	30	702.1	136.5711198	32	901.3	134.8999192
1200	27	521.8	163.2300430	30	718.6	154.6101424	32	918.0	152.8540613

Table 2 *Optimal n^*, k^* and $C(n^*, k^*, L)$ at different combination of λ and μ_x*

	λ								
	1.2			1.6			2.0		
μ_x	n^*	k^*	$C(n^*, k^*, L)$	n^*	k^*	$C(n^*, k^*, L)$	n^*	k^*	$C(n^*, k^*, L)$
40	33	691.2	105.9452085	33	691.9	107.0501990	33	692.4	107.8173807
60	30	701.6	145.5288997	30	702.6	147.5896420	30	703.3	149.0296303
80	26	710.1	185.7779308	26	711.4	189.1442743	27	712.3	191.5123869
100	26	716.6	224.2445394	26	718.2	229.1672234	26	719.2	232.6525186

Table 3 *Optimal n^*, k^* and $C(n^*, k^*, L)$ at different combination of L/μ_x and K/μ_d*

	L/μ_x								
	8			10			12		
K/μ_d	n^*	k^*	$C(n^*, k^*, L)$	n^*	k^*	$C(n^*, k^*, L)$	n^*	k^*	$C(n^*, k^*, L)$
16	27	688.4	157.3016842	27	680.7	141.9676416	30	675.4	132.8197040
20	29	709.5	153.9901750	30	702.1	136.5711198	32	696.6	125.3011255
25	29	727.3	153.2032777	30	721.1	134.8999192	33	716.0	122.4285467

the optimal n^* and k^* increase, i.e., the mean time to replacement will increase and the minimum $C(n^*, k^*, L)$ will be smaller.

(2) When preventive replacement cost C_0 increases, we can know that the optimal n^*, k^* and the minimum $C(n^*, k^*, L)$ increase synchronously in Table 1. The greater C_0 implies that the gap between C_0 and C_1 is smaller, and then the possibility of preventive replacement will be reduced, then the optimal n^* and k^* will increase. And, the minimum $C(n^*, k^*, L)$ will increase because preventive replacement cost C_0 increases.

(3) If the parameter λ increases from 1.2 to 2.0, in other words, the rate of recurrence of unit 1 failures is larger, hence unit 1 failures happen quickly. When the parameter λ increases, we can know that the optimal k^* and the minimum $C(n^*, k^*, L)$ increase synchronously in Table 2, but the variation of the optimal n^* has not an significiant change. The variation of the optimal k^* and the minimum $C(n^*, k^*, L)$ is relatively smaller than the increment of λ.

(4) When the mean cost of minimal repair μ_x increases, we can know that the optimal k^* and the minimum $C(n^*, k^*, L)$ increase synchronously in Table 2, but the optimal n^* decreases. The greater μ_x implies that the possibility of preventive replacement at the nth unit 1 failure increases, then the optimal n^* decreases. As the optimal n^* decreases, the mean time to replacement is shorter, and then the minimum $C(n^*, k^*, L)$ increases.

(5) When the ratio L/μ_x increases, that is, it allows more minimal repairs for unit 1 failures, we can see that the optimal n^* increases, but the optimal k^* and the minimum C(n^*, k^*, L) decrease synchronously in Table 3. The greater L/μ_x implies that there are more unit 1 failures to occur, that is, the total damage of unit 2 will be accumulated rapidly, so the optimal k^* will be shorter to prevent total failure of the system.

(6) When the ratio K/μ_d increases, that is, it allows more damages from unit 1 failures before system failure, we can see that the optimal n^* and k^* increase synchronously, but the minimum $C(n^*, k^*, L)$ decreases in Table 3. The greater K/μ_d implies that the system can allow more damages from unit 1 failures before system failure, so the optimal k^* can be larger.

(7) In Table 3, we can see that the variations in the optimal k^* and the minimum $C(n^*, k^*, L)$ with regard to K/μ_d are significantly larger than that of L/u_x. So, the ratio K/μ_d is more important than the ratio L/μ_x when we are going to determine the optimal replacement policy.

Although these conclusions are intuitive, our model provides a quantitative tool for practitioners in designing the most cost effective PM policies. This PM policy proposed is not too complicated so that the implementation is still feasible in a real life situation. However, before using this approach, we must estimate the parameters like λ, μ_x and μ_d. But, the parameters like λ, μ_x and μ_d are unknown in practice, so collecting the relevant data (for example, the failure times, the repair costs and the amount of damages due to unit 1) to the parameters λ, μ_x and

μ_d are necessary. These parameters can be estimated using the general statistical inference procedure.

5 Conclusions

In this paper, an optimal bivariate PM policy with cumulative repair-cost limit for a two-unit system under a shock damage interaction between units was presented and studied. The long-term expected cost per unit time was developed, incorporating costs related to replacement and repair. We have shown the existence and uniqueness of the optimal PM policy that minimizes the long-term expected cost per unit time. The optimal PM schedule n^* and k^* can be determined by the numerical search. Furthermore, we can find that the ratio K/μ_d is more important for determining the optimal PM policy. This model provided a more general framework for analyzing the maintenance policies for a two-unit system subject to shock damage interactions. These results and methods would be useful practically for the maintenance of general systems by suitable modification and extension.

However, some future directions for this research can be considered. First, the intensity rate of unit 1 failure may be permitted to depend on the number of failures or its age, which will be more general than an increasing intensity rate function. This research introduces a PM model for a two-unit system with shock damage interactions between units. In real situations, the multi-unit systems having more applicable cases can be another future research.

Appendix: Proof of Lemma 1

(i) First, we want to show that $M_n = \int_0^k \overline{H}(K-x)\,dH^{(n)}(x)/H^{(n)}(k)$ is increasing in n. Let

$$\begin{aligned} M_n &= \frac{\int_0^k \overline{H}(K-x)\,dH^{(n)}(x)}{H^{(n)}(k)} = 1 - \frac{\int_0^k H(K-x)\,dH^{(n)}(x)}{H^{(n)}(k)} \\ &= 1 - \frac{H^{(n+1)}(k)}{H^{(n)}(k)}. \end{aligned} \tag{A.1}$$

From assumptions (a1) and (a2), we can know that $H^{(n)}(k)/H^{(n-1)}(k)$ is decreasing in n for all $k > 0$, because $H^{(n)}(k)$ is PF2. Thus, $M_n = 1 - \frac{H^{(n+1)}(k)}{H^{(n)}(k)}$ is increasing in n.

(ii) From Nakagawa and Kowada (1983), we know that if $r_1(t)$ is (strictly) increasing, then $\int_0^T P_n(t)\,dt$ is (strictly) decreasing in n, and then, $Q_n = 1/\int_0^\infty P_n(t)\,dt$ is increasing in n.

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