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Research Article

Tracking Control Based on Recurrent Neural Networks for Nonlinear Systems with Multiple Inputs and Unknown Deadzone

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This paper deals with the problem of trajectory tracking for a broad class of uncertain nonlinear systems with multiple inputs each one subject to an unknown symmetric deadzone. On the basis of a model of the deadzone as a combination of a linear term and a disturbance-like term, a continuous-time recurrent neural network is directly employed in order to identify the uncertain dynamics. By using a Lyapunov analysis, the exponential convergence of the identification error to a bounded zone is demonstrated. Subsequently, by a proper control law, the state of the neural network is compelled to follow a bounded reference trajectory. This control law is designed in such a way that the singularity problem is conveniently avoided and the exponential convergence to a bounded zone of the difference between the state of the neural identifier and the reference trajectory can be proven. Thus, the exponential convergence of the tracking error to a bounded zone and the boundedness of all closed-loop signals can be guaranteed. One of the main advantages of the proposed strategy is that the controller can work satisfactorily without any specific knowledge of an upper bound for the unmodeled dynamics and/or the disturbance term.

1. Introduction

After more than a half century of ongoing research, the adaptive control of linear and nonlinear systems with linearly parameterized unknown constants is currently a solid area within an automatic control theory. In order to extend these results to more general classes

of systems, during the last twenty years, intense research has been carried out relying on the universal approximation capability of the artificial neural networks [1–7].

An artificial neural network can be simply considered as a nonlinear generic mathematical formula whose parameters are adjusted in order to represent the behavior of a static or dynamic system [5]. These parameters are called weights. Generally speaking, ANN can be classified as feedforward (static) ones, based on the back propagation technique [2], or as recurrent (dynamic) ones [4]. In the first network type, system dynamics is approximated by a static mapping. These networks have two major disadvantages: a slow learning rate and a high sensitivity to training data. The second approach (recurrent ANN) incorporates feedback into its structure. Due to this feature, recurrent neural networks can overcome many problems associated with static ANN, such as global extrema search, and consequently have better approximation properties [8]. Depending on their structure, recurrent neural networks can be classified as discrete-time ones or continuous-time ones.

Much of the first effort of research about the theory and application of the control based on continuous-time recurrent neural networks was synthesized in [4, 6, 9, 10]. In [9], a strategy of indirect adaptive control based on a parallel recurrent neural network was presented. In that study, the asymptotic convergence of the average integral identification error to a bounded zone was guaranteed. In order to prove this result, a Riccati matrix equation was employed. Based on the neural model of the uncertain system, a local optimal-type controller was developed. In spite of the significant contributions presented in that study, the usage of the Riccati matrix equation can be some restrictive and certain important questions such as the possible singularity of the control law were not considered. On the basis of this work, the exponential convergence of the identification error to a bounded zone could be guaranteed in [11–13]. However, the need of a Riccati matrix equation could not be avoided. In [10], a tracking controller based on a series-parallel neural network model was proposed. In that study, the assumptions about the uncertain system were less restrictive than in [9], Riccati matrix equation was not necessary, and the possibility of the singularity problem for the control law was conveniently avoided. In contrast, the control law proposed by [10] is some complex. In spite of the importance of the aforementioned works, the case when the presence of a deadzone degrades the performance of an automatic control system was not taken into account.

The deadzone is a nonsmooth nonlinearity commonly found in many practical systems such as hydraulic positioning systems [14], pneumatic servo systems [15], and DC servo motors and so on. When the deadzone is not considered explicitly during the design process, the performance of the control system could be degraded due to an increase of the steady-state error, the presence of limit cycles, or inclusive instability [16–19]. A direct way of compensating the deleterious effect of the deadzone is by calculating its inverse. However, this is not an easy question because in many practical situations, both the parameters and the output of the deadzone are unknown. To overcome this problem, in a pioneer work [16], Tao and Kokotović proposed to employ an adaptive inverse of the deadzone. This scheme was applied to linear systems in a transfer function form. Cho and Bai [20] extended this work and achieved a perfect asymptotic adaptive cancellation of the deadzone. However, their work assumed that the deadzone output was measurable. In [21], the work of Tao and Kokotović was extended to linear systems in a state space form with nonmeasurable deadzone output. In [22], a new smooth parameterization of the deadzone was proposed and a class of SISO systems with completely known nonlinear functions and with linearly parameterized unknown constants was controlled by using backstepping technique. In order to avoid the construction of the adaptive inverse, in [23], the same class of nonlinear systems

as in [22] was controlled by means of a robust adaptive approach and by modeling the deadzone as a combination of a linear term and a disturbance-like term. The controller design in [23] was based on the assumption that maximum and minimum values for the deadzone parameters are a priori known. However, a specific procedure to find such bounds was not provided. Based on the universal approximation property of the neural networks, a wider class of SISO systems in Brunovsky canonical form with completely unknown nonlinear functions and unknown constant control gain was considered in [24–26]. Apparently, the generalization of these results to the case when the control gain is varying, state dependent is trivial. Nevertheless, the solution to this problem is not so simple due to the singularity possibility for the control law. In [27, 28], this problem was overcome satisfactorily.

All the aforementioned works about deadzone studied a very particular class of systems, that is, systems in strict Brunovsky canonical form with a unique input. In this paper, by combining, in an original way, the design strategies from [9, 10, 23], we can handle a broad class of uncertain nonlinear systems with multiple inputs each one subject to an unknown symmetric deadzone. On the basis of a model of the deadzone as a combination of a linear term and a disturbance-like term, a continuous-time recurrent neural network is directly employed in order to identify the uncertain dynamics. By using a Lyapunov analysis, the exponential convergence of the identification error to a bounded zone is demonstrated. Subsequently, by a proper control law, the state of the neural network is compelled to follow a bounded reference trajectory. This control law is designed in such a way that the singularity problem is conveniently avoided as in [10] and the exponential convergence to a bounded zone of the difference between the state of the neural identifier and the reference trajectory can be proven. Thus, the exponential convergence of the tracking error to a bounded zone and the boundedness of all closed-loop signals can be guaranteed. This is the first time, up to the best of our knowledge, that recurrent neural networks are utilized in the context of uncertain system control with deadzone.

2. Preliminaries

In this study, the system to be controlled consists of an unknown multi-input nonlinear plant with unknown deadzones in the following form:

$$\text{Plant: } \dot{x}(t) = f(x(t)) + g(x(t))u(t) + \xi(t), \tag{2.1}$$

$$\text{Deadzone: } u_i(t) = DZ_i(v_i(t)) = \begin{cases} m_i(v_i(t) - b_{i,r}) & v_i(t) \geq b_{i,r}, \\ 0 & b_{i,l} < v_i(t) < b_{i,r}, \\ m_i(v_i(t) - b_{i,l}) & v_i(t) \leq b_{i,l}, \end{cases} \tag{2.2}$$

where $x(t) \in \Re^n$ is the measurable state vector for $t \in \Re^+ := \{t : t \geq 0\}$, $f : \Re^n \to \Re^n$ is an unknown but continuous nonlinear vector function, $g : \Re^n \to \Re^{n\times q}$ is an unknown but continuous nonlinear matrix function, $\xi(t) \in \Re^n$ represents an unknown but bounded deterministic disturbance, the ith element of the vector $u(t) \in \Re^q$, that is, $u_i(t)$, represents the output of the ith deadzone, $v_i(t)$ is the input to the ith deadzone, $b_{i,r}$ and $b_{i,l}$ represent the right and left constant breakpoints of the ith deadzone, and m_i is the constant slope of the ith deadzone. In accordance with [16, 17], the deadzone model (2.2) is a static simplification of diverse physical phenomena with negligible fast dynamics. Note that $v(t) \in \Re^q$ is the actual

control input to the global system described by (2.1) and (2.2). Hereafter it is considered that the following assumptions are valid.

Assumption 2.1. The plant described by (2.1) is controllable.

Assumption 2.2. The ith deadzone output, that is, $u_i(t)$ is not available for measurement.

Assumption 2.3. Although the ith deadzone parameters $b_{i,r}$, $b_{i,l}$, and m_i are unknown constants, we can assure that $b_{i,r} > 0$, $b_{i,l} < 0$, and $m_i > 0$ for all $i \in \{1, 2, \ldots q\}$.

2.1. Statement of the Problem

The objective that we are trying to achieve is to determine a control signal $v(t)$ such that the state $x(t)$ follows a given bounded reference trajectory $x_r(t)$, and, at the same time, all closed-loop signals stay bounded.

Assumption 2.4. Without the loss of generality, we consider that $x_r(t)$ is generated by the following exosystem:

$$\dot{x}_r(t) = B(x_r(t)), \tag{2.3}$$

where $B : \Re^n \to \Re^n$ is an unknown but continuous nonlinear vector function.

2.2. Deadzone Representation as a Linear Term and a Disturbance-Like Term

The deadzone model (2.2) can alternatively be described as [23, 29]:

$$u_i(t) = m_i v_i(t) + d_i(t), \tag{2.4}$$

where $d_i(t)$ is given by

$$d_i(t) = \begin{cases} -m_i b_{i,r}, & v_i(t) \geq b_{i,r}, \\ -m_i v_i(t), & b_{i,l} < v_i(t) < b_{i,r}, \\ -m_i b_{i,l}, & v_i(t) \leq b_{i,l}. \end{cases} \tag{2.5}$$

Note that (2.5) is the negative of a saturation function. Thus, although $d_i(t)$ could not be exactly known, its boundedness can be assured. Consider that the positive constant $\overline{d}_i$ is an upper bound for $d_i(t)$, that is, $|d_i(t)| \leq \overline{d}_i$.

Based on (2.4), the relationship between $u(t)$ and $v(t)$ can be expressed as

$$u(t) = Mv(t) + d(t), \tag{2.6}$$

where $M := \text{diag}(m_1, m_2, \ldots m_q)$ and $d(t) \in R^q$ is given by $d(t) := [d_1(t), d_2(t), \ldots, d_q(t)]^T$. Clearly, $d(t) \in L_\infty$. Consider that the positive constant $\overline{d}$ is an upper bound for $d(t)$.

3. Neural Identifier

In this section, the identification problem of the unknown global dynamics described by (2.1) and (2.2) using a recurrent neural network is considered.

Note that an alternative representation for (2.1) is given by

$$\dot{x}(t) = Ax(t) + W_1^*\sigma(x(t)) + W_2^*\phi(x(t))u(t) + \omega(x(t), u(t)) + \xi(t), \tag{3.1}$$

where $A \in \Re^{n\times n}$ is a Hurwitz matrix, $W_1^* \in \Re^{n\times m}$ and $W_2^* \in \Re^{n\times r}$ are unknown constant weight matrices, and $\sigma(\cdot)$ is the activation vector function with sigmoidal components, that is, $\sigma(\cdot) := [\sigma_1(\cdot), \ldots, \sigma_m(\cdot)]^\top$

$$\sigma_j(x(t)) := \frac{a_{\sigma j}}{1 + \exp\left(-\sum_{i=1}^{n} c_{\sigma j,i} x_i(t)\right)} - d_{\sigma j} \quad \text{for } j = 1, \ldots, m, \tag{3.2}$$

where $a_{\sigma j}$, $c_{\sigma j,i}$, and $d_{\sigma j}$ are positive constants which can be specified by the designer, $\phi(\cdot) : \Re^n \to \Re^{r\times q}$ is a sigmoidal function, that is,

$$\phi_{ij}(x(t)) := \frac{a_{\phi ij}}{1 + \exp\left(-\sum_{l=1}^{n} c_{\phi ij,l} x_l(t)\right)} - d_{\phi ij} \quad \text{for } i = 1, \ldots, r,\ j = 1, \ldots, q, \tag{3.3}$$

where $a_{\phi ij}$, $c_{\phi ij,l}$, and $d_{\phi ij}$ are positive constants which can be specified by the designer, and $\omega : \Re^n \times \Re^q \to \Re^n$ is the unmodeled dynamics which can be defined simply as $\omega(x(t), u(t)) := f(x(t)) + g(x(t))u(t) - Ax(t) - W_1^*\sigma(x(t)) - W_2^*\phi(x(t))u(t)$.

Assumption 3.1. On a compact set $\Omega \subset \Re^n$, unmodeled dynamics $\omega(x(t), u(t))$ is bounded by $\overline{\omega}$, that is, $|\omega(x(t), u(t))| \leq \overline{\omega}$. The disturbance $\xi(t)$ is also bounded, that is, $|\xi(t)| \leq \Upsilon$. Both $\overline{\omega}$ and Υ are positive constants not necessarily a priori known.

By substituting (2.6) into (3.1), we get

$$\dot{x}(t) = Ax(t) + W_1^*\sigma(x(t)) + W_2^*\phi(x(t))Mv(t) + W_2^*\phi(x(t))d(t) + \omega(x(t), u(t)) + \xi(t). \tag{3.4}$$

Remark 3.2. It can be observed that by using the model (2.6), the actual control input $v(t)$ appears now directly into the dynamics.

Since, by construction, $\phi(x(t))$ is bounded, the term $W_2^*\phi(x(t))d(t)$ is also bounded. Let us define the following expression: $\zeta(t) := W_2^*\phi(x(t))d(t) + \omega(x(t), u(t)) + \xi(t)$. Clearly, this expression is bounded. Let us denote an upper bound for $\zeta(t)$ as $\overline{\zeta}$. This bound is a positive constant not necessarily a priori known. Now, note that the term $W_2^*\phi(x(t))Mv(t)$ can be alternatively expressed as $S^*\phi(x(t))v(t)$, where $S^* \in \Re^{n\times r}$ is an unknown weight matrix. In view of the above, (3.4) can be rewritten as

$$\dot{x}(t) = Ax(t) + W_1^*\sigma(x(t)) + S^*\phi(x(t))v(t) + \zeta(t). \tag{3.5}$$

Now, consider the following series-parallel structure for a continuous-time recurrent neural network

$$\dot{\hat{x}}(t) = A\hat{x}(t) + W_1(t)\sigma(x(t)) + S(t)\phi(x(t))v(t), \tag{3.6}$$

where $\hat{x}_t \in \Re^n$ is the state of the neural network, $v(t) \in \Re^q$ is the control input, and $W_1(t) \in \Re^{n\times m}$ and $S(t) \in \Re^{n\times r}$ are the time-varying weight matrices. The problem of identifying system (2.1)-(2.2) based on the recurrent neural network (3.6) consists of, given the measurable state $x(t)$ and the input $v(t)$, adjusting online the weights $W_1(t)$ and $S(t)$ by proper learning laws such that the identification error $\Delta(t) := \hat{x}(t) - x(t)$ can be reduced. Specifically, the following learning laws are here used:

$$\dot{W}_1(t) = -k_1\Delta(t)\sigma^T(x(t)) - \ell_1 W_1(t), \tag{3.7}$$

$$\dot{S}(t) = -k_2\Delta(t)v^T(t)\phi^T(x(t)) - \ell_2 S(t), \tag{3.8}$$

where k_1, ℓ_1, k_2, and ℓ_2 are positive constants selectable by the designer.

Based on the learning laws (3.7) and (3.8), the following result is here established.

Theorem 3.3. *If the Assumptions 2.2, 2.3, and 3.1 are satisfied, the constant a is selected greater than 0.5, and the weight matrices $W_1(t)$, $S(t)$ of the neural network (3.6) are adjusted by the learning laws (3.7) and (3.8), respectively, then*

(a) *the identification error and the weights of the neural network (3.6) are bounded:*

$$\Delta(t), W_1(t), S(t) \in L_\infty, \tag{3.9}$$

(b) *the norm of the identification error, that is, $|\hat{x}(t) - x(t)|$ converges exponentially fast to a zone bounded by the term*

$$\sqrt{\frac{2\beta}{\alpha}}, \tag{3.10}$$

where $\alpha := \min\{(2a - 1), \ell_1, \ell_2\}$, $\beta := (1/2)\bar{\zeta}^2 + (\ell_1/2k_1)\,\mathrm{tr}\{W_1^{*T}W_1^*\} + (\ell_2/2k_2)\,\mathrm{tr}\{S^{*T}S^*\}$.

Proof of Theorem 3.3. First, let us determine the dynamics of the identification error. The first derivative of $\Delta(t)$ is simply

$$\dot{\Delta}(t) = \dot{\hat{x}}(t) - \dot{x}(t). \tag{3.11}$$

Substituting (3.6) and (3.5) into (3.11) yields

$$\begin{aligned}\dot{\Delta}_t &= A\hat{x}(t) + W_1(t)\sigma(x(t)) + S(t)\phi(x(t))v(t) - Ax(t) - W_1^*\sigma(x(t)) \\ &\quad - S^*\phi(x(t))v(t) - \zeta(t) \\ &= A\Delta(t) + \widetilde{W}_1(t)\sigma(x(t)) + \widetilde{S}(t)\phi(x(t))v(t) - \zeta(t),\end{aligned} \tag{3.12}$$

where $\widetilde{W}_1(t) := W_1(t) - W_1^*$ and $\widetilde{S}(t) := S(t) - S^*$.

Consider the following Lyapunov function candidate

$$V(t) = \frac{1}{2}\Delta^T(t)\Delta(t) + \frac{1}{2k_1}\operatorname{tr}\left\{\widetilde{W}_1^T(t)\widetilde{W}_1(t)\right\} + \frac{1}{2k_2}\operatorname{tr}\left\{\widetilde{S}^T(t)\widetilde{S}(t)\right\}. \tag{3.13}$$

The first derivative of $V(t)$ is

$$\dot{V}(t) = \Delta^T(t)\dot{\Delta}(t) + \frac{1}{k_1}\operatorname{tr}\left\{\dot{\widetilde{W}}_1^T(t)\widetilde{W}_1(t)\right\} + \frac{1}{k_2}\operatorname{tr}\left\{\dot{\widetilde{S}}^T(t)\widetilde{S}(t)\right\}. \tag{3.14}$$

Substituting (3.12) into (3.14) and taking into account that, for simplicity, A can be selected as $A = -aI$, where a is a positive constant greater than 0.5 and $I \in \Re^{n\times n}$ is the identity matrix, yields

$$\begin{aligned}\dot{V}(t) &= -a|\Delta(t)|^2 + \Delta^T(t)\widetilde{W}_1(t)\sigma(x(t)) + \Delta^T(t)\widetilde{S}(t)\phi(x(t))v(t) - \Delta^T(t)\zeta(t) \\ &\quad + \frac{1}{k_1}\operatorname{tr}\left\{\dot{\widetilde{W}}_1^T(t)\widetilde{W}_1(t)\right\} + \frac{1}{k_2}\operatorname{tr}\left\{\dot{\widetilde{S}}^T(t)\widetilde{S}(t)\right\}.\end{aligned} \tag{3.15}$$

Since $\widetilde{W}_1(t) := W_1(t) - W_1^*$ and $\widetilde{S}(t) := S(t) - S^*$, the first derivatives for $\dot{\widetilde{W}}_1(t)$ and $\dot{\widetilde{S}}(t)$ are clearly $\dot{\widetilde{W}}_1(t) = \dot{W}_1(t)$ and $\dot{\widetilde{S}}(t) = \dot{S}(t)$, respectively. However, $\dot{W}_1(t)$ and $\dot{S}(t)$ are given by the learning laws (3.7) and (3.8). Therefore, by substituting (3.7) into $\dot{\widetilde{W}}_1(t) = \dot{W}_1(t)$ and (3.8) into $\dot{\widetilde{S}}(t) = \dot{S}(t)$ and the corresponding expressions into the right-hand side of (3.15), it is possible to obtain

$$\begin{aligned}\dot{V}(t) &= -a|\Delta(t)|^2 + \Delta^T(t)\widetilde{W}_1(t)\sigma(x(t)) + \Delta^T(t)\widetilde{S}(t)\phi(x(t))v(t) - \Delta^T(t)\zeta(t) \\ &\quad + \operatorname{tr}\left\{-\sigma(x(t))\Delta^T(t)\widetilde{W}_1(t)\right\} - \frac{\ell_1}{k_1}\operatorname{tr}\left\{W_1^T(t)\widetilde{W}_1(t)\right\} + \operatorname{tr}\left\{-\phi(x(t))v(t)\Delta^T(t)\widetilde{S}(t)\right\} \\ &\quad - \frac{\ell_2}{k_2}\operatorname{tr}\left\{S^T(t)\widetilde{S}(t)\right\}.\end{aligned} \tag{3.16}$$

We can see that

$$\begin{aligned}
\operatorname{tr}\left\{-\sigma(x(t))\Delta^T(t)\widetilde{W}_1(t)\right\} &= -\operatorname{tr}\left\{\sigma(x(t))\Delta^T(t)\widetilde{W}_1(t)\right\} = -\operatorname{tr}\left\{\Delta^T(t)\widetilde{W}_1(t)\sigma(x(t))\right\} \\
&= -\Delta^T(t)\widetilde{W}_1(t)\sigma(x(t)), \\
\operatorname{tr}\left\{-\phi(x(t))v(t)\Delta^T(t)\widetilde{S}(t)\right\} &= -\operatorname{tr}\left\{\phi(x(t))v(t)\Delta^T(t)\widetilde{S}(t)\right\} = -\operatorname{tr}\left\{\Delta^T(t)\widetilde{S}(t)\phi(x(t))v(t)\right\} \\
&= -\Delta^T(t)\widetilde{S}(t)\phi(x(t))v(t).
\end{aligned} \tag{3.17}$$

Substituting (3.17) into (3.16) and reducing the like terms yields

$$\dot{V}(t) = -a|\Delta(t)|^2 - \Delta^T(t)\zeta(t) - \frac{\ell_1}{k_1}\operatorname{tr}\left\{W_1^T(t)\widetilde{W}_1(t)\right\} - \frac{\ell_2}{k_2}\operatorname{tr}\left\{S^T(t)\widetilde{S}(t)\right\}. \tag{3.18}$$

Now, it can be proven that [10]

$$\begin{aligned}
\operatorname{tr}\left\{W_1^T(t)\widetilde{W}_1(t)\right\} &= \frac{1}{2}\operatorname{tr}\left\{W_1^T(t)W_1(t)\right\} + \frac{1}{2}\operatorname{tr}\left\{\widetilde{W}_1^T(t)\widetilde{W}_1(t)\right\} - \frac{1}{2}\operatorname{tr}\left\{W_1^{*T}W_1^*\right\}, \\
\operatorname{tr}\left\{S^T(t)\widetilde{S}(t)\right\} &= \frac{1}{2}\operatorname{tr}\left\{S^T(t)S(t)\right\} + \frac{1}{2}\operatorname{tr}\left\{\widetilde{S}^T(t)\widetilde{S}(t)\right\} - \frac{1}{2}\operatorname{tr}\left\{S^{*T}S^*\right\}.
\end{aligned} \tag{3.19}$$

Likewise, it is easy to show that

$$-\Delta^T(t)\zeta(t) \le \frac{1}{2}|\Delta(t)|^2 + \frac{1}{2}|\zeta(t)|^2 \le \frac{1}{2}|\Delta(t)|^2 + \frac{1}{2}\bar{\zeta}^2. \tag{3.20}$$

If (3.19) and the inequality (3.20) are substituted into (3.18), we obtain

$$\begin{aligned}
\dot{V}(t) \le{}& -a|\Delta(t)|^2 + \frac{1}{2}|\Delta(t)|^2 + \frac{1}{2}\bar{\zeta}^2 - \frac{\ell_1}{2k_1}\operatorname{tr}\left\{W_1^T(t)W_1(t)\right\} - \frac{\ell_1}{2k_1}\operatorname{tr}\left\{\widetilde{W}_1^T(t)\widetilde{W}_1(t)\right\} \\
&+ \frac{\ell_1}{2k_1}\operatorname{tr}\left\{W_1^{*T}W_1^*\right\} - \frac{\ell_2}{2k_2}\operatorname{tr}\left\{S^T(t)S(t)\right\} - \frac{\ell_2}{2k_2}\operatorname{tr}\left\{\widetilde{S}^T(t)\widetilde{S}(t)\right\} + \frac{\ell_2}{2k_2}\operatorname{tr}\left\{S^{*T}S^*\right\}
\end{aligned} \tag{3.21}$$

or

$$\begin{aligned}
\dot{V}(t) \le{}& -(2a-1)\left\{\frac{1}{2}|\Delta(t)|^2\right\} - \ell_1\left(\frac{1}{2k_1}\operatorname{tr}\left\{\widetilde{W}_1^T(t)\widetilde{W}_1(t)\right\}\right) - \ell_2\left(\frac{1}{2k_2}\operatorname{tr}\left\{\widetilde{S}^T(t)\widetilde{S}(t)\right\}\right) \\
&+ \frac{1}{2}\bar{\zeta}^2 + \frac{\ell_1}{2k_1}\operatorname{tr}\left\{W_1^{*T}W_1^*\right\} + \frac{\ell_2}{2k_2}\operatorname{tr}\left\{S^{*T}S^*\right\}.
\end{aligned} \tag{3.22}$$

In view of $\alpha := \min\{(2a-1), \ell_1, \ell_2\}$, $\beta := (1/2)\bar{\zeta}^2 + (\ell_1/2k_1)\operatorname{tr}\{W_1^{*T}W_1^*\} + (\ell_2/2k_2)\operatorname{tr}\{S^{*T}S^*\}$, the following bound as a function of $V(t)$ can finally be determined for $\dot{V}(t)$,

$$\dot{V}(t) \le -\alpha V(t) + \beta. \tag{3.23}$$

Equation (3.23) can be rewritten in the following form

$$\dot{V}(t) + \alpha V(t) \leq \beta. \tag{3.24}$$

Multiplying both sides of the last inequality by $\exp(\alpha t)$, it is possible to obtain

$$\exp(\alpha t)\dot{V}(t) + \alpha \exp(\alpha t)V(t) \leq \beta \exp(\alpha t). \tag{3.25}$$

The left-hand side of (3.25) can be rewritten as

$$\frac{d}{dt}\left(\exp(\alpha t)V(t)\right) \leq \beta \exp(\alpha t) \tag{3.26}$$

or equivalently as

$$d\left(\exp(\alpha t)V(t)\right) \leq \beta \exp(\alpha t)dt. \tag{3.27}$$

Integrating both sides of the last inequality from 0 to t yields

$$\exp(\alpha t)V(t) - V(0) \leq \int_0^t \beta \exp(\alpha \tau)d\tau. \tag{3.28}$$

Adding $V(0)$ to both sides of the last inequality, we obtain

$$\exp(\alpha t)V(t) \leq V(0) + \int_0^t \beta \exp(\alpha \tau)d\tau. \tag{3.29}$$

Multiplying both sides of the inequality (3.29) by $\exp(-\alpha t)$ yields

$$V(t) \leq \exp(-\alpha t)V(0) + \exp(-\alpha t)\int_0^t \beta \exp(\alpha \tau)d\tau \tag{3.30}$$

and, consequently

$$V(t) \leq V(0)\exp(-\alpha t) + \frac{\beta}{\alpha}\left(1 - \exp(-\alpha t)\right). \tag{3.31}$$

As by definition α and β are positive constants, the right-hand side of the last inequality can be bounded by $V(0) + (\beta/\alpha)$. Thus, $V(t) \in L_\infty$ and since by construction $V(t)$ is a nonnegative function, the boundedness of $\Delta(t)$, $\widetilde{W}_1(t)$, and $\widetilde{S}(t)$ can be guaranteed. Because W_1^* and S^* are bounded, $W_1(t) = \widetilde{W}_1(t) + W_1^*$, and $S(t) = \widetilde{S}(t) + S^*$ must be bounded too and the first part of

Theorem 3.3 has been proven. With respect to the second part of this theorem, from (3.13), it is evident that $(1/2)|\Delta(t)|^2 \leq V(t)$. Taking into account this fact and from (3.31), we get

$$|\Delta(t)| \leq \sqrt{2V(0)\exp(-\alpha t) + \frac{2\beta}{\alpha}\left(1 - \exp(-\alpha t)\right)}. \tag{3.32}$$

By taking the limit as $t \to \infty$ of the inequality (3.32), we can guarantee that $|\Delta(t)|$ converges exponentially fast to a zone bounded by the term $\sqrt{2\beta/\alpha}$ and the last part of Theorem 3.3 has been proven. □

Remark 3.4. It is very important to mention that the identification process based on Theorem 3.3 can be accomplished without the a priori knowledge about W_1^*, S^*, and $\overline{\zeta}$.

4. Controller Design

In this section, a proper control law $v(t)$ in order to solve the tracking problem is determined.

Note that the dynamics of the exosystem (2.3) can be alternatively represented as

$$\dot{x}_r(t) = Ax_r(t) + W_r^*\sigma_r(x_r(t)) + \omega_r(x_r(t)), \tag{4.1}$$

where $A \in \Re^{n\times n}$ is the same Hurwitz matrix as in (3.6), $W_r^* \in \Re^{n\times m_r}$ is an unknown constant weight matrix, $\sigma_r(\cdot)$ is an activation vector function with sigmoidal components, that is, $\sigma_r(\cdot) := [\sigma_{r1}(\cdot), \ldots, \sigma_{rm_r}(\cdot)]^\top$

$$\sigma_{rj}(x(t)) := \frac{a_{\sigma_r j}}{1 + \exp\left(-\sum_{i=1}^{n} c_{\sigma_r j,i} x_i(t)\right)} - d_{\sigma_r j} \quad \text{for } j = 1, \ldots, m_r, \tag{4.2}$$

where $a_{\sigma_r j}$, $c_{\sigma_r j,i}$, and $d_{\sigma_r j}$ are positive constants which can be specified by the designer, and $\omega_r : \Re^n \to \Re^n$ is an error term which can be defined simply as

$$\omega_r(x(t)) := B(x_r(t)) - Ax_r(t) - W_r^*\sigma_r(x_r(t)). \tag{4.3}$$

Assumption 4.1. On a compact set $\Omega \subset \Re^n$, the error term $\omega_r(x_r(t))$ is bounded by the positive constant not necessarily a priori known $\overline{\omega}_r$, that is, $|\omega_r(x_r(t))| \leq \overline{\omega}_r$.

Let us define the virtual tracking error $e(t)$ as

$$e(t) := \hat{x}(t) - x_r(t). \tag{4.4}$$

The first derivative of (4.4) is simply

$$\dot{e}(t) = \dot{\hat{x}}(t) - \dot{x}_r(t). \tag{4.5}$$

Substituting (3.6) and (4.1) into (4.5) yields

$$\dot{e}(t) = A\hat{x}(t) + W_1(t)\sigma(x(t)) + S(t)\phi(x(t))v(t) - Ax_r(t) - W_r^*\sigma_r(x_r(t)) - \omega_r(x_r(t)). \tag{4.6}$$

By adding and subtracting the term $W_r(t)\sigma_r(x_r(t))$ into (4.6), we obtain

$$\begin{aligned}\dot{e}(t) = {} & Ae(t) + W_1(t)\sigma(x(t)) + S(t)\phi(x(t))v(t) + \widetilde{W}_r(t)\sigma_r(x_r(t)) \\ & - W_r(t)\sigma_r(x_r(t)) - \omega_r(x_r(t)),\end{aligned} \tag{4.7}$$

where $\widetilde{W}_r(t) := W_r(t) - W_r^*$.

Consider the following Lyapunov function candidate:

$$V_2(t) = \frac{1}{2}\gamma e^T(t)e(t) + \frac{1}{2}\operatorname{tr}\left\{\widetilde{W}_r^T(t)\widetilde{W}_r(t)\right\}, \tag{4.8}$$

where γ is a positive constant. The first derivative of $V_2(t)$ is

$$\dot{V}_2(t) = \gamma e^T(t)\dot{e}(t) + \operatorname{tr}\left\{\dot{\widetilde{W}}_r^T(t)\widetilde{W}_r(t)\right\}. \tag{4.9}$$

Substituting (4.7) into (4.9) and taking into account that A was selected in Section 3 as $A = -aI$ yields

$$\begin{aligned}\dot{V}_2(t) = {} & -\gamma a|e(t)|^2 + \gamma e^T(t)W_1(t)\sigma(x(t)) + \gamma e^T(t)S(t)\phi(x(t))v(t) + \gamma e^T(t)\widetilde{W}_r(t)\sigma_r(x_r(t)) \\ & - \gamma e^T(t)W_r(t)\sigma_r(x_r(t)) - \gamma e^T(t)\omega_r(x_r(t)) + \operatorname{tr}\left\{\dot{\widetilde{W}}_r^T(t)\widetilde{W}_r(t)\right\}.\end{aligned} \tag{4.10}$$

If the learning law for $W_r(t)$ is selected as

$$\dot{W}_r(t) = -\gamma e(t)\sigma_r^T(x_r(t)) - \ell_r W_r(t), \tag{4.11}$$

where ℓ_r is a positive constant and the control law $v(t)$ is chosen as

$$v(t) = \frac{1}{\lambda_r}\frac{\phi^T(x(t))S^T(t)W_r(t)\sigma_r(x_r(t))}{1 + \|S(t)\|^2\left\|\phi(x(t))\right\|^2} - ke(t), \tag{4.12}$$

where λ_r and k are positive constants and taking into account that $\dot{\widetilde{W}}_r(t) = \dot{W}_r(t)$ then, (4.10) becomes

$$\begin{aligned}\dot{V}_2(t) = {} & -\gamma a|e(t)|^2 + \gamma e^T(t)W_1(t)\sigma(x(t)) + \frac{\gamma}{\lambda_r}\frac{e^T(t)S(t)\phi(x(t))\phi^T(x(t))S^T(t)W_r(t)\sigma_r(x_r(t))}{1+\|S(t)\|^2\left\|\phi^T(x(t))\right\|^2} \\ & - \gamma k e^T(t)S(t)\phi(x(t))e(t) + \gamma e^T(t)\widetilde{W}_r(t)\sigma_r(x_r(t)) - \gamma e^T(t)W_r(t)\sigma_r(x_r(t)) \\ & - \gamma e^T(t)\omega_r(x_r(t)) - \gamma\,\mathrm{tr}\left\{\sigma_r(x_r(t))e^T(t)\widetilde{W}_r(t)\right\} - \ell_r\,\mathrm{tr}\left\{W_r^T(t)\widetilde{W}_r(t)\right\}.\end{aligned} \tag{4.13}$$

It can be proven that

$$\begin{aligned}\mathrm{tr}\left\{\sigma_r(x_r(t))e^T(t)\widetilde{W}_r(t)\right\} &= \mathrm{tr}\left\{e^T(t)\widetilde{W}_r(t)\sigma_r(x_r(t))\right\} = e^T(t)\widetilde{W}_r(t)\sigma_r(x_r(t)), \\ \mathrm{tr}\left\{W_r^T(t)\widetilde{W}_r(t)\right\} &= \frac{1}{2}\mathrm{tr}\left\{W_r^T(t)W_r(t)\right\} + \frac{1}{2}\mathrm{tr}\left\{\widetilde{W}_r^T(t)\widetilde{W}_r(t)\right\} - \frac{1}{2}\mathrm{tr}\left\{W_r^{*T}W_r^*\right\}.\end{aligned} \tag{4.14}$$

By substituting (4.14) into (4.13) and reducing the like terms, we obtain

$$\begin{aligned}\dot{V}_2(t) = {} & -\gamma a|e(t)|^2 + \gamma e^T(t)W_1(t)\sigma(x(t)) + \frac{\gamma}{\lambda_r}\frac{e^T(t)S(t)\phi(x(t))\phi^T(x(t))S^T(t)W_r(t)\sigma_r(x_r(t))}{1+\|S(t)\|^2\left\|\phi^T(x(t))\right\|^2} \\ & - \gamma k e^T(t)S(t)\phi(x(t))e(t) - \gamma e^T(t)W_r(t)\sigma_r(x_r(t)) - \gamma e^T(t)\omega_r(x_r(t)) \\ & - \frac{\ell_r}{2}\mathrm{tr}\left\{W_r^T(t)W_r(t)\right\} - \frac{\ell_r}{2}\mathrm{tr}\left\{\widetilde{W}_r^T(t)\widetilde{W}_r(t)\right\} + \frac{\ell_r}{2}\mathrm{tr}\left\{W_r^{*T}W_r^*\right\}.\end{aligned} \tag{4.15}$$

Taking into account that $\pm y^T z \le |y|\,|z|$ for $y \in \Re^n$, $z \in \Re^n$ and $\|Y\|^2 = \mathrm{tr}\{Y^TY\}$ for $Y \in \Re^{L_1\times L_2}$, (4.15) becomes

$$\begin{aligned}\dot{V}_2(t) \le {} & -\gamma a|e(t)|^2 + \gamma|e(t)|\,\|W_1(t)\|\,|\sigma(x(t))| + \frac{\gamma}{\lambda_r}\frac{|e(t)|\,\|S(t)\|^2\left\|\phi(x(t))\right\|^2\|W_r(t)\|\,|\sigma_r(x_r(t))|}{1+\|S(t)\|^2\left\|\phi^T(x(t))\right\|^2} \\ & + \gamma k|e(t)|^2\|S(t)\|\left\|\phi(x(t))\right\| + \gamma|e(t)|\,\|W_r(t)\|\,|\sigma_r(x_r(t))| \\ & + \gamma|e(t)|\,|\omega_r(x_r(t))| - \frac{\ell_r}{2}\|W_r(t)\|^2 - \frac{\ell_r}{2}\left\|\widetilde{W}_r(t)\right\|^2 + \frac{\ell_r}{2}\|W_r^*\|^2.\end{aligned} \tag{4.16}$$

Note that

$$\frac{\|S(t)\|^2\left\|\phi(x(t))\right\|^2}{1+\|S(t)\|^2\left\|\phi^T(x(t))\right\|^2} \le 1. \tag{4.17}$$

On the other hand, by construction, $\sigma(x(t))$ and $\sigma_r(x_r(t))$ are bounded. Consider that s_1 and s_r are the corresponding upper bounds, that is, $|\sigma(x(t))| \leq s_1$ and $|\sigma_r(x_r(t))| \leq s_r$ (both s_1 and s_r can be calculated). Likewise, by construction, $\phi(x(t))$ is bounded and $S(t)$ is bounded from Theorem 3.3. Consider that μ is an upper bound for $\|S(t)\| \|\phi(x(t))\|$, that is, $\|S(t)\| \|\phi(x(t))\| \leq \mu$. In view of the above and selecting $a > \mu k$ and

$$\gamma(a - \mu k) = \gamma_1 + \gamma_2, \tag{4.18}$$

where $\gamma_1 > 0.5$ and γ_2 are two positive constants, we can obtain

$$\begin{aligned}\dot{V}_2(t) \leq {} & -\gamma_1|e(t)|^2 - \gamma_2|e(t)|^2 + \gamma|e(t)|\,\|W_1(t)\|s_1 + \frac{\gamma}{\lambda_r}|e(t)|\,\|W_r(t)\|s_r + \gamma|e(t)|\,\|W_r(t)\|s_r \\ & + \gamma|e(t)|\,|\omega_r(x_r(t))| - \frac{\ell_r}{2}\|W_r(t)\|^2 - \frac{\ell_r}{2}\left\|\widetilde{W}_r(t)\right\|^2 + \frac{\ell_r}{2}\|W_r^*\|^2\end{aligned} \tag{4.19}$$

or

$$\begin{aligned}\dot{V}_2(t) \leq {} & -\gamma_1|e(t)|^2 - \gamma_2|e(t)|^2 + \gamma s_r\left(1 + \frac{1}{\lambda_r}\right)|e(t)|\,\|W_r(t)\| - \frac{\ell_r}{2}\|W_r(t)\|^2 \\ & + \gamma|e(t)|\{\|W_1(t)\|s_1 + |\omega_r(x_r(t))|\} - \frac{\ell_r}{2}\left\|\widetilde{W}_r(t)\right\|^2 + \frac{\ell_r}{2}\|W_r^*\|^2.\end{aligned} \tag{4.20}$$

Now, in accordance with Theorem 3.3, $W_1(t) \in L_\infty$. Based on this fact together with the Assumption 4.1, the boundedness of the term $\|W_1(t)\|s_1 + |\omega_r(x_r(t))|$ can be concluded. Consider that the unknown positive constant ε is an upper bound for that term, that is, $\|W_1(t)\|s_1 + |\omega_r(x_r(t))| \leq \varepsilon$. Thus, it is easy to show that

$$\gamma|e(t)|\{\|W_1(t)\|s_1 + |\omega_r(x_r(t))|\} \leq \gamma|e(t)|\varepsilon \leq \frac{1}{2}|e(t)|^2 + \frac{1}{2}\gamma^2\varepsilon^2. \tag{4.21}$$

On the other hand, if the constants ℓ_r and λ_r are selected in such a way that

$$\begin{aligned}\ell_r &> \frac{\gamma^2 s_r^2}{2\gamma_2}, \\ \lambda_r &\geq \frac{\gamma s_r}{\sqrt{2\gamma_2\ell_r} - \gamma s_r}\end{aligned} \tag{4.22}$$

then the following can be established

$$\gamma s_r\left(1 + \frac{1}{\lambda_r}\right) \leq \sqrt{2\gamma_2\ell_r}. \tag{4.23}$$

Based on (4.23), it can be proven that

$$-\gamma_2|e(t)|^2+\gamma s_r\left(1+\frac{1}{\lambda_r}\right)|e(t)|\,\|W_r(t)\|-\frac{\ell_r}{2}\|W_r(t)\|^2\leq-\left(\sqrt{\gamma_2}|e(t)|-\sqrt{\frac{\ell_r}{2}}\|W_r(t)\|\right)^2\leq 0. \tag{4.24}$$

Substituting (4.21) and (4.24) into (4.20) yields

$$\dot{V}_2(t)\leq-\gamma_1|e(t)|^2+\frac{1}{2}|e(t)|^2-\frac{\ell_r}{2}\left\|\widetilde{W}_r(t)\right\|^2+\frac{1}{2}\gamma^2\varepsilon^2+\frac{\ell_r}{2}\|W_r^*\|^2. \tag{4.25}$$

Defining $\alpha_r:=\min\{(2\gamma_1-1)/\gamma,\ell_r\}$, $\beta_r:=(1/2)\gamma^2\varepsilon^2+(\ell_r/2)\|W_r^*\|^2$, (4.25) becomes

$$\dot{V}_2(t)\leq-\alpha_r V_2(t)+\beta_r. \tag{4.26}$$

This means that

$$V_2(t)\leq V_2(0)\exp(-\alpha_r t)+\frac{\beta_r}{\alpha_r}\left(1-\exp(-\alpha_r t)\right). \tag{4.27}$$

As by definition α_r and β_r are positive constants, the right-hand side of the last inequality is bounded by $V(0)+(\beta_r/\alpha_r)$. Next, $V_2(t)\in L_\infty$ and consequently $e(t),\widetilde{W}_r(t)$, and $W_r(t)\in L_\infty$.

As by hypothesis $x_r(t)\in L_\infty$, the boundedness of $e(t)$ guarantees the boundedness of $\hat{x}(t)$. Remember that Theorem 3.3 guarantees that $\Delta(t)\in L_\infty$. By the definition of $\Delta(t)$, that is, $\Delta(t)=\hat{x}(t)-x(t)$ and considering that $\hat{x}(t)\in L_\infty$, the boundedness of $x(t)$ can be concluded. From (4.12), we can see that the control law $v(t)$ is selected in such a way that the denominator is never equal to zero although $\|S(t)\|=0$ and/or $\|\phi(x(t))\|=0$. Besides, we can verify that $v(t)$ is formed by bounded elements. Next, the control input $v(t)$ must be bounded too. On the other hand, note that the following is true:

$$\frac{1}{2}\gamma|e(t)|^2\leq V_2(t). \tag{4.28}$$

Taking into account (4.28) and from (4.27), we get

$$|e(t)|\leq\sqrt{\frac{2}{\gamma}V_2(0)\exp(-\alpha_r t)+\frac{2\beta_r}{\gamma\alpha_r}\left(1-\exp(-\alpha_r t)\right)}. \tag{4.29}$$

Now, the ultimate objective is to achieve that the state $x(t)$ of the unknown system (2.1)-(2.2) follows the reference trajectory $x_r(t)$. Thus, we need to know if the actual tracking error $x(t)-x_r(t)$ converges or not to a some value. Note that

$$|x(t)-x_r(t)|=|x(t)-\hat{x}(t)+\hat{x}(t)-x_r(t)|\leq|\hat{x}(t)-x(t)|+|\hat{x}(t)-x_r(t)|=|\Delta(t)|+|e(t)|. \tag{4.30}$$

Clearly, $|x(t) - x_r(t)| \in L_\infty$. Finally, by substituting (3.32) and (4.29) into (4.30), we have

$$|x(t) - x_r(t)| \leq \sqrt{2V(0)\exp(-\alpha t) + \frac{2\beta}{\alpha}\left(1 - \exp(-\alpha t)\right)} + \sqrt{\frac{2}{\gamma}V_2(0)\exp(-\alpha_r t) + \frac{2\beta_r}{\gamma\alpha_r}\left(1 - \exp(-\alpha_r t)\right)}. \tag{4.31}$$

By taking the limit as $t \to \infty$ of the last inequality, we can guarantee that $|x(t) - x_r(t)|$ converges exponentially fast to a zone bounded by the term $\sqrt{2\beta/\alpha} + \sqrt{2\beta_r/\gamma\alpha_r}$. Thus, the following theorem has been proven

Theorem 4.2. *Given the Assumptions 2.1–4.1, if the control law* (4.12) *is used together with the learning laws* (3.8) *and* (4.11) *then it can be guaranteed that*

(a) *the weight matrix $W_r(t)$, the virtual tracking error, the actual tracking error, the state of the neural network, the system state, and the control input are bounded:*

$$W_r(t), e(t), x(t) - x_r(t), \widehat{x}(t), x(t), v(t) \in L_\infty, \tag{4.32}$$

(b) *the actual tracking error $|x(t) - x_r(t)|$ converges exponentially to a zone bounded by the term*

$$\sqrt{\frac{2\beta}{\alpha}} + \sqrt{\frac{2\beta_r}{\gamma\alpha_r}}, \tag{4.33}$$

where α and β are defined as in Theorem 3.3 and $\alpha_r := \min\{(2\gamma_1 - 1)/\gamma, \ell_r\}$, $\beta_r := (1/2)\gamma^2\varepsilon^2 + (\ell_r/2)\|W_r^*\|^2$.

5. Numerical Example

In this section, a simple but illustrative simulation example is presented in order to show the feasibility of the suggested approach. Consider the first order nonlinear system given by

$$\dot{x}(t) = -x(t)\sin(x(t)) + \left(0.2 + \cos^2(x(t))\right)u(t) + \xi(t). \tag{5.1}$$

The initial condition for system (5.1) is $x(0) = 1$; $u(t)$ is the deadzone output; the parameters of the deadzone are $m = 1.6$, $b_r = 2.5$, and $b_l = -2$; $\xi(t)$, the disturbance term is selected as $\xi(t) = 0.5\sin(13t)$. The following reference trajectory is employed $y_r(t) = \sin(t) - 1.5\sin(2t)$. The parameters for the neural identifier and the control law are selected by trial and error as $\widehat{x}(0) = 0$, $a = 2000$, $k_1 = 500000$, $l_1 = 1$, $W_1(0) = 0$, $k_2 = 200$, $l_2 = 50$, $S(0) = 0.5$, $\sigma(x(t)) = \phi(x(t)) = 2/(1+\exp(-x(t)))-1$, $\gamma = 300$, $l_r = 31$, $W_r(0) = -1$, $\sigma_r(x_r(t)) = 2/(1+\exp(-x_r(t)))-1$, $\gamma_1 = 1$, $\gamma_2 = 1499$, $s_r = 1$, $\mu = 8$, $k = 249.375$, and $\lambda_r = 62$. The simulation is carried out by means of Simulink with ode45 method, relative tolerance equal to $1e-7$, and absolute tolerance

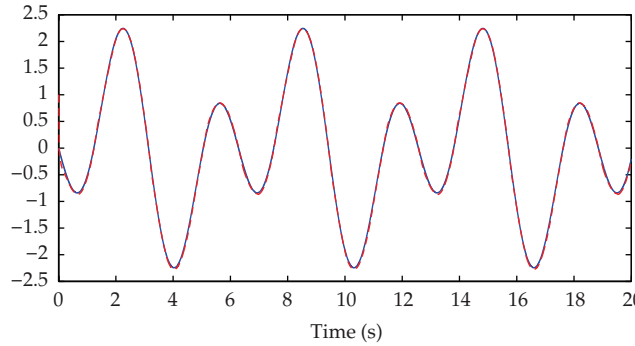


Figure 1: Tracking process: reference trajectory: solid line; system output: dashed line.

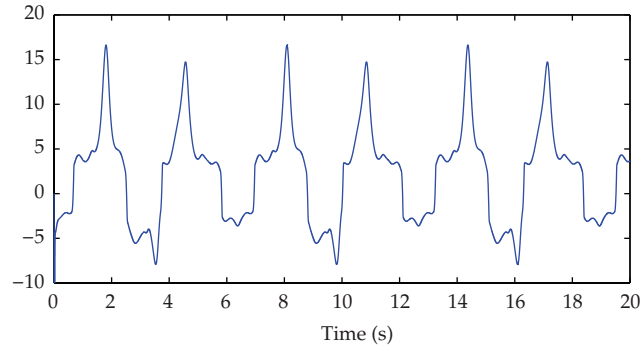


Figure 2: Control signal $v(t)$.

equal to $1e-9$. The results of the tracking process are presented in Figures 1–3 for the first 20 seconds. In Figure 1, the output of the nonlinear system (5.1), $x(t)$, is represented by a dashed line whereas the reference trajectory $x_r(t)$ is represented by a solid line. In Figure 2, the control signal $v(t)$ acting as the input of the deadzone is shown. In Figure 3, a zoom of Figure 2 is presented. From Figure 3, we can appreciate that the control law $v(t)$ avoids properly the deadzone.

6. Conclusions

In this paper, an adaptive scheme based on a continuous-time recurrent neural network is proposed in order to handle the tracking problem for a broad class of nonlinear systems with multiple inputs each one subject to an unknown symmetric deadzone. The need of an inverse adaptive commonly required in many previous works is conveniently avoided by considering the deadzone as a combination of a linear term and a disturbance-like term. Thus, the identification of the unknown dynamics together with the deadzone can be carried out directly by using a recurrent neural network. The exponential convergence of the identification error norm to a bounded zone is thoroughly proven by a Lyapunov analysis. Subsequently, the state of the neural network is compelled to follow a reference trajectory by using a control law designed in such a way that the singularity problem is conveniently avoided without the need of any projection strategy. By another Lyapunov

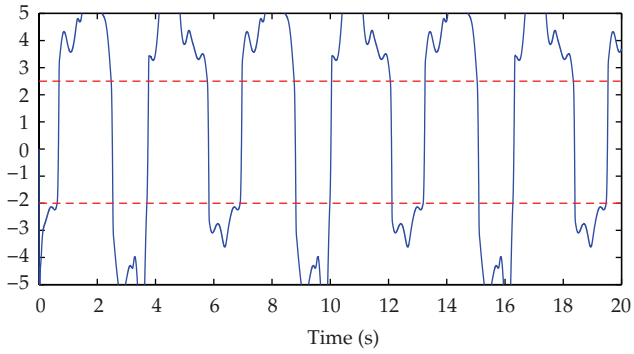


Figure 3: Zoom of Figure 2.

analysis, the exponential convergence of the difference between the neural network state and the reference trajectory is demonstrated. As the tracking error is bounded by the identification error and the difference between the neural network state and the reference trajectory, the exponential convergence of the tracking error to a bounded zone is also proven. Besides, the boundedness of the system state, the neural network state, the weights, and the control signal can be guaranteed. The proposed control scheme presents two important advantages:

(i) the specific knowledge of a bound for the unmodeled dynamics and/or the disturbance term is not necessary,

(ii) the determination of the first derivative for the reference trajectory is not required.

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