

Research Article

Finite-Time Stabilization of Stochastic Nonholonomic Systems and Its Application to Mobile Robot

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This paper investigates the problem of finite-time stabilization for a class of stochastic nonholonomic systems in chained form. By using stochastic finite-time stability theorem and the method of adding a power integrator, a recursive controller design procedure in the stochastic setting is developed. Based on switching strategy to overcome the uncontrollability problem associated with $x_0(0) = 0$, global stochastic finite-time regulation of the closed-loop system states is achieved. The proposed scheme can be applied to the finite-time control of nonholonomic mobile robot subject to stochastic disturbances. The simulation results demonstrate the validity of the presented algorithm.

1. Introduction

The nonholonomic systems, which can model many classes of mechanical systems such as mobile robots and wheeled vehicles, have attracted intensive attention over the past decades. From Brockett's necessary condition [1], it is well known that the nonholonomic systems cannot be stabilized to the origin by any static continuous state feedback although it is controllable. As a consequence, the classical smooth control theory cannot be applied directly used to such systems. In order to overcome this obstruction, several approaches have been proposed for the problem, such as discontinuous time-invariant stabilization [2, 3], smooth time-varying stabilization [4–6], and hybrid stabilization [7]. Using these valid approaches, many fruitful results have been developed [8–15]. Particularly, considering the unavailability of stochastic disturbance, the asymptotic stabilization for stochastic nonholonomic systems was achieved in [16–18]. However, it should be mentioned that those

aforementioned papers consider the feedback stabilizer that makes the trajectories of the systems converge to the equilibrium as the time goes to infinity.

Compared to the asymptotic stabilization, the finite-time stabilization, which renders the trajectories of the closed-loop systems convergent to the origin in a finite time, has many advantages such as fast response, high tracking precision, and disturbance-rejection properties. In many practical situations, the finite-time stabilization problem is more meaningful than the classical asymptotical stability. For the deterministic case, a sufficient and necessary condition for finite-time stability has been proposed in [19]. Its improvements and extensions have been given in [20, 21], for continuous systems satisfying uniqueness of solutions in forward time and for nonautonomous continuous systems, respectively. Reference [22] defined finite-time input-to-state stability for continuous systems with locally essentially bounded input. Accordingly, the problem of finite-time stabilization for nonlinear systems has been studied and numerous theoretical control design methods were presented and developed for various types of nonlinear systems over the last years [23–27]. Especially with help of time-rescaling and Lyapunov based method [28] proposed a novel switching finite time control strategy to nonholonomic chained systems in the deterministic setting.

However, the finite-time stabilization for stochastic nonholonomic systems cannot be solved by simply extending the methods for deterministic systems because of the presence of stochastic disturbance. As pointed out by Yin et al. [29], the existence of a unique solution and the nonsatisfaction of local Lipschitz condition are the preconditions of discussing the finite-time stability for a stochastic nonlinear system. Therefore, the finite-time controller design for stochastic nonholonomic systems in this paper should solve the following questions. Under what conditions, the stochastic nonholonomic systems exist possibly finite-time stabilizer? Under these conditions, how can one design a finite-time state-feedback stabilizing controller? Inspired by the works [25, 28], we generalize adding a power integrator design method to a stochastic system and based on stochastic finite-time stability theorem, by skillfully constructing C^2 Lyapunov functions, a state feedback controller is successfully achieved to guarantee that the closed-loop system states are globally regulated to zero within a given settling time almost surely.

The remainder of this paper is organized as follows. Section 2 presents some necessary notations, definitions and preliminary results. Section 3 describes the systems to be studied and formulates the control problem. Section 4 gives the main contributions of this paper and presents the design scheme to the controller. Section 5 gives a practical example, the model of which falls into our class of uncertain nonlinear system (3.1) via some technical transformations, to demonstrate the effectiveness of the theoretical results. Finally, concluding remarks are proposed in Section 6.

2. Notations and Preliminary Results

The following notations, definitions, and lemmas are to be used throughout the paper. R^+ denotes the set of all nonnegative real numbers and R^n denotes the real n -dimensional space. For a given vector or matrix X , X^T denotes its transpose, $\text{Tr}\{X\}$ denotes its trace when X is square, and $|X|$ is the Euclidean norm of a vector X . C^i denotes the set of all functions with continuous i th partial derivatives. K denotes the set of all functions: $R^+ \rightarrow R^+$, which are continuous, strictly increasing, and vanishing at zero; K_∞ denotes the set of all functions which are of class K and unbounded.

Consider the stochastic nonlinear system

$$dx = f(t, x)dt + g(t, x)d\omega, \quad (2.1)$$

where $x \in R^n$ is the system state with the initial condition $x(0) = x_0$, ω is an m -dimensional independent standard Wiener process defined on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, P)$ with Ω being a sample space, \mathcal{F} being a σ -field, $\{\mathcal{F}_t\}_{t \geq 0}$ being a filtration, and P being a probability measure. The functions: $f : R^+ \times R^n \rightarrow R^n$ and $g : R^+ \times R^n \rightarrow R^{n \times m}$ are piecewise continuous and continuous with respect to the first and second arguments, respectively, and satisfy $f(t, 0) \equiv 0$ and $g(t, 0) \equiv 0$.

The following Lemma is a corollary of Theorem 170 in [30], which provides a sufficient condition to ensure the existence and uniqueness of solution for the system (2.1).

Lemma 2.1. Assume that $f(t, x)$ and $g(t, x)$ are continuous in x . Further, for any $0 < \delta < 1$, each $N = 1, 2, \dots$, and each $0 \leq T < \infty$, if the following conditions hold:

$$\begin{aligned} |f(t, x)| &\leq c(t)(1 + |x|), & |g(t, x)|^2 &\leq c(t)(1 + |x|^2), \\ |f(t, x_1) - f(t, x_2)| &\leq c_T^N(t)|x_1 - x_2|, & |g(t, x_1) - g(t, x_2)| &\leq c_T^N(t)|x_1 - x_2|, \end{aligned} \quad (2.2)$$

as $0 < \delta \leq |x_i| \leq N$, $i = 1, 2$, $t \in [0, T]$, where $c(t)$ and $c_T^N(t)$ are nonnegative functions such that $\int_0^T c(t)dt < \infty$ and $\int_0^T c_T^N(t)dt < \infty$. Then for any given $x_0 \in R^n$, system (2.1) has a pathwise unique strong solution.

Definition 2.2 (see [31]). For system (2.1), define $\tau(0, x_0) = \inf \{T \geq 0 : x(t, x_0) = 0, \text{ for all } t \geq T\}$, which is called the stochastic settling time function of system (2.1), where $x_0 \in R^n$.

Definition 2.3 (see [31]). The equilibrium $x \equiv 0$ of the system (2.1) is said to be a stochastic finite-time stable equilibrium if

- (i) it is stable in probability: for every pair of $\varepsilon \in (0, 1)$ and $r > 0$, there exists $\delta > 0$ such that $P\{|x(t, x_0)| < r, \text{ for all } t \geq 0\} \geq 1 - \varepsilon$, whenever $|x| < \delta$.
- (ii) its stochastic settling-time function $\tau(t_0, x_0)$ exists finitely with probability and $E[\tau(0, x_0)] < \infty$.

Lemma 2.4 (see [32]). Consider the stochastic nonlinear system described in (2.1). Suppose there exists a C^2 function $V(x)$, class K_∞ functions μ_1 and μ_2 , real numbers $c > 0$ and $0 < \alpha < 1$, such that

$$\begin{aligned} \mu_1(|x|) &\leq V(x) \leq \mu_2(|x|), \\ \mathcal{L}V(x) &= \frac{\partial V}{\partial x}f + \frac{1}{2} \text{Tr} \left\{ g^T \frac{\partial^2 V}{\partial x^2} g \right\} \leq -cV^\alpha(x). \end{aligned} \quad (2.3)$$

Then it is globally finite-time stable in probability and the stochastic settling time function $\tau(0, x_0)$ satisfies

$$E[\tau(0, x_0)] \leq \frac{V^{1-\alpha}(x_0)}{c(1-\alpha)}. \quad (2.4)$$

Lemma 2.5 (see [25]). For any real numbers $x_i, i = 1, \dots, n$ and $0 < b < 1$, the following inequality holds:

$$(|x_1| + \dots + |x_n|)^b \leq |x_1|^b + \dots + |x_n|^b, \quad (2.5)$$

when $b = p/q < 1$, where $p > 0$ and $q > 0$ are odd integers,

$$|x^b - y^b| \leq 2^{1-b} |x - y|^b. \quad (2.6)$$

Lemma 2.6 (see [25]). Let c, d be positive real numbers and $\pi(x, y) > 0$ be a real-valued function. Then,

$$|x|^c |y|^d \leq \frac{c\pi(x, y)|x|^{c+d}}{c+d} + \frac{d\pi^{-c/d}(x, y)|y|^{c+d}}{c+d}. \quad (2.7)$$

3. Problem Formulation

In this paper, we focus our attention on the following class of stochastic nonholonomic systems:

$$\begin{aligned} dx_0 &= d_0(t)u_0 dt, \\ dx_i &= d_i(t)x_{i+1}u_0 dt + g_i^T(x_0, x_{[i]})d\omega, \quad i = 1, \dots, n-1, \\ dx_n &= d_n(t)u_1 dt + g_n^T(x_0, x_{[n]})d\omega, \end{aligned} \quad (3.1)$$

where $x_0 \in R$ and $x = (x_1, \dots, x_n)^T \in R^n$ are system states, $u_0 \in R$ and $u_1 \in R$ are control inputs, respectively; $x_{[i]} = (x_1, \dots, x_i)^T$, $x_{[n]} = x$; $d_i, i = 1, \dots, n$ represent the possible modeling error, referred to as disturbed virtual control coefficients; $g_i : R \times R^i \rightarrow R^m, i = 1, \dots, n$, are uncertain continuous functions satisfying $g_i(0, 0) = 0$; and ω is an m -dimensional independent standard Wiener process defined on a complete probability space (Ω, F, P) with Ω being a sample space, F being a filtration, and P being a probability measure.

Remark 3.1. It should be mentioned that the system investigated in this paper, which emphasizes the effect of stochastic disturbance on the x -subsystem, is a special one; however it can be found in many real systems, such as the angular velocity of mobile robot subject to stochastic disturbances (see Section 5).

The objective of this paper is to find a robust state feedback controller of the form

$$u_0 = u_0(x_0), \quad u_1 = u_1(x_0, x), \quad (3.2)$$

such that the stochastic finite-time regulation of closed-loop system states is achieved, that is, $P\{\lim_{t \rightarrow T}(|x_0(t)| + |x(t)|) = 0\} = 1$ and $P\{(x_0(t), x(t)) = (0, 0)\} = 1$ for any $t \geq T$, where T is a given settling time.

To achieve the above control objective, we need the following assumptions.

Assumption 3.2. For $i = 0, 1, \dots, n$, there are known positive constants c_{i1} and c_{i2} such that

$$c_{i1} \leq d_i(t) \leq c_{i2}. \quad (3.3)$$

Assumption 3.3. For $i = 1, \dots, n$, there are constants b and $\tau \in (-2/(4n+1), 0)$ such that

$$|g_i(x_0, x_{[i]}) - g_i(x_0, \hat{x}_{[i]})| \leq b(|x_1 - \hat{x}_1|^{(2m_i+\tau)/2m_1} + \dots + |x_i - \hat{x}_i|^{(2m_i+\tau)/2m_i}), \quad (3.4)$$

where $m_i = 1 + (i-1)\tau$.

For simplicity, in this paper we assume $\tau = -p/q$ with p being any even integer and q being any odd integer, under which and the definition of m_i in Assumption 3.3, we know that m_i is an odd number.

Remark 3.4. Noting that $g_i(0, 0) = 0$ is assumed, Assumption 3.3 implies that

$$|g_i(t, x, u)| \leq b(|x_1|^{(2m_i+\tau)/2m_1} + \dots + |x_i|^{(2m_i+\tau)/2m_i}). \quad (3.5)$$

In fact, Assumption 3.3 is a generalization of the homogeneous growth condition introduced in [33] where $\hat{x}_{[i]} = 0$ and $\tau \geq 0$. The assumption is necessary, which plays an essential role in ensuring the existence of finite-time stabilizer for stochastic nonholonomic system (3.1). Furthermore, it is worthwhile to point out that there exist some nonlinearities such as $\sin x$ that can be bounded by a function $|x|^m$ for any constant $m \in (0, 1)$ and satisfies Assumption 3.3.

4. Finite-Time Stabilization

In this section, we give a constructive procedure for the finite-time stabilizing control of system (3.1) within any given settling time T . For clarity, the case that $x_0(0) \neq 0$ is considered first. Then, the case where the initial $x_0(0) = 0$ is dealt later. The inherently triangular structure of system (3.1) suggests that we should design the control inputs u_0 and u_1 in two separate stages.

4.1. Control for $x_0(0) \neq 0$

For x_0 -subsystem, we take the following control law:

$$u_0 = -k_0 x_0^{\alpha_0}, \quad 0 < \alpha_0 = \frac{p}{q} < 1, \quad (4.1)$$

where k_0 is a positive design parameter, and p, q are positive odd numbers.

Taking the Lyapunov function $V_0 = x_0^2/2$, a simple computation gives

$$-c_{02}k_0x_0^{1+\alpha_0} \leq \dot{V}_0 \leq -c_{01}k_0x_0^{1+\alpha_0} \leq 0, \quad (4.2)$$

which implies $|x_0(t)| \leq |x_0(0)|$.

Furthermore, we have

$$-c_{02}k_0(2V_0)^{(1+\alpha_0)/2} \leq \dot{V}_0 \leq -c_{01}k_0(2V_0)^{(1+\alpha_0)/2}. \quad (4.3)$$

Thus, x_0 tends to 0 within a settling time denoted by T_0 . Moreover,

$$\frac{|x_0(0)|^{1-\alpha_0}}{c_{02}k_0(1-\alpha_0)} \leq T_0 \leq \frac{|x_0(0)|^{1-\alpha_0}}{c_{01}k_0(1-\alpha_0)}. \quad (4.4)$$

To secure finite-time convergence within T for any $x_0(0) \neq 0$, we need to keep $T_0 \leq |x_0(0)|^{1-\alpha_0}/c_{01}k_0(1-\alpha_0) \leq T$ by taking $k_0 > |x_0(0)|^{1-\alpha_0}/c_{01}T(1-\alpha_0)$. If we take $T_* = (c_{01}T_0)/(2c_{02})$, then we obtain $x_0(t) \in R$ does not change its sign when $t < T_*$, $x_0(0) \neq 0$ and moreover

$$|x_0(0)| \geq |x_0(t)| \geq \frac{|x_0(0)|}{2}, \quad \forall t \in [0, T_*]. \quad (4.5)$$

Therefore, u_0 is bounded and does not change sign during $[0, T_*]$. Furthermore, from this and Assumption 3.2, the following result can be obtained.

Lemma 4.1. *For $i = 1, \dots, n$, there are positive constants λ_i and μ_i such that*

$$\lambda_i \leq d_i(t)\alpha_i u_0 \leq \mu_i, \quad \lambda_n \leq d_n(t) \leq \mu_n, \quad (4.6)$$

where $\alpha_i = -\text{sgn}(x_0(0))$. Besides, for the simplicity of expression in later use, let $\alpha_n = 1$.

Since we have already proven that x_0 can be globally finite-time regulated to zero as $t \rightarrow T_0$. Next, we only need to stabilize the time-varying x -subsystem

$$\begin{aligned} dx_i &= d_i(t)x_{i+1}u_0dt + g_i^T(x_0, x_{[i]})dw, \quad i = 1, \dots, n-1, \\ dx_n &= d_n(t)u_1dt + g_n^T(x_0, x_{[n]})dw, \end{aligned} \quad (4.7)$$

within the given settling time T_* . The control law u_1 can be recursively constructed by applying the method of adding a power integrator.

Step 1. Let $\xi_1 = x_1^\sigma$, where $\sigma > 2$ is a odd number and choose $V_1 = x_1^{4\sigma-\tau}/(4\sigma-\tau)$ to be the candidate Lyapunov function for this step. Then, along the trajectories of system (4.7), we have

$$\mathcal{L}V_1 \leq d_1u_0x_1^{4\sigma-\tau-1}(x_2 - x_2^*) + d_1u_0x_1^{4\sigma-\tau-1}x_2^* + \frac{1}{2}(4\sigma-\tau-1)b^2x_1^{4\sigma}. \quad (4.8)$$

Obviously, the first virtual controller

$$\begin{aligned} x_2^* &= -\frac{1}{\lambda_1} \alpha_1 x_1^{1+\tau} \left[M + n - 1 + \frac{1}{2}(4\sigma - \tau - 1)b^2 \right] \\ &\triangleq -\alpha_1 \beta_1 \xi_1^{m_2/\sigma}, \end{aligned} \quad (4.9)$$

with design constant $M > 0$, results in

$$\mathcal{L}V_1 \leq -(M + n - 1)\xi_1^4 + d_1 u_0 x_1^{4\sigma-\tau-1} (x_2 - x_2^*). \quad (4.10)$$

Inductive Step ($2 \leq k \leq n$)

Suppose at step $k - 1$, there is a C^2 Lyapunov function V_{k-1} , which is positive definite and proper, satisfying

$$V_{k-1} \leq 2 \left(\xi_1^{(4\sigma-\tau)/\sigma} + \dots + \xi_{k-1}^{(4\sigma-\tau)/\sigma} \right), \quad (4.11)$$

and a set of virtual controllers x_1^*, \dots, x_{k-1}^* defined by

$$\begin{aligned} x_1^* &= 0, & \xi_1 &= x_1^{\sigma/m_1} - x_1^{*\sigma/m_1}, \\ x_2^* &= -\alpha_1 \beta_1 \xi_1^{m_2/\sigma}, & \xi_2 &= x_2^{\sigma/m_2} - x_2^{*\sigma/m_2}, \\ &\vdots & &\vdots \\ x_k^* &= -\alpha_{k-1} \beta_{k-1} \xi_{k-1}^{m_k/\sigma}, & \xi_k &= x_k^{\sigma/m_k} - x_k^{*\sigma/m_k}, \end{aligned} \quad (4.12)$$

with constants $\beta_1 > 0, \dots, \beta_{k-1} > 0$, such that

$$\mathcal{L}V_{k-1} \leq -(M + n - k + 1) \left(\xi_1^4 + \dots + \xi_{k-1}^4 \right) + d_{k-1} u_0 \xi_{k-1}^{(4\sigma-\tau-m_{k-1})/\sigma} (x_k - x_k^*). \quad (4.13)$$

We claim that (4.11) and (4.13) also hold at step k . To prove this claim, consider

$$V_k(x_{[k]}) = V_{k-1}(x_{[k-1]}) + W_k(x_{[k]}), \quad (4.14)$$

where

$$W_k(x_{[k]}) = \int_{x_k^*}^{x_k} \left(s^{\sigma/m_k} - x_k^{*\sigma/m_k} \right)^{(4\sigma-\tau-m_k)/\sigma} ds. \quad (4.15)$$

Noting that

$$\begin{aligned} x_k^{*\sigma/m_k} &= -\alpha_{k-1}^{\sigma/m_k} \beta_{k-1}^{\sigma/m_k} \xi_{k-1}, \\ \xi_k &= c_k x_k^{\sigma/m_k} + c_{k-1} x_{k-1}^{\sigma/m_{k-1}} + \dots + c_1 x_1^{\sigma/m_1}, \end{aligned} \quad (4.16)$$

where

$$c_i = \begin{cases} \alpha_{k-1}^{\sigma/m_k} \beta_{k-1}^{\sigma/m_k} \dots \alpha_i^{\sigma/m_{i+1}} \beta_i^{\sigma/m_{i+1}}, & i = 1, \dots, k-1, \\ 1, & i = k. \end{cases} \quad (4.17)$$

Similar to the corresponding proof in [25], it is easy to verify that the Lyapunov function $V_k(x_{[k]})$ thus defined has several useful properties collected in the following propositions.

Proposition 4.2. $W_k(x_{[k]})$ is C^2 . Moreover

$$\begin{aligned} \frac{\partial W_k}{\partial x_i} &= -c_i \frac{4\sigma - \tau - m_k}{\sigma} x_i^{(\sigma-m_i)/m_i} \times \int_{x_k^*}^{x_k} \left(s^{\sigma/m_k} - x_k^{*\sigma/m_k} \right)^{(3\sigma-\tau-m_k)/\sigma} ds, \\ \frac{\partial W_k}{\partial x_k} &= \xi_k^{(4\sigma-\tau-m_k)/\sigma}, \\ \frac{\partial^2 W_k}{\partial x_i \partial x_j} &= c_i c_j \frac{4\sigma - \tau - m_k}{\sigma} \cdot \frac{3\sigma - \tau - m_k}{\sigma} x_i^{(\sigma-m_i)/m_i} x_j^{(\sigma-m_j)/m_j} \\ &\quad \times \int_{x_k^*}^{x_k} \left(s^{\sigma/m_k} - x_k^{*\sigma/m_k} \right)^{(2\sigma-\tau-m_k)/\sigma} ds, \\ \frac{\partial^2 W_k}{\partial x_i^2} &= c_i^2 \frac{4\sigma - \tau - m_k}{\sigma} \cdot \frac{3\sigma - \tau - m_k}{\sigma} x_i^{2(\sigma-m_i)/m_i} \times \int_{x_k^*}^{x_k} \left(s^{1/m_k} - x_k^{*1/m_k} \right)^{(2\sigma-\tau-m_k)/\sigma} ds \\ &\quad - c_i^2 \frac{4\sigma - \tau - m_k}{\sigma} x_i^{(\sigma-2m_i)/m_i} \times \int_{x_k^*}^{x_k} \left(s^{\sigma/m_k} - x_k^{*\sigma/m_k} \right)^{(3\sigma-\tau-m_k)/\sigma} ds, \\ \frac{\partial^2 W_k}{\partial x_k^2} &= c_k \frac{4\sigma - \tau - m_k}{\sigma} \xi_k^{(3\sigma-\tau-m_k)/\sigma} x_k^{(\sigma-m_k)/m_k}, \\ \frac{\partial^2 W_k}{\partial x_k \partial x_i} &= \frac{\partial^2 W_k}{\partial x_i \partial x_k} = c_i \frac{4\sigma - \tau - m_k}{\sigma} \xi_k^{(3\sigma-\tau-m_k)/\sigma} x_i^{(\sigma-m_i)/m_i}, \end{aligned} \quad (4.18)$$

where $i, j = 1, \dots, k-1, i \neq j$.

Proposition 4.3. $V_k(x_{[k]})$ is C^2 , positive definite and proper, satisfying

$$V_k \leq 2 \left(\xi_1^{(4\sigma-\tau)/\sigma} + \dots + \xi_k^{(4\sigma-\tau)/\sigma} \right). \quad (4.19)$$

Using Proposition 4.2, it is deduced from (4.13) that

$$\begin{aligned} \mathcal{L}V_k &\leq -(M + n - k + 1) \left(\xi_1^4 + \dots + \xi_{k-1}^4 \right) + d_{k-1} u_0 \xi_{k-1}^{(4\sigma-\tau-m_{k-1})/\sigma} (x_k - x_k^*) \\ &\quad + d_k u_0 \xi_k^{(4\sigma-\tau-m_k)/\sigma} x_{k+1} + \sum_{i=1}^{k-1} \frac{\partial W_k}{\partial x_i} d_i u_0 x_{i+1} \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} \sum_{i,j=1, i \neq j}^{k-1} \left| \frac{\partial^2 W_k}{\partial x_i \partial x_j} \right| |g_i^T| |g_j^T| + \frac{1}{2} \sum_{i=1}^{k-1} \left| \frac{\partial^2 W_k}{\partial x_i^2} \right| |g_i^T|^2 \\
& + \frac{1}{2} \sum_{i=1}^{k-1} \left| \frac{\partial^2 W_k}{\partial x_k \partial x_i} \right| |g_k^T| |g_i^T| + \frac{1}{2} \left| \frac{\partial^2 W_k}{\partial x_k^2} \right| |g_k^T|^2.
\end{aligned} \tag{4.20}$$

To estimate the second term in (4.20), by Lemma 2.5, we have

$$|x_k - x_k^*| = \left| \left(x_k^{\sigma/m_k} \right)^{m_k/\sigma} - \left(x_k^{*\sigma/m_k} \right)^{m_k/\sigma} \right| \leq 2^{(\sigma-m_k)/\sigma} |\xi_k|^{m_k/\sigma}. \tag{4.21}$$

Noting that $m_k = m_{k-1} + \tau$, by applying (4.6) and Lemma 2.6, we have

$$d_{k-1} u_0 \xi_{k-1}^{(4\sigma-\tau-m_{k-1})/\sigma} (x_k - x_k^*) \leq \frac{1}{6} \xi_{k-1}^4 + l_{k1} \xi_k^4, \tag{4.22}$$

where l_{k1} is a positive constant.

Based on Proposition 4.2 and Lemma 2.6, the following propositions are given to estimate the other terms on the right hand side of inequality (4.20), whose proofs are included in the appendix.

Proposition 4.4. *There exists a positive constant l_{k2} such that*

$$\sum_{i=1}^{k-1} \frac{\partial W_k}{\partial x_i} d_i u_0 x_{i+1} \leq \frac{1}{6} (\xi_1^4 + \cdots + \xi_{k-1}^4) + l_{k2} \xi_k^4. \tag{4.23}$$

Proposition 4.5. *There exists a positive constant l_{k3} such that*

$$\frac{1}{2} \sum_{i,j=1, i \neq j}^{k-1} \left| \frac{\partial^2 W_k}{\partial x_i \partial x_j} \right| |g_i^T| |g_j^T| \leq \frac{1}{6} (\xi_1^4 + \cdots + \xi_{k-1}^4) + l_{k3} \xi_k^4. \tag{4.24}$$

Proposition 4.6. *There exists a positive constant l_{k4} such that*

$$\frac{1}{2} \sum_{i=1}^{k-1} \left| \frac{\partial^2 W_k}{\partial x_i^2} \right| |g_i^T|^2 \leq \frac{1}{6} (\xi_1^4 + \cdots + \xi_{k-1}^4) + l_{k4} \xi_k^4. \tag{4.25}$$

Proposition 4.7. *There exists a positive constant l_{k5} such that*

$$\frac{1}{2} \sum_{i=1}^{k-1} \left| \frac{\partial^2 W_k}{\partial x_k \partial x_i} \right| |g_k^T| |g_i^T| \leq \frac{1}{6} (\xi_1^4 + \cdots + \xi_{k-1}^4) + l_{k5} \xi_k^4. \tag{4.26}$$

Proposition 4.8. *There exists a positive constant l_{k6} such that*

$$\frac{1}{2} \left| \frac{\partial^2 W_k}{\partial x_k^2} \right| \left| \mathcal{G}_k^T \right|^2 \leq \frac{1}{6} \left(\xi_1^4 + \dots + \xi_{k-1}^4 \right) + l_{k6} \xi_k^4. \quad (4.27)$$

Substituting (4.22)–(4.27) into (4.20) yields

$$\mathcal{L}V_k \leq -(M + n - k) \left(\xi_1^4 + \dots + \xi_{k-1}^4 \right) + d_k u_0 \xi_k^{(4\sigma - \tau - m_k)/\sigma} x_{k+1} + \xi_k^4 (l_{k1} + \dots + l_{k6}). \quad (4.28)$$

Clearly, the C^0 virtual controller

$$\begin{aligned} x_{k+1}^* &= -\frac{1}{\lambda_k} \alpha_k \xi_k^{(m_k + \tau)/\sigma} (M + n - k + l_{k1} + \dots + l_{k6}) \\ &\triangleq -\alpha_k \beta_k \xi_k^{m_{k+1}}, \end{aligned} \quad (4.29)$$

with $\beta_k > 0$ being constant, results in

$$\mathcal{L}V_k \leq -(M + n - k) \left(\xi_1^4 + \dots + \xi_k^4 \right) + d_k u_0 \xi_k^{(4\sigma - \tau - m_k)/\sigma} (x_{k+1} - x_{k+1}^*). \quad (4.30)$$

This completes the proof of the inductive step.

The inductive argument shows that (4.30) holds for $k = n$. Hence, we conclude that at the last step the actual controller

$$u_1 = x_{n+1}^* = -\beta_n \xi_n^{m_{n+1}/\sigma}, \quad (4.31)$$

with $\beta_n > 0$ being constant and a C^2 positive definite and proper Lyapunov function $V_n(x_{[n]})$ of the form (4.14) and (4.15), such that

$$\mathcal{L}V_n \leq -M \left(\xi_1^4 + \dots + \xi_n^4 \right). \quad (4.32)$$

We have thus far completed the controller design procedure for $x_0(0) \neq 0$.

4.2. Control for $x_0(0) = 0$

In the last subsection, we gave the controller expressions (4.1) and (4.31) for u_0 and u_1 of system (3.1) in the case of $x_0(0) \neq 0$. Now, we consider finite-time control laws for the case of $x_0(0) = 0$. In the absence of the disturbances, most of the commonly used control strategies use constant control $u_0 = u_0^* \neq 0$ in time interval $[0, t_s)$. In this paper, we also use this method when $x_0(0) = 0$, with u_0 chosen as follows:

$$u_0 = u_0^*, \quad u_0^* > 0. \quad (4.33)$$

Since $x_0(0) = 0$, from the x_0 -subsystem we know that

$$\dot{x}(0) = u_0^*(0) = u_0^* \neq 0. \quad (4.34)$$

We have x_0 does not escape and $x(t_s) \neq 0$, for given any finite $t_s > 0$.

During the time period $[0, t_s)$, using u_0 defined in (4.33), new control law $u_1 = u_1^*(x_0, x)$ can be obtained by the control procedure described above to the original x -subsystem in (3.1). Then we can conclude that the x -state of (3.1) cannot blow up during the time period $[0, t_s)$. Since $x_0(t_s) \neq 0$ at $t = t_s$, we can switch the control inputs u_0 and u_1 to (4.1) and (4.31), respectively.

The following theorem summarizes the main result of this paper.

Theorem 4.9. *Under Assumptions 3.2 and 3.3, if the proposed control design procedure together with the above switching control strategy is applied to system (3.1), then, for any initial conditions in the state space $(x_0, x) \in \mathbb{R}^{n+1}$, the closed-loop system is globally finite-time regulated at origin in probability.*

Proof. According to the above analysis, it suffices to prove the statement in the case where $x_0(0) \neq 0$.

Since we have already proven that x_0 can be globally finite-time regulated to zero in Section 4.1, we just need to show that $x(t)$ is globally stochastically convergence to zero in a finite time. For the system (4.7)+(4.31), it is not hard to verify that all conditions in Lemma 2.1 are satisfied, which means that the closed-loop system admits a unique solution. In this case, choose the Lyapunov function $V = V_n$, from (4.32), its time derivative is given by

$$\mathcal{L}V \leq -M(\xi_1^4 + \cdots + \xi_n^4). \quad (4.35)$$

Let $\alpha = 4\sigma/(4\sigma - \tau)$, by Proposition 4.3 and Lemma 2.5, we have

$$V^\alpha \leq 2(\xi_1^4 + \cdots + \xi_n^4). \quad (4.36)$$

Then, putting (4.36) back to (4.35) gives

$$\mathcal{L}V \leq -MV^\alpha/2. \quad (4.37)$$

By Lemma 2.4, system (4.7) under control law (4.31) is finite-time stable in probability with its settling time T_x satisfying

$$T_x \leq \frac{2V^{(1-\alpha)}(0)}{M(1-\alpha)}. \quad (4.38)$$

Hence with the choice of M satisfying $M > 2V^{(1-\alpha)}(0)/[T_*(1-\alpha)]$, $T_x < T_*$ is guaranteed. Thus, the conclusion follows. \square

Remark 4.10. As seen from (4.31) and (4.1), the control law u_1 may exhibit extremely large value when $x_0(0) \neq 0$ is sufficiently small. This is unacceptable from a practical point of view. It is therefore recommended to apply (4.33) in order to enlarge the initial value of x_0 before we appeal to the finite-time converging controllers (4.1) and (4.31).

5. Application to Mobile Robot

In this section, we illustrate systematic controller design method proposed above by means of the example of mobile robot.

Consider the tricycle-type mobile robot with parametric uncertainty [34], which is described by

$$\begin{aligned}\dot{x}_c &= p_1^* v \cos \theta, \\ \dot{y}_c &= p_1^* v \sin \theta, \\ \dot{\theta} &= p_2^* \omega,\end{aligned}\tag{5.1}$$

where p_1^* and p_2^* are unknown parameters taking values in a known interval $[p_{\min}, p_{\max}]$ with $0 < p_{\min} < p_{\max} < \infty$, v and ω are two control inputs to denote the linear velocity and angular velocity, respectively.

When the angular velocity ω is subject to some stochastic disturbances, that is,

$$\omega(x_c, y_c, \theta) = \omega_1(x_c, y_c, \theta) + \omega_2(x_c, y_c, \theta) \dot{B}(t),\tag{5.2}$$

where $B(t)$ is the so-called white noise. Then system (5.1) is transformed into

$$\begin{aligned}dx_c &= p_1^* v \cos \theta dt, \\ dy_c &= p_1^* v \sin \theta dt, \\ d\theta &= p_2^* \omega_1 dt + p_2^* \omega_2 dB.\end{aligned}\tag{5.3}$$

For system (5.3), by taking the following state and input transformation:

$$x_0 = x_c, \quad x_1 = y_c, \quad x_2 = \tan \theta, \quad u_0 = v \cos \theta, \quad u_1 = \omega_1 \sec^2 \theta,\tag{5.4}$$

we obtain

$$\begin{aligned}dx_0 &= p_1^* u_0 dt, \\ dx_1 &= p_1^* x_2 u_0 dt, \\ dx_2 &= p_2^* u_1 dt + p_2^* (1 + x_2^2) \omega_2 dB.\end{aligned}\tag{5.5}$$

Clearly, system (5.5) is a special case of system (3.1). As discussed in Remark 3.4, there always exist some nonlinearities satisfy Assumption 3.3. For simplicity, it is assumed the $\omega_2 =$

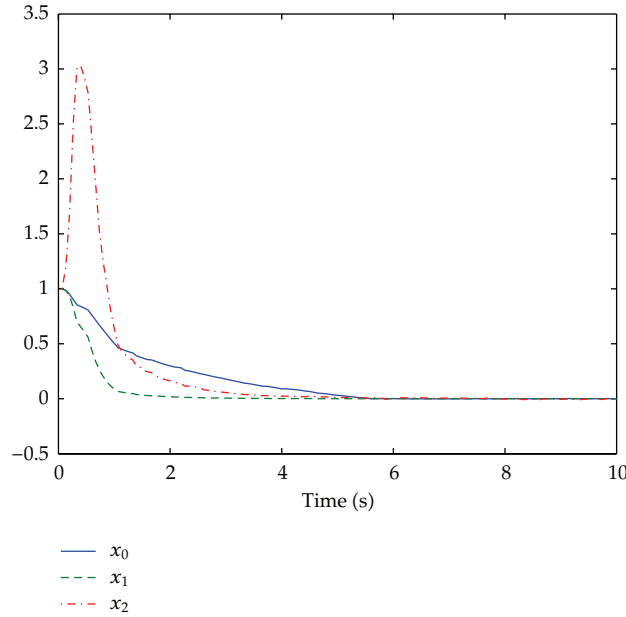


Figure 1: Time history of system states.

$(x_2 \sin x_1)/(1 + x_2^2)^2$. And as in [8], $p_{\min} = 1$, $p_{\max} = 2$, which are known for us in constructing control laws to make (5.5) finite-time stable, and parameters $p_1^* = p_1^* = 1.5$ are unknown. When $(x_0(0), x_1(0), x_2(0)) = (1, 1, 1)$, by choosing $\tau = -2/11$ and $\sigma = 3$, according to the design procedure proposed in Section 4, the following controllers can be obtained for a given settling time $T = 6$:

$$\begin{aligned} u_0 &= -\frac{1}{2}x_0^{1/3}, \\ u_1 &= -\left[x_2^{11/3} - (2M + 2)^{11/3}x_1^3\right]^{7/33}(M + l_{21} + l_{22} + l_{24}), \end{aligned} \quad (5.6)$$

where l_{21} , l_{22} and l_{24} are known positive constants. Choosing design parameter as $M = 1$, the simulation results in Figures 1 and 2 show that the effectiveness of the controller.

6. Conclusion

In this paper, the finite-time state feedback stabilization problem has been investigated for a class of nonholonomic systems with stochastic disturbances. With the help of adding a power integrator technique, a systematic control design procedure is developed in the stochastic setting. To get around the stabilization burden associated with nonholonomic systems, a switching control strategy is proposed. It is shown that the designed control laws can guarantee that the closed-loop system states are globally finite-time regulated to zero in probability. In addition, the proposed approach can be applied to mobile robot with stochastic disturbances.

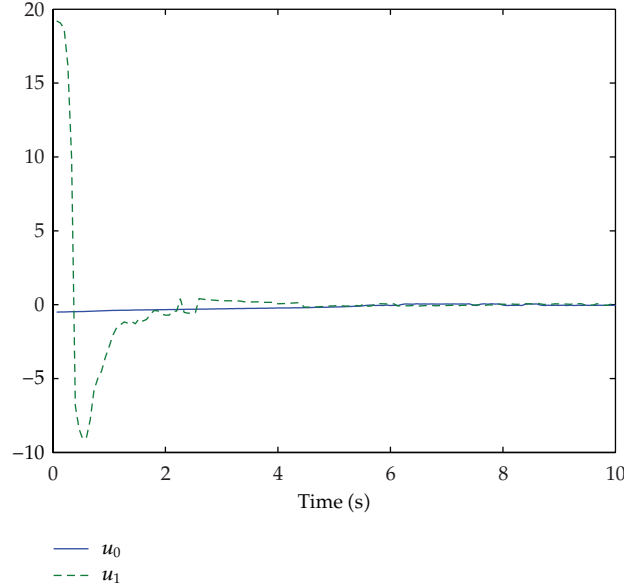


Figure 2: Time history of control inputs.

There are some related problems to investigate, for example, how to design a finite-time state-feedback stabilizing controller for stochastic nonholonomic systems when the x_0 -subsystem contains stochastic disturbances. Furthermore, if only partial state vector being measurable, how to design a finite-time output feedback stabilizing controller for stochastic nonholonomic systems.

Appendix

Proof of Proposition 4.4. Recall that $\xi_i = x_i^{\sigma/m_i} - x_i^{*\sigma/m_i}$ and $x_i^* = -\alpha_{i-1}\beta_{i-1}\xi_{i-1}^{m_i/\sigma}$. By Lemma 2.5, for $i = 2, \dots, k$,

$$|x_i| \leq \left| \xi_i + x_i^{*\sigma/m_i} \right|^{m_i/\sigma} \leq h(|\xi_{i-1}| + |\xi_i|)^{m_i/\sigma}, \quad (\text{A1})$$

where $h = \max\{1, \beta_1^{\sigma/m_1}, \dots, \beta_n^{\sigma/m_n}\}$.

With (4.17), (4.18), and (A1), we get

$$\begin{aligned} \sum_{i=1}^{k-1} \frac{\partial W_k}{\partial x_i} d_i u_0 x_{i+1} &= \sum_{i=1}^{k-1} \left[-c_i \frac{4\sigma - \tau - m_k}{\sigma} x_i^{(\sigma-m_i)/m_i} \times \int_{x_k^*}^{x_k} \left(s^{\sigma/m_k} - x_k^{*\sigma/m_k} \right)^{(3\sigma-\tau-m_k)/\sigma} ds \right] \\ &\quad \times d_i u_0 x_{i+1} \leq a_k \sum_{i=1}^{k-1} |\xi_k|^{(3\sigma-\tau)/\sigma} |x_i|^{(\sigma-m_i)/m_i} |x_{i+1}| \\ &\leq \hat{a}_k \sum_{i=1}^{k-1} |\xi_k|^{(3\sigma-\tau)/\sigma} (|\xi_{i-1}| + |\xi_i|)^{(\sigma-m_i)/\sigma} (|\xi_i| + |\xi_{i+1}|)^{m_{i+1}/\sigma}, \end{aligned} \quad (\text{A2})$$

where a_k and \hat{a}_k are positive constants.

Applying Lemma 2.6 ($3\sigma - \tau + \sigma - m_i + m_{i+1} = 4\sigma$) to (A2), we can find a positive constant l_{k2} such that

$$\sum_{i=1}^{k-1} \frac{\partial W_k}{\partial x_i} d_i(t) u_0 x_{i+1} \leq \frac{1}{3} (\xi_1^4 + \cdots + \xi_{k-1}^4) + l_{k2} \xi_k^d. \quad (\text{A3})$$

□

Proof of Proposition 4.5. According to (4.17), (4.18), (A1), and Remark 3.4, we have

$$\begin{aligned} & \frac{1}{2} \sum_{i,j=1, i \neq j}^{k-1} \left| \frac{\partial^2 W_k}{\partial x_i \partial x_j} \right| |g_i^T| |g_j^T| \\ &= \frac{1}{2} \sum_{i,j=1, i \neq j}^{k-1} \left| c_i c_j \frac{4\sigma - \tau - m_k}{\sigma} \cdot \frac{3\sigma - \tau - m_k}{\sigma} x_i^{(\sigma-m_i)/m_i} x_j^{(\sigma-m_j)/m_j} \right. \\ & \quad \times \left. \int_{x_k^*}^{x_k} \left(s^{\sigma/m_k} - x_k^{*\sigma/m_k} \right)^{(2\sigma-\tau-m_k)/\sigma} \right| |g_i^T| |g_j^T| \\ &\leq b_k \sum_{i,j=1, i \neq j}^{k-1} |\xi_k|^{(2\sigma-\tau)/\sigma} |x_i|^{(\sigma-m_i)/m_i} |x_j|^{(\sigma-m_j)/m_j} |g_i^T| |g_j^T| \\ &\leq \widehat{b}_k \sum_{i,j=1, i \neq j}^{k-1} \sum_{l=1}^i \sum_{r=1}^j |\xi_k|^{(2\sigma-\tau)/\sigma} x_i^{(\sigma-m_i)/m_i} x_j^{(\sigma-m_j)/m_j} \times |x_l|^{(2m_i+\tau)/2m_l} |x_r|^{(2m_j+\tau)/2m_r} \\ &\leq \widetilde{b}_k \sum_{i,j=1, i \neq j}^{k-1} \sum_{l=1}^i \sum_{r=1}^j |\xi_k|^{(2\sigma-\tau)/\sigma} (|\xi_{i-1}| + |\xi_i|)^{(\sigma-m_i)/\sigma} \times (|\xi_{j-1}| + |\xi_j|)^{(\sigma-m_j)/\sigma} \\ & \quad \times (|\xi_{l-1}| + |\xi_l|)^{(2m_i+\tau)/2\sigma} \times (|\xi_{r-1}| + |\xi_r|)^{(2m_j+\tau)/2\sigma}, \end{aligned} \quad (\text{A4})$$

where b_k , \widehat{b}_k and \widetilde{b}_k are positive constants.

Applying Lemma 2.6 to the above inequality, we have

$$\frac{1}{2} \sum_{i,j=1, i \neq j}^{k-1} \left| \frac{\partial^2 W_k}{\partial x_i \partial x_j} \right| |g_i^T| |g_j^T| \leq \frac{1}{6} (\xi_1^4 + \cdots + \xi_{k-1}^4) + l_{k3} \xi_k^4, \quad (\text{A5})$$

where l_{k3} is a positive constant.

□

Proof of Proposition 4.6. From (4.17), (4.18), (A1), and Remark 3.4, with Lemma 2.6, it is deduced that

$$\begin{aligned}
& \frac{1}{2} \sum_{i=1}^{k-1} \left| \frac{\partial^2 W_k}{\partial x_i^2} \right| \left| g_i^T \right|^2 \\
&= \frac{1}{2} \sum_{i=1}^{k-1} \left| c_i^2 \frac{4\sigma - \tau - m_k}{\sigma} \cdot \frac{3\sigma - \tau - m_k}{\sigma} x_i^{2(\sigma - m_i)/m_i} \times \int_{x_k^*}^{x_k} \left(s^{1/m_k} - x_k^{*1/m_k} \right)^{(2\sigma - \tau - m_k)/\sigma} ds \right. \\
&\quad \left. - c_i^2 \frac{4\sigma - \tau - m_k}{\sigma} x_i^{(\sigma - 2m_i)/m_i} \times \int_{x_k^*}^{x_k} \left(s^{\sigma/m_k} - x_k^{*\sigma/m_k} \right)^{(3\sigma - \tau - m_k)/\sigma} ds \right| \left| g_i^T \right|^2 \\
&\leq d_k \sum_{i=1}^{k-1} \left(|\xi_k|^{(2\sigma - \tau)/\sigma} |x_i|^{2(\sigma - m_i)/m_i} + |\xi_k|^{(3\sigma - \tau)/\sigma} x_i^{(\sigma - 2m_i)/m_i} \right) \left| g_i^T \right|^2 \\
&\leq \hat{d}_k \sum_{i=1}^{k-1} \sum_{j=1}^i \left(|\xi_k|^{(2\sigma - \tau)/\sigma} |x_i|^{2(\sigma - m_i)/m_i} + |\xi_k|^{(3\sigma - \tau)/\sigma} x_i^{(\sigma - 2m_i)/m_i} \right) x_j^{(2m_i + \tau)/m_j} \\
&\leq \tilde{d}_k \sum_{i=1}^{k-1} \sum_{j=1}^i \left(|\xi_k|^{(2\sigma - \tau)/\sigma} (|\xi_{i-1}| + |\xi_i|)^{2(\sigma - m_i)/\sigma} + |\xi_k|^{(3\sigma - \tau)/\sigma} (|\xi_{i-1}| + |\xi_i|)^{(\sigma - 2m_i)/\sigma} \right) \\
&\quad \times (|\xi_{j-1}| + |\xi_j|)^{(2m_i + \tau)/\sigma} \leq \frac{1}{6} \left(\xi_1^4 + \cdots + \xi_{k-1}^4 \right) + l_{k4} \xi_k^4,
\end{aligned} \tag{A6}$$

where $d_k, \hat{d}_k, \tilde{d}_k$ and l_{k4} are positive constants. \square

Proof of Proposition 4.7. From (4.17), (4.18), (A1), and Remark 3.4, with Lemma 2.6, it follows that

$$\begin{aligned}
& \frac{1}{2} \sum_{i=1}^{k-1} \left| \frac{\partial^2 W_k}{\partial x_k \partial x_i} \right| \left| g_k^T \right| \left| g_i^T \right| = \frac{1}{2} \sum_{i=1}^{k-1} \left| c_i \frac{4\sigma - \tau - m_k}{\sigma} \xi_k^{(3\sigma - \tau - m_k)/\sigma} x_i^{(\sigma - m_i)/m_i} \right| \left| g_k^T \right| \left| g_i^T \right| \\
&\leq e_k \sum_{i=1}^{k-1} |\xi_k|^{(3\sigma - \tau - m_k)/\sigma} |x_i|^{(\sigma - m_i)/m_i} \left| g_k^T \right| \left| g_i^T \right| \\
&\leq \hat{e}_k \sum_{i=1}^{k-1} \sum_{l=1}^k \sum_{r=1}^i |\xi_k|^{(3\sigma - \tau - m_k)/\sigma} |x_i|^{(\sigma - m_i)/m_i} |x_l|^{(2m_k + \tau)/2m_l} |x_r|^{(2m_i + \tau)/2m_r} \\
&\leq \tilde{e}_k \sum_{i=1}^{k-1} \sum_{l=1}^k \sum_{r=1}^i |\xi_k|^{(3\sigma - \tau - m_k)/\sigma} (|\xi_{i-1}| + |\xi_i|)^{(\sigma - m_i)/\sigma} \\
&\quad \times (|\xi_{l-1}| + |\xi_l|)^{(2m_k + \tau)/2\sigma} \times (|\xi_{r-1}| + |\xi_r|)^{(2m_i + \tau)/2\sigma} \\
&\leq \frac{1}{6} \left(\xi_1^4 + \cdots + \xi_{k-1}^4 \right) + l_{k5} \xi_k^4,
\end{aligned} \tag{A7}$$

where $e_k, \hat{e}_k, \tilde{e}_k$ and l_{k5} are positive constants. \square

Proof of Proposition 4.8. By (4.17), (4.18), (A1), and Remark 3.4, with Lemma 2.6, we can obtain

$$\begin{aligned}
\frac{1}{2} \left| \frac{\partial^2 W_k}{\partial x_k^2} \right| \left| g_k^T \right|^2 &= \frac{1}{2} \left| c_k \frac{4\sigma - \tau - m_k}{\sigma} \xi_k^{(3\sigma - \tau - m_k)/\sigma} x_k^{(\sigma - m_k)/m_k} \right| \left| g_k^T \right|^2 \\
&\leq f_k \sum_{i=1}^k |\xi_k|^{(3\sigma - \tau - m_k)/\sigma} |x_k|^{(\sigma - m_k)/m_k} |x_i|^{(2m_k + \tau)/2m_i} \\
&\leq \hat{f}_k \sum_{i=1}^k |\xi_k|^{(3\sigma - \tau - m_k)/\sigma} (|\xi_{k-1}| + |\xi_k|)^{(\sigma - m_k)/\sigma} \times (|\xi_{i-1}| + |\xi_i|)^{(2m_k + \tau)/\sigma} \\
&\leq \frac{1}{6} \left(\xi_1^4 + \cdots + \xi_{k-1}^4 \right) + l_{k6} \xi_k^4,
\end{aligned} \tag{A8}$$

where f_k , \hat{f}_k , and l_{k6} are positive constants. \square

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