Bayesian Analysis (2014) **9**, Number 4, pp. 805–808

Comment on Article by Windle and Carvalho

Catherine Scipione Forbes *

1 Introduction

I congratulate the authors on their state-space model for covariance matrices associated with multivariate return vectors subject to stochastic volatility. Despite the apparent restrictions imposed on the proposed model, its tractability offers exciting possibilities for modelling and forecasting the covariance of high dimensional multivariate financial asset price returns.

2 The proposed model requires strong assumptions

Conditionally normal price returns. To obtain the computational advantages delivered by the proposed model, one is required to make strong distributional assumptions regarding both the measurement and transition equations in the state-space model. As the authors detail in Section 5, the measurement equation imposed is equivalent to assuming that the m-vector of prices evolves over day t according to a geometric Brownian motion process. This relates to a conditional Gaussian distribution for the daily (log) price return. Yet, when modelling price returns in a univariate setting, the conditional distribution of the return is often taken to have 'fatter tails' than a normal distribution (see, for example, Chib et al. (2002)). That is, even when accounting for changes in the latent volatility process, the conditional normal distribution is not usually sufficient to generate the marginal skewness and leptokurtosis evident in observed daily returns.

A diffusive volatility process. In addition, the latent covariances in the proposed multivariate model (and indeed in the discussed competing multivariate models) are implicitly assumed to be 'locally smooth', and thereby will change only slowly over time in a manner akin to a random walk (see Prado and West (2010), p. 272). Arguably, this assumption would be similar to assuming that the stochastic variances evolve from day to day according to a (potentially nonlinear, multivariate) diffusive process. However, again in the univariate setting, diffusive volatility alone is generally viewed as a restrictive assumption, with jumps in the price process (and indeed, in the latent stochastic variance process) deemed important for the prediction of derivative asset prices (e.g. Bates (2000), Duffie et al. (2000), Eraker et al. (2003) and Maneesoonthorn et al. (2012)).

*Monash University, Australia catherine.forbes@monash.edu

 DOI:10.1214/14-BA920

3 Relaxing the assumptions

Introducing 'fat tails'. Despite these limitations, I think the proposed state-space model offers exciting possibilities for modelling high dimensional multivariate returns. Not only does the model now explicitly take advantage of the more informative and robust realized kernel covariance measures, the tractability of the model suggests that computation even with *both* 'fat tails' and price jumps included in the model may be feasible, via Markov chain Monte Carlo (MCMC) methods. Already the 'fat tails' feature appears to be accommodated by the authors' suggestion, detailed in Section 6, that the conditional measurement distribution of $\mathbf{Y}_t$ be augmented by a gamma distributed auxiliary variable ϕ_t. While the suggestion is not implemented, and would imply an additional computational cost, conditional on the set of auxiliary variables $\{\phi_t, t = 1, 2, ..., T\}$ the proposed forward filtering, backwards sampling (FFBS) method is essentially preserved. Consequently, it seems plausible that the computational demands of this extension, while raised, would not be too onerous.

Introducing price jumps. Regarding the potential for multivariate price jumps, suppose the vector of intraday stock prices evolves as a geometric Brownian motion with independent compound Bernoulli jumps, such that on day t the log price vector at time s on day t may be represented as

$$\mathbf{p}_{t+s} = \mathbf{p}_t + \mu s + \mathbf{V}_t^{1/2}\left(\mathbf{w}_{t+s} - \mathbf{w}_t\right) + \mathbf{Z}_t\left(\mathbf{N}_{t+s} - \mathbf{N}_t\right), \tag{1}$$

where $\mathbf{p}_t, \mu, \mathbf{V}$ and $\mathbf{w}_t$ are all defined as in Section 5[1], $\{\mathbf{N}_s\}_{s\geq 0}$ is an $m-$dimensional Bernoulli process comprised of m stacked independent univariate Bernoulli processes $\{N_{i,s}\}_{s\geq 0}$ having corresponding intensity parameter δ_i, for $i = 1, 2, ..., m$, and with the $(m \times m)$ random diagonal matrix $\mathbf{Z}_t$ having i^{th} diagonal element

$$Z_{i,t} \overset{independent}{\sim} N\left(\alpha_i, \tau_i^2\right), \text{ for } i = 1, 2, ..., m. \tag{2}$$

Now, *conditional* on the jump process $\mathbf{Z}_t\Delta\mathbf{N}_t = \mathbf{Z}_t\left(\mathbf{N}_{t+1} - \mathbf{N}_t\right)$, and assuming $\mu = \mathbf{0}$, we have

$$\begin{aligned}
\mathbf{r}_t &\sim N\left(\mathbf{Z}_t\Delta\mathbf{N}_t, \mathbf{X}_t^{-1}\right), \\
\mathbf{X}_t &= \mathbf{T}_{t-1}'\boldsymbol{\Psi}_t\mathbf{T}_{t-1}/\lambda, \qquad \boldsymbol{\Psi}_t \sim \beta_m\left(\frac{n}{2}, \frac{k}{2}\right), \\
T_{t-1} &= \text{upper chol } \mathbf{X}_{t-1}.
\end{aligned} \tag{3}$$

Again the notation in (3) comforms to that of Section 5. The specification is such that *conditional* on the set of price jumps $\{(\mathbf{Z}_t\Delta\mathbf{N}_t)\}_{t=1}^{T}$, the model for the adjusted

[1] To avoid inconsistencies, I have replaced $\mathbf{p}_{t,s}$ with $\mathbf{p}_{t+s}$ and $\mathbf{p}_{t,0}$ with $\mathbf{p}_t$.

measurement covariance matrix $\mathbf{Y}_t^* = \mathbf{Y}_t - (\mathbf{Z}_t \Delta \mathbf{N}_t)(\mathbf{Z}_t \Delta \mathbf{N}_t)'$ retains the original form

$$\mathbf{Y}_t^* \sim W_m\left(k, (k\mathbf{X}_t)^{-1}\right), \tag{4}$$

$$\mathbf{X}_t = \mathbf{T}_{t-1}' \mathbf{\Psi}_t \mathbf{T}_{t-1}/\lambda, \qquad \mathbf{\Psi}_t \sim \beta_m\left(\frac{n}{2}, \frac{k}{2}\right), \tag{5}$$

$$T_{t-1} = \text{upper chol } \mathbf{X}_{t-1}. \tag{6}$$

Accordingly, the proposed FFBS algorithm may therefore be applied to this conditionally adjusted measurement covariance, enabling relatively efficient sampling of the latent symmetric positive-definite matrices $\{\mathbf{X}_t\}_{t=1}^T$ from the relevant (joint) conditional posterior distribution. Joint sampling of the latent price jump series $\{\Delta\mathbf{N}_t\}_{t=1}^T$ is also easily achieved, as the individual ΔN_t components are independent Bernoulli random variables, when conditioned upon the observed data $\{\Delta\mathbf{Y}_t\}_{t=1}^T$, the jump sizes $\{\mathbf{Z}_t\}_{t=1}^T$, and the latent stochastic variances $\left\{\Delta\mathbf{X}_t^{-1}\right\}_{t=1}^T$. In addition, the latent price jump sizes $\{\mathbf{Z}_t\}_{t=1}^T$ are conditionally independent given the data $\{\Delta\mathbf{Y}_t\}_{t=1}^T$, the jump events $\{\Delta\mathbf{N}_t\}_{t=1}^T$, and the latent stochastic variances $\left\{\Delta\mathbf{X}_t^{-1}\right\}_{t=1}^T$.

Note that rather than necessarily impose independent jump processes, one could explore the use of *dependent* jump processes, as in Aït-Sahalia et al. (2014), and Maneesoonthorn et al. (2013).

Of course, whether the tractability of the proposed model ultimately delivers on its promise to enable feasible MCMC computation of an expanded multivariate model incorporating both 'fat tails' and price jumps remains to be determined. If it does, however, I would expect forecasts from such a model to outperform those based on either a comparable model based on daily returns alone (e.g. Chib et al. (2006)) and to outperform those that ignore 'fat tails' and price jumps altogether.

References

Aït-Sahalia, Y., Cacho-Diaz, J., and Laeven, R. (2014). "Modeling Financial Contagion Using Mutually Exciting Jump Processes." Forthcoming in *Journal of Financial Economics*.
URL http://www.princeton.edu/~yacine/contagion.pdf 807

Bates, D. S. (2000). "Post-'87 Crash Fears in the S&P 500 Futures Option Market." *Journal of Econometrics*, 94: 181–238. 805

Chib, S., Nardari, F., and Shephard, N. (2002). "Markov Chain Monte Carlo Methods for Stochastic Volatility Models." *Journal of Econometrics*, 108: 281–316. 805

— (2006). "Analysis of High Dimensional Multivariate Stochastic Volatility Models." *Journal of Econometrics*, 134: 341–371. 807

Duffie, D., Pan, J., and Singleton, K. (2000). "Transform Analysis and Asset Pricing for Affine Jump-Diffusions." *Econometrica*, 68 (6): 1343–1376. 805

Eraker, B., Johannes, M., and Polson, N. (2003). "The Impact of Jumps in Volatility and Returns." *Biometrics*, 58 (3): 1269–1300. 805

Maneesoonthorn, W., Forbes, C., and Martin, G. (2013). "Inference on Self-Exciting Jumps in Prices and Volatility using High Frequency Measures." Technical Report Working paper wp28-13, Monash University Department of Econometrics and Business Statistics. 807

Maneesoonthorn, W., Martin, G., Forbes, C., and Grose, S. (2012). "Probabilistic Forecasts of Volatility and Its Risk Premia." *Journal of Econometrics*, 171: 217–236. 805

Prado, R. and West, M. (2010). *Time Series: Modeling, Computation, and Inference*. Taylor & Francis, Ltd. 805