Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2012, Article ID 815870, 19 pages
doi:10.1155/2012/815870

Research Article

Some Fixed-Point Results for a G-Weak Contraction in G-Metric Spaces

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Received 7 August 2012; Accepted 4 September 2012

Academic Editor: Yonghong Yao

We introduce the concepts of a G-weak contraction mapping of types A and B and we establish some fixed point theorems for a G-weak contraction mapping of types A and B in complete G-metric spaces. Our results generalize several well-known comparable results in the literature.

1. Introduction

As it is well known, one of the most useful theorems in nonlinear analysis is the Banach contraction principle [1]. Many authors generalized this famous result in different ways. In their outstanding article [2], Mustafa and Sims introduced a new notion of generalized metric space, called G-metric space and gave a modification to the contraction principle of Banach. After then, several authors studied various fixed and common fixed point problems for adequate classes of contractive mappings in generalized metric spaces (for an up to date reference list, see [3–28]).

Despite these important advances in nonlinear analysis, our problem, imposed by theoretical and practical reasons as well, had not been studied so far. In this paper, we introduce the concepts of a G-weak contraction mapping of types A and B and we establish some fixed point theorems for a G-weak contraction mapping of types A and B in complete G-metric spaces. Our study is encouraged by its possible application, especially in discrete models for numerical analysis, where iterative processes are extensively used due to their versatility for computer simulation. These models play an essential role in applied mathematical studies of certain nonlinear processes in relation with economics, biology, numerical physics, and tribology. Also, we are motivated by several scientists, who pay attention to such kind of problems. On one hand, Berinde [29–31], Berinde and Păcurar [32], and Păcurar [33, 34] studied many existing results for almost contraction mappings in metric spaces. On other hand, Samet and Vetro [35] and Shatanawi [36] studied some existing results for almost contraction mappings in G metric spaces.

This paper aims to establish new results for almost contraction mappings on nonlinear analysis on G-metric spaces. It is organized as follows. Next, in Section 2 our framework is introduced, in Section 3 our main results are given, while in Section 4 we introduce a nontrivial example and application to support the use ability of our results. Finally, we conclude the paper.

2. Previous Definitions and Results

To make our presentation self-contained, in this section we give basic definitions and previous results, which are used throughout the paper. This background is organized as a whole, given credit to [2] by Mustafa and Sims.

Definition 2.1. Let X be a nonempty set and let $G : X \times X \times X \to \mathbb{R}^{+}$ be a function satisfying the following properties:

(G1) $G(x, y, z) = 0$ if $x = y = z$,

(G2) $0 < G(x, x, y)$, for all $x, y \in X$ with $x \neq y$,

(G3) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $z \neq y$,

(G4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \cdots$, symmetry in all three variables,

(G5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$.

Then the function G is called a generalized metric, or, more specifically, a G-metric on X, and the pair (X, G) is called a G-metric space.

Definition 2.2. Let (X, G) be a G-metric space, and let $\{x_n\}$ be a sequence of points of X. A point $x \in X$ is said to be the limit of the sequence $\{x_n\}$, if $\lim_{n,m\to+\infty} G(x, x_n, x_m) = 0$, and we say that the sequence $\{x_n\}$ is G-convergent to x or $\{x_n\}$ G-converges to x.

Thus, $x_n \to x$ in a G-metric space (X, G) if for any $\varepsilon > 0$, there exists $k \in \mathbb{N}$ such that $G(x, x_n, x_m) < \varepsilon$ for all $m, n \geq k$.

Proposition 2.3. *Let (X, G) be a G-metric space. Then the following statements are equivalent:*

(1) *$\{x_n\}$ is G-convergent to x;*

(2) *$G(x_n, x_n, x) \to 0$ as $n \to +\infty$;*

(3) *$G(x_n, x, x) \to 0$ as $n \to +\infty$;*

(4) *$G(x_n, x_m, x) \to 0$ as $n, m \to +\infty$.*

Definition 2.4. Let (X, G) be a G-metric space, a sequence $\{x_n\}$ is called G-Cauchy if for every $\varepsilon > 0$, there is $k \in \mathbb{N}$ such that $G(x_n, x_m, x_l) < \varepsilon$, for all $n, m, l \geq k$, that is, $G(x_n, x_m, x_l) \to 0$ as $n, m, l \to +\infty$.

Proposition 2.5. *Let (X, G) be a G-metric space. Then the following statements are equivalent.*

(1) *The sequence $\{x_n\}$ is G-Cauchy.*

(2) *For every $\varepsilon > 0$, there is $k \in \mathbb{N}$ such that $G(x_n, x_m, x_m) < \varepsilon$, for all $n, m \geq k$.*

Definition 2.6. A G-metric space (X, G) is called G-complete if every G-Cauchy sequence in (X, G) is G-convergent in (X, G).

Proposition 2.7. *Let (X, G) be a G-metric space. Then, for any $x, y, z, a \in X$ it follows that:*

(i) *if $G(x, y, z) = 0$, then $x = y = z$;*

(ii) $G(x, y, z) \leq G(x, x, y) + G(x, x, z)$;

(iii) $G(x, y, y) \leq 2\, G(y, x, x)$;

(iv) $G(x, y, z) \leq G(x, a, z) + G(a, y, z)$;

(v) $G(x, y, z) \leq (2/3)[G(x, y, a) + G(x, a, z) + G(a, y, z)]$;

(vi) $G(x, y, z) \leq G(x, a, a) + G(y, a, a) + G(z, a, a)$.

3. Main Results

We start with the following definition.

Definition 3.1 (see [37]). The function $\phi : [0, +\infty) \to [0, +\infty)$ is called an altering distance function, if the following properties are satisfied.

(1) ϕ is continuous and nondecreasing.

(2) $\phi(t) = 0$ if and only if $t = 0$.

Let (X, G) be a G-metric space and $F : X \to X$ be a mapping. We set

$$\begin{aligned} M(x, y, y) &= \max\Bigg\{G(x, y, y), G(x, Fx, Fx), G(y, Fy, Fy), \\ &\qquad \frac{G(x, Fy, Fy) + G(y, Fy, Fy) + G(y, Fx, Fx)}{3}\Bigg\}, \\ M_1(x, y, y) &= \max\{G(x, y, y), G(y, Fy, Fy)\}, \\ N(x, y, y) &= \min\{G(x, Fx, Fx), G(y, Fy, Fy), G(y, Fx, Fx)\}. \end{aligned} \tag{3.1}$$

With this setting, we introduce the following definitions.

Definition 3.2. Let (X, G) be a G-metric space. A mapping $F : X \to X$ is called a G-weak contraction of type A if and only if there exist two constants $a \in (0, 1)$ and $L \geq 0$ such that

$$G(Fx, Fy, Fy) \leq aM(x, y, y) + LN(x, y, y), \tag{3.2}$$

for all $x, y \in X$.

Definition 3.3. Let (X, G) be a G-metric space and let ψ, ϕ, γ be altering distance functions. A mapping $F : X \to X$ is called a (ψ, ϕ, γ, G)-weak contraction of type A if and only if there exist a constant $L \geq 0$ such that

$$\psi(G(Fx, Fy, Fy)) \leq \psi(M(x, y, y)) - \phi(M_1(x, y, y)) + L\gamma(N(x, y, y)), \tag{3.3}$$

for all $x, y \in X$.

Now, we can introduce and prove our main result.

Theorem 3.4. *Let (X, G) be a complete G-metric space and let $F : X \to X$ be a (ψ, ϕ, G)-weak contraction of type A. Then F has a unique fixed point.*

Proof. Consider x_0 in X and define a sequence $\{x_n\}$ in X such that

$$x_{n+1} = Fx_n, \quad \text{for any } n \in \mathbb{N}. \tag{3.4}$$

If for some $n \in \mathbb{N}$, $x_{n+1} = x_n$, then $x_n = Fx_n$, that is, F has a fixed point. Thus, we may assume that $x_{n+1} \neq x_n$ for all $n \in \mathbb{N}$. Now, we finish our proof by the following steps.

Step One. We will show that

$$\lim_{n \to 0} G(x_n, x_{n+1}, x_{n+1}) = 0. \tag{3.5}$$

Given $n \in \mathbb{N}$. Since F is (ψ, ϕ, γ, G)-weak contraction, we have

$$\begin{aligned}
\psi(G(x_n, x_{n+1}, x_{n+1})) &= \psi(G(Fx_{n-1}, Fx_n, Fx_n)) \\
&\leq \psi(M(x_{n-1}, x_n, x_n)) - \phi(M_1(x_{n-1}, x_n, x_n)) \\
&\quad + L\gamma(N(x_{n-1}, x_n, x_n)),
\end{aligned} \tag{3.6}$$

where

$$\begin{aligned}
M(x_{n-1}, x_n, x_n) &= \max\Big\{G(x_{n-1}, x_n, x_n), G(x_{n-1}, Fx_{n-1}, Fx_{n-1}), G(x_n, Fx_n, Fx_n), \\
&\qquad \frac{G(x_{n-1}, Fx_n, Fx_n) + G(x_n, Fx_n, Fx_n) + G(x_n, Fx_{n-1}, Fx_{n-1})}{3}\Big\} \\
&= \max\Big\{G(x_{n-1}, x_n, x_n), G(x_n, x_{n+1}, x_{n+1}), \\
&\qquad \frac{G(x_{n-1}, x_{n+1}, x_{n+1}) + G(x_n, x_{n+1}, x_{n+1}) + G(x_n, x_n, x_n)}{3}\Big\} \\
&= \max\Big\{G(x_{n-1}, x_n, x_n), G(x_n, x_{n+1}, x_{n+1}), \\
&\qquad \frac{G(x_{n-1}, x_{n+1}, x_{n+1}) + G(x_n, x_{n+1}, x_{n+1})}{3}\Big\},
\end{aligned}$$

$$\begin{aligned}
M_1(x_{n-1}, x_n, x_n) &= \max\{G(x_{n-1}, Fx_{n-1}, Fx_{n-1}), G(x_n, Fx_n, Fx_n)\} \\
&= \max\{G(x_{n-1}, x_n, x_n), G(x_n, x_{n+1}, x_{n+1})\}, \\
N(x_{n-1}, x_n, x_n) &= \min\{G(x_{n-1}, Fx_{n-1}, Fx_{n-1}), G(x_n, Fx_n, Fx_n), G(x_n, Fx_{n-1}, Fx_{n-1})\} \\
&= \min\{G(x_{n-1}, x_n, x_n), G(x_n, x_{n+1}, x_{n+1}), G(x_n, x_n, x_n)\} = 0.
\end{aligned} \tag{3.7}$$

From (3.6)-(3.7), we obtain

$$\begin{aligned}
\psi(G(x_n, x_{n+1}, x_{n+1})) \leq \psi\Bigg(&\max\Bigg\{G(x_{n-1}, x_n, x_n), G(x_n, x_{n+1}, x_{n+1}), \\
&\frac{G(x_{n-1}, x_{n+1}, x_{n+1}) + G(x_n, x_{n+1}, x_{n+1})}{3}\Bigg\}\Bigg) \\
&- \phi(\max\{G(x_{n-1}, x_n, x_n), G(x_n, x_{n+1}, x_{n+1})\}).
\end{aligned} \tag{3.8}$$

By G_5, we have

$$G(x_{n-1}, x_{n+1}, x_{n+1}) \leq G(x_{n-1}, x_n, x_n) + G(x_n, x_{n+1}, x_{n+1}). \tag{3.9}$$

Thus

$$\frac{1}{3}(G(x_{n-1}, x_{n+1}, x_{n+1}) + G(x_n, x_{n+1}, x_{n+1})) \leq \max\{G(x_{n-1}, x_n, x_n), G(x_n, x_{n+1}, x_{n+1})\}. \tag{3.10}$$

If for some n in $\mathbb{N}$,

$$\max\{G(x_{n-1}, x_n, x_n), G(x_n, x_{n+1}, x_{n+1})\} = G(x_n, x_{n+1}, x_{n+1}), \tag{3.11}$$

then

$$\psi(G(x_n, x_{n+1}, x_{n+1})) \leq \psi(G(x_n, x_{n+1}, x_{n+1})) - \phi(G(x_n, x_{n+1}, x_{n+1})). \tag{3.12}$$

Thus $\phi(G(x_n, x_{n+1}, x_{n+1})) = 0$ and hence $G(x_n, x_{n+1}, x_{n+1}) = 0$. Therefore, $x_n = x_{n+1}$ which is a contradiction. So,

$$\max\{G(x_{n-1}, x_n, x_n), G(x_n, x_{n+1}, x_{n+1})\} = G(x_{n-1}, x_n, x_n). \tag{3.13}$$

Thus, we get

$$G(x_n, x_{n+1}, x_{n+1}) \leq G(x_{n-1}, x_n, x_n) \quad \forall n \in \mathbb{N}, \tag{3.14}$$

$$\psi(G(x_n, x_{n+1}, x_{n+1})) \leq \psi(G(x_{n-1}, x_n, x_n)) - \phi(G(x_{n-1}, x_n, x_n)) \quad \forall n \in \mathbb{N}. \tag{3.15}$$

By (3.14), $\{G(x_n, x_{n+1}, x_{n+1}) : n \in \mathbb{N}\}$ is a nonincreasing sequence. Hence, there is $r \geq 0$ such that

$$\lim_{n \to +\infty} G(x_n, x_{n+1}, x_{n+1}) = r. \tag{3.16}$$

Taking limit as $n \to +\infty$ in (3.15), we obtain

$$\psi(r) \leq \psi(r) - \phi(r). \tag{3.17}$$

Therefore $\phi(r) = 0$ and hence $r = 0$. Thus

$$\lim_{n \to +\infty} G(x_n, x_{n+1}, x_{n+1}) = 0. \tag{3.18}$$

Step Two. We show that $\{x_n\}$ is a G-Cauchy sequence in X. Assume on contrary, then there exists $\epsilon > 0$ for which we can find two subsequences $(x_{m(i)})$ and $(x_{n(i)})$ of (x_n) such that $n(i)$ is the smallest index for which

$$n(i) > m(i) > i, \qquad G\left(x_{m(i)}, x_{n(i)}, x_{n(i)}\right) \geq \epsilon. \tag{3.19}$$

This means that

$$G\left(x_{m(i)}, x_{n(i)-1}, x_{n(i)-1}\right) < \epsilon. \tag{3.20}$$

From (3.19), (3.20), and (G_5), we get

$$\begin{aligned} \epsilon \leq G\left(x_{m(i)}, x_{n(i)}, x_{n(i)}\right) &\leq G\left(x_{m(i)}, x_{n(i)-1}, x_{n(i)-1}\right) + G\left(x_{n(i)-1}, x_{n(i)}, x_{n(i)}\right) \\ &< \epsilon + G\left(x_{n(i)-1}, x_{n(i)}, x_{n(i)}\right). \end{aligned} \tag{3.21}$$

Taking limit as $i \to +\infty$ and using (3.18), we have

$$\lim_{i \to +\infty} G\left(x_{m(i)}, x_{n(i)}, x_{n(i)}\right) = \epsilon. \tag{3.22}$$

Using (G_5) again, we obtain

$$\begin{aligned} &G\left(x_{m(i)}, x_{n(i)}, x_{n(i)}\right) \\ &\quad\leq G\left(x_{m(i)}, x_{m(i)+1}, x_{m(i)+1}\right) + G\left(x_{m(i)+1}, x_{n(i)+1}, x_{n(i)+1}\right) + G\left(x_{n(i)+1}, x_{n(i)}, x_{n(i)}\right) \\ &\quad\leq G\left(x_{m(i)}, x_{m(i)+1}, x_{m(i)+1}\right) + G\left(x_{m(i)+1}, x_{m(i)}, x_{m(i)}\right) \\ &\qquad + G\left(x_{m(i)}, x_{n(i)}, x_{n(i)}\right) + G\left(x_{n(i)}, x_{n(i)+1}, x_{n(i)+1}\right) + G\left(x_{n(i)+1}, x_{n(i)}, x_{n(i)}\right). \end{aligned} \tag{3.23}$$

Taking $i \to +\infty$ in the above inequality and using (3.18) and (3.22), we arrive at

$$\lim_{i \to +\infty} G\left(x_{m(i)+1}, x_{n(i)+1}, x_{n(i)+1}\right) = \epsilon. \tag{3.24}$$

Again applying (G_5), we have

$$\begin{aligned} G\left(x_{m(i)}, x_{n(i)+1}, x_{n(i)+1}\right) &\leq G\left(x_{m(i)}, x_{n(i)}, x_{n(i)}\right) + G\left(x_{n(i)}, x_{n(i)+1}, x_{n(i)+1}\right) \\ &\leq G\left(x_{m(i)}, x_{n(i)+1}, x_{n(i)+1}\right) + G\left(x_{n(i)+1}, x_{n(i)}, x_{n(i)}\right) \\ &\quad + G\left(x_{n(i)}, x_{n(i)+1}, x_{n(i)+1}\right). \end{aligned} \tag{3.25}$$

By using (3.18), (3.22) and on taking limit as $i \to +\infty$ in the above inequality, we get

$$\lim_{i \to +\infty} G\left(x_{m(i)}, x_{n(i)+1}, x_{n(i)+1}\right) = \epsilon. \tag{3.26}$$

Also,

$$\begin{aligned} G\left(x_{m(i)+1}, x_{n(i)}, x_{n(i)}\right) &\leq G\left(x_{m(i)+1}, x_{n(i)+1}, x_{n(i)+1}\right) + G\left(x_{n(i)+1}, x_{n(i)}, x_{n(i)}\right) \\ &\leq G\left(x_{m(i)+1}, x_{n(i)}, x_{n(i)}\right) + G\left(x_{n(i)}, x_{n(i)+1}, x_{n(i)+1}\right) \\ &\quad + G\left(x_{n(i)+1}, x_{n(i)}, x_{n(i)}\right). \end{aligned} \tag{3.27}$$

By taking the limit in the above inequalities and using (3.18), (3.24), we obtain

$$\lim_{i \to +\infty} G\left(x_{m(i)+1}, x_{n(i)}, x_{n(i)}\right) = \epsilon. \tag{3.28}$$

Now, we have

$$\begin{aligned} &\psi\left(G\left(x_{m(i)+1}, x_{n(i)+1}, x_{n(i)+1}\right)\right) \\ &\quad = \psi\left(G\left(Fx_{m(i)}, Fx_{n(i)}, Fx_{n(i)}\right)\right) \\ &\quad \leq \psi\Bigg(\max\Bigg\{G\left(x_{m(i)}, x_{n(i)}, x_{n(i)}\right), G\left(x_{m(i)}, Fx_{m(i)}, Fx_{m(i)}\right), G\left(x_{n(i)}, Fx_{n(i)}, Fx_{n(i)}\right), \\ &\qquad\quad \frac{1}{3}\left(G\left(x_{m(i)}, Fx_{n(i)}, Fx_{n(i)}\right) + G\left(x_{n(i)}, Fx_{n(i)}, Fx_{n(i)}\right) + G\left(x_{n(i)}, Fx_{m(i)}, Fx_{m(i)}\right)\right)\Bigg\}\Bigg) \\ &\qquad\quad - \phi\left(\max\left\{G\left(x_{m(k)}, x_{n(k)}, x_{n(k)}\right), G\left(x_{n(k)}, Tx_{n(k)}, Tx_{n(k)}\right)\right\}\right) \\ &\qquad\quad + L\gamma\left(\min\left\{G\left(x_{m(i)}, x_{n(i)}, x_{n(i)}\right), G\left(x_{n(i)}, Fx_{n(i)}, Fx_{n(i)}\right), G\left(x_{n(i)}, Fx_{m(i)}, Fx_{m(i)}\right)\right\}\right) \end{aligned}$$

$$\begin{aligned}
&\leq \psi\Bigg(\max\bigg\{G\big(x_{m(i)},x_{n(i)},x_{n(i)}\big),G\big(x_{m(i)},x_{m(i)+1},x_{m(i)+1}\big),G\big(x_{n(i)},x_{n(i)+1},x_{n(i)+1}\big),\\
&\qquad \frac{1}{3}\big(G\big(x_{m(i)},x_{n(i)+1},x_{n(i)+1}\big)+G\big(x_{n(i)},x_{n(i)+1},x_{n(i)+1}\big)+G\big(x_{n(i)},x_{m(i)+1},x_{m(i)+1}\big)\big)\bigg\}\Bigg)\\
&\qquad -\phi\big(\max\big\{G\big(x_{m(k)},x_{n(k)},x_{n(k)}\big),G\big(x_{n(k)},x_{n(k)+1},x_{n(k)+1}\big)\big\}\big)\\
&\qquad +L\gamma\big(\min\big\{G\big(x_{m(i)},x_{n(i)},x_{n(i)}\big),G\big(x_{n(i)},x_{n(i)+1},x_{n(i)+1}\big),G\big(x_{n(i)},x_{m(i)+1},x_{m(i)+1}\big)\big\}\big)\\
&\leq \psi\Bigg(\max\bigg\{G\big(x_{m(i)},x_{n(i)},x_{n(i)}\big),G\big(x_{m(i)},x_{m(i)+1},x_{m(i)+1}\big),G\big(x_{n(i)},x_{n(i)+1},x_{n(i)+1}\big),\\
&\qquad \frac{1}{3}\big(G\big(x_{m(i)},x_{n(i)+1},x_{n(i)+1}\big)+G\big(x_{n(i)},x_{n(i)+1},x_{n(i)+1}\big)+2G\big(x_{n(i)},x_{n(i)},x_{m(i)+1}\big)\big)\bigg\}\Bigg)\\
&\qquad -\phi\big(\max\big\{G\big(x_{m(k)},x_{n(k)},x_{n(k)}\big),G\big(x_{n(k)},x_{n(k)+1},x_{n(k)+1}\big)\big\}\big)\\
&\qquad +L\gamma\big(\min\big\{G\big(x_{m(i)},x_{n(i)},x_{n(i)}\big),G\big(x_{n(i)},x_{n(i)+1},x_{n(i)+1}\big),G\big(x_{n(i)},x_{m(i)+1},x_{m(i)+1}\big)\big\}\big).
\end{aligned} \tag{3.29}$$

Letting $i \to +\infty$, using (3.18), (3.22), (3.24), (3.26), (3.28), and the continuity of ψ, ϕ, and γ, we have

$$\psi(\epsilon) \leq \psi(\epsilon) - \phi(\epsilon). \tag{3.30}$$

Thus $\psi(\epsilon) = 0$ and hence $\epsilon = 0$, a contradiction. Thus $\{x_n\}$ is a G-Cauchy sequence in X.

Step Three. We show that F has a fixed point.

Since $\{x_n\}$ is a G-Cauchy sequence in the complete G-metric space X, there is $u \in X$ such that

$$\lim_{n\to+\infty} G(x_n,u,u) = \lim_{n\to+\infty} G(x_n,x_n,u) = 0. \tag{3.31}$$

Since F is (ψ,ϕ,γ,G)-weak contraction, we have

$$\begin{aligned}
\psi(G(x_{n+1},Fu,Fu)) &= \psi(G(Fx_n,Fu,Fu))\\
&\leq \psi(M(x_n,u,u)) - \phi(M_1(x_n,u,u)) + L\gamma(N(x_n,u,u)),
\end{aligned} \tag{3.32}$$

where

$$\begin{aligned}
M(x_n,u,u) = \max\bigg\{&G(x_n,u,u),G(x_n,Fx_n,Fx_n),G(u,Fu,Fu),\\
&\frac{1}{3}(G(x_n,Fu,Fu)+G(u,Fu,Fu)+G(u,Fx_n,Fx_n))\bigg\}
\end{aligned}$$

$$
\begin{aligned}
&= \max\Big\{G(x_n,u,u),G(x_n,x_{n+1},x_{n+1}),G(u,Fu,Fu),\\
&\qquad \frac{1}{3}\left(G(x_n,Fu,Fu)+G(u,Fu,Fu)+G(u,x_{n+1},x_{n+1})\right)\Big\},\\
M_1(x_n,u,u) &= \max\{G(x_n,u,u),G(u,Tu,Tu)\},\\
N(x_n,u,u) &= \min\{G(x_n,Fx_n,Fx_n),G(u,Fu,Fu),G(u,Fx_n,Fx_n)\}\\
&= \min\{G(x_n,x_{n+1},x_{n+1}),G(u,Fu,Fu),G(u,x_{n+1},x_{n+1})\}.
\end{aligned}
\tag{3.33}
$$

Letting $n \to +\infty$ in (3.33) we get

$$
\begin{aligned}
M(x_n,u,u) &\longrightarrow G(u,Fu,Fu),\\
M_1(x_n,u,u) &\longrightarrow G(u,Fu,Fu),\\
N(x_n,u,u) &\longrightarrow 0.
\end{aligned}
\tag{3.34}
$$

On letting $n \to +\infty$ in (3.32) and using the continuity of ψ, ϕ, and γ, we get

$$
\psi(G(u,Fu,Fu)) \le \psi(G(u,Fu,Fu)) - \phi(G(u,Fu,Fu)).
\tag{3.35}
$$

Therefore $\psi(G(u,Fu,Fu)) = 0$ and hence $u = Fu$. Thus u is a fixed point of F.

Step Four. We prove the uniqueness of the fixed point.

In this respect, we proceed by *reductio ad absurdum*. Suppose that there are two fixed points of F, say $u, v \in X$ such that $u \ne v$. Since F is (ψ,ϕ,γ,G)-weak contraction, we have

$$
\psi(G(Fu,Fv,Fv)) \le \psi(M(u,v,v)) - \phi(M_1(u,v,v)) + L\gamma(N(u,v,v)),
\tag{3.36}
$$

where

$$
\begin{aligned}
M(u,v,v) &= \max\Big\{G(u,v,v),G(u,Fu,Fu),G(v,Fv,Fv),\\
&\qquad \frac{G(u,Fv,Fv)+G(v,Fv,Fv)+G(v,Fu,Fu)}{3}\Big\}\\
&= \max\left\{G(u,v,v),\frac{G(u,v,v)+G(v,u,u)}{3}\right\}\\
&\le \max\left\{G(u,v,v),\frac{G(u,v,v)+2G(v,v,u)}{3}\right\}\\
&= G(u,v,v),
\end{aligned}
$$

$$\begin{aligned} M_1(u,v,v) &= \max\{G(u,v,v),G(v,Fv,Fv)\} = G(u,v,v),\\ N(u,v,v) &= \min\{G(u,Fu,Fu),G(v,Fv,Fv),G(v,Fu,Fu)\}\\ &= \min\{G(u,u,u),G(v,v,v),G(v,u,u)\} = 0. \end{aligned} \tag{3.37}$$

By (3.36)-(3.37), we obtain

$$G(u,v,v) = G(Fu,Fv,Fv) \le \phi(G(u,v,v)) - \phi(G(u,v,v)) < \psi(G(u,v,v)), \tag{3.38}$$

a contradiction. Thus, $\phi(G(u,v,v)) = 0$, and hence $u = v$. Therefore the fixed point of F is unique. □

Corollary 3.5. *Let (X,G) be a complete G-metric space and let $F : X \to X$ be a G-weak contraction of type A. Then F has a unique fixed point.*

Proof. Since F is a G-weak contraction, there are two constant $a \in [0,1)$ and $L \ge 0$ such that

$$G(Fx,Fy,Fy) \le aM(x,y,y) + LN(x,y,y). \tag{3.39}$$

Define altering distance functions $\psi,\phi,\gamma : [0,+\infty) \to [0,+\infty)$ by $\psi(t) = \gamma(t) = t$ and $\phi(t) = (1-a)t$. Then

$$\begin{aligned} \psi(G(Fx,Fy,Fy)) &\le \psi(M(x,y,y)) - \phi(M(x,y,y)) + L(\psi(N(x,y,y)))\\ &\le \psi(M(x,y,y)) - \phi(M_1(x,y,y)) + L(\psi(N(x,y,y))). \end{aligned} \tag{3.40}$$

Thus F is (ψ,ϕ,γ,G)-weak contraction. Hence by Theorem 3.4, we conclude that F has a unique fixed point. □

Corollary 3.6. *Let (X,G) be a complete G-metric space and $F : X \to X$ a mapping. Suppose there exists $a \in [0,1)$ such that*

$$\begin{aligned} G(Fx,Fy,Fy) \le a\max\Bigg\{ &G(x,y,y),G(x,Fx,Fx),G(y,Fy,Fy),\\ &\frac{G(x,Fy,Fy)+G(y,Fy,Fy)+G(y,Fx,Fx)}{3}\Bigg\}, \end{aligned} \tag{3.41}$$

for all $x,y \in X$. Then F has a unique fixed point.

Corollary 3.7. *Let (X, G) be a complete G-metric space and let $F : X \to X$ be a mapping. Suppose there exist nonnegative real numbers $a, b, c,$ and d with $a + b + c + d < 1$ such that*

$$\begin{aligned} G(Fx, Fy, Fy) \leq{}& aG(x, y, y) + bG(x, Fx, Fx) + cG(y, Fy, Fy) \\ &+ \frac{d}{3}\left(G(x, Fy, Fy) + G(y, Fy, Fy) + G(y, Fx, Fx)\right), \end{aligned} \tag{3.42}$$

for all $x, y \in X$. Then F has a unique fixed point.

Proof. Follows from Corollary 3.6, by noting that

$$\begin{aligned} &aG(x, y, y) + bG(x, Fx, Fx) + cG(y, Fy, Fy) \\ &\quad + \frac{d}{3}\left(G(x, Fy, Fy) + G(y, Fy, Fy) + G(y, Fx, Fx)\right) \\ &\quad \leq (a + b + c + d) \max\Bigg\{G(x, y, y), G(x, Fx, Fx), G(y, Fy, Fy), \\ &\qquad \frac{G(x, Fy, Fy) + G(y, Fy, Fy) + G(y, Fx, Fx)}{3}\Bigg\}. \end{aligned} \tag{3.43}$$

$\square$

Consider again (X, G) be a G-metric space, and $F : X \to X$ a mapping. We set

$$\begin{aligned} m(x, x, y) ={}& \max\Bigg\{G(x, x, y), G(x, x, Fx), G(y, y, Fy), \\ &\frac{G(x, x, Fy) + G(y, y, Fy) + G(y, y, Fx)}{3}\Bigg\}, \\ m_1(x, x, y) ={}& \max\{G(x, x, y), G(y, y, Fy)\}, \\ n(x, x, y) ={}& \min\{G(x, x, Fx), G(y, y, Fy), G(y, y, Fx)\}. \end{aligned} \tag{3.44}$$

Now, we introduce the following definitions.

Definition 3.8. Let (X, G) be a G-metric space. A mapping $F : X \to X$ is called a G-weak contraction of type B if and only if there exist two constants $a \in (0, 1)$ and $L \geq 0$ such that

$$G(Fx, Fx, Fy) \leq am(x, x, y) + Ln(x, x, y), \tag{3.45}$$

for all $x, y \in X$.

Definition 3.9. Let (X,G) be a G-metric space and let ψ,ϕ,γ be altering distance functions. A mapping $F : X \to X$ is called a (ψ,ϕ,γ,G)-weak contraction of type B if and only if there exist a constant $L \geq 0$ such that

$$\psi(G(Fx,Fx,Fy)) \leq \psi(m(x,x,y)) - \phi(m_1(x,x,y)) + L\gamma(n(x,x,y)) \tag{3.46}$$

for all $x,y \in X$.

Following the same arguments as those in the proof of Theorem 3.4, we obtain the following.

Theorem 3.10. *Let (X,G) be a complete G-metric space and let $\psi,\phi,\gamma : [0,+\infty) \to [0,+\infty)$ be altering distance function. If $F : X \to X$ be a (ψ,ϕ,γ,G)-weak contraction of type B, then F has a unique fixed point.*

The following results are direct consequences of Theorem 3.10.

Corollary 3.11. *Let (X,G) be a complete G-metric space and let $F : X \to X$ be a G-weak contraction of type B. Then F has a unique fixed point.*

Corollary 3.12. *Let (X,G) be a complete G-metric space and let $F : X \to X$ be a mapping. Suppose there exists $a \in [0,1)$ such that*

$$\begin{aligned} G(Fx,Fx,Fy) \leq a \max\Big\{ & G(x,x,y), G(x,x,Fx), G(y,y,Fy), \\ & \frac{G(x,x,Fy)+G(y,y,Fy)+G(y,y,Fx)}{3}\Big\}, \end{aligned} \tag{3.47}$$

for all $x,y \in X$. Then F has a unique fixed point.

Corollary 3.13. *Let (X,G) be a complete G-metric space and let $F : X \to X$ be a mapping. Suppose there exist nonnegative real numbers $a,b,c,$ and d with $a+b+c+d<1$, such that*

$$\begin{aligned} G(Fx,Fx,Fy) \leq{} & aG(x,x,y) + bG(x,x,Fx) + cG(y,y,Fy) \\ & + \frac{d}{3}(G(x,x,Fy) + G(y,y,Fy) + G(y,y,Fx)), \end{aligned} \tag{3.48}$$

for all $x,y \in X$. Then F has a unique fixed point.

4. Example and Application

In this section, we introduce an example and application to support the validity of our results.

Example 4.1. Let $X = \{0,1,2,3,\ldots\}$. Define $G : X \times X \times X \to X$ by

$$G(x,y,z) = \begin{cases} x+y+z, & \text{if } x,y,z \text{ are all distinct and different from zero,} \\ x+z, & \text{if } x = y \neq z \text{ and all are different from zero,} \\ y+z+1, & \text{if } x = 0,\ y \neq z \text{ and } y,z \text{ are different from zero,} \\ y+2, & \text{if } x = 0,\ z = y \neq 0, \\ z+1, & \text{if } x = 0,\ y = 0,\ z \neq 0, \\ 0, & \text{if } x = y = x, \end{cases} \tag{4.1}$$

and $F : X \to X$ by

$$Fx = \begin{cases} 0, & \text{if } x = 0, \\ x-1, & \text{if } x \geq 1. \end{cases} \tag{4.2}$$

Also, define $\psi, \phi, \gamma : [0,+\infty) \to [0,+\infty)$ by $\psi(t) = t^2$ and $\phi(t) = \gamma(t) = t$. Then

(a) $(X, G, \leq)$ is a complete nonsymmetric G-metric space,

(b) for all $x, y \in X$ and $L \geq 0$, we have

$$\psi(G(Fx,Fy,Fy)) \leq \psi(M(x,y,y)) - \phi(M_1(x,y,y)) + L\gamma(N(x,y,y)), \tag{4.3}$$

that is, F is (ψ,ϕ,γ,G)-weak contraction of type A.

Proof. The proof of part (a) follows from [12]. Now, given $x, y \in X$. We divide the proof of part (b) into the following cases.

Case 1 $(x = y)$. Here, we have $\psi(G(Fx,Fy,Fy)) = 0$ and hence

$$\psi(G(Fx,Fy,Fy)) \leq \psi(M(x,y,y)) - \phi(M_1(x,y,y)) + L\gamma(N(x,y,y)). \tag{4.4}$$

Case 2 $(x > y = 1)$. Here $Fx = x-1$ and $Fy = 0$. Thus

$$\begin{aligned} G(Fx,Fy,Fy) &= G(x-1,0,0) = x, \\ M(x,1,1) &= \max\Big\{G(x,1,1), G(x,x-1,x-1), G(1,0,0), \\ &\qquad \frac{1}{3}(G(x,0,0) + G(1,0,0) + G(1,x-1,x-1))\Big\} \end{aligned}$$

$$
\begin{aligned}
&= \max\left\{x+1, 2x-1, 2, \frac{1}{3}(x+1+2+x)\right\} \\
&= \max\left\{x+1, 2x-1, \frac{1}{3}(2x+3)\right\} = 2x-1, \\
M_1(x,1,1) &= \max\{G(x,1,1), G(1,0,0)\} \\
&= \max\{x+1, 2\} = x+1.
\end{aligned}
\tag{4.5}
$$

Since

$$x^2 \le (2x-1)^2 - (x+1), \tag{4.6}$$

we have

$$\psi(G(Fx,Fy,Fy)) \le \psi(M(x,y,y)) - \phi(M_1(x,y,y)). \tag{4.7}$$

Hence

$$\psi(G(Fx,Fy,Fy)) \le \psi(M(x,y,y)) - \phi(M_1(x,y,y)) + L\gamma(N(x,y,y)). \tag{4.8}$$

Case 3 ($x > y \ge 2$). Here $Fx = x-1$ and $Fy = y-1$.
Subcase 3.1 ($x = y+1$). Here

$$
\begin{aligned}
G(F(y+1), Fy, Fy) &= G(y, y-1, y-1) = 2y-1, \\
M(y+1,y,y) &= \max\Big\{G(y+1,y,y), G(y+1,y,y), G(y,y-1,y-1), \\
&\qquad \frac{1}{3}(G(y+1,y-1,y-1) + G(y,y-1,y-1) + G(y,y,y))\Big\} \\
&= \max\left\{2y+1, 2y-1, \frac{1}{3}(2y+2y-1+0)\right\} = 2y+1, \\
M_1(y+1,y,y) &= \max\{G(y+1,y,y), G(y,y-1,y-1)\} \\
&= \max\{2y+1, 2y-1\} = 2y+1.
\end{aligned}
\tag{4.9}
$$

Since

$$(2y-1)^2 \le (2y+1)^2 - (2y+1), \tag{4.10}$$

we have

$$\psi(G(Fx,Fy,Fy)) \le \psi(M(x,y,y)) - \phi(M_1(x,y,y)). \tag{4.11}$$

Hence

$$\psi(G(Fx,Fy,Fy)) \le \psi(M(x,y,y)) - \phi(M_1(x,y,y)) + L\gamma(N(x,y,y)). \tag{4.12}$$

Subcase 3.2 ($x > y+1$). Here

$$\begin{aligned}
G(Fx,Fy,Fy) &= G(x-1,y-1,y-1) = x+y-2,\\
M(x,y,y) &= \max\Big\{G(x,y,y), G(x,x-1,x-1), G(y,y-1,y-1),\\
&\qquad \frac{1}{3}(G(x,y-1,y-1)+G(y,y-1,y-1)+G(y,x-1,x-1))\Big\}\\
&= \max\Big\{x+y, 2x-1, 2y-1, \frac{1}{3}(x+y-1+2y-1+y+x-1)\Big\}\\
&= \max\Big\{x+y, 2x-1, 2y-1, \frac{1}{3}(2x+4y-3)\Big\} = 2x-1,\\
M_1(x,y,y) &= \max\{G(x,y,y), G(y,y-1,y-1)\}\\
&= \max\{x+y, 2y-1\} = x+y.
\end{aligned} \tag{4.13}$$

Since

$$(x+y-2)^2 \le (2x-3)^2 \le (2x-1)^2 - (x+y), \tag{4.14}$$

we have

$$\psi(G(Fx,Fy,Fy)) \le \psi(M(x,y,y)) - \phi(M_1(x,y,y)). \tag{4.15}$$

Hence

$$\psi(G(Fx,Fy,Fy)) \le \psi(M(x,y,y)) - \phi(M_1(x,y,y)) + L\gamma(N(x,y,y)). \tag{4.16}$$

Case 4 ($x > y = 0$). Here, we have $Fx = x-1$ and $Fy = 0$.
Subcase 4.1 ($x = 1$). Here, we have

$$G(Fx,Fy,Fy) = G(0,0,0) = 0. \tag{4.17}$$

Hence

$$\psi(G(Fx,Fy,Fy)) \le \psi(M(x,y,y)) - \phi(M_1(x,y,y)) + L\gamma(N(x,y,y)). \tag{4.18}$$

Subcase 4.2 ($x \geq 2$). Here,

$$\begin{aligned}
G(Fx, Fy, Fy) &= G(x-1, 0, 0) = x, \\
M(x, 0, 0) &= \max\Big\{ G(x,0,0), G(x, x-1, x-1), G(0,0,0), \\
&\qquad \frac{1}{3}(G(x,0,0) + G(0,0,0) + G(0, x-1, x-1)) \Big\} \\
&= \max\Big\{ x+1, 2x-1, 0, \frac{1}{3}(x+1+0+x-1+2) \Big\} \\
&= \max\Big\{ x+1, 2x-1, 0, \frac{1}{3}(2x+2) \Big\} = 2x-1, \\
M_1(x, 0, 0) &= \max\{G(x,0,0), G(0,0,0)\} = x+1.
\end{aligned} \tag{4.19}$$

Since

$$x^2 \leq (2x-1)^2 - (x+1), \tag{4.20}$$

we have

$$\psi(G(Fx, Fy, Fy)) \leq \psi(M(x, y, y)) - \phi(M_1(x, y, y)). \tag{4.21}$$

Hence

$$\psi(G(Fx, Fy, Fy)) \leq \psi(M(x, y, y)) - \phi(M_1(x, y, y)) + L\gamma(N(x, y, y)). \tag{4.22}$$

Thus F satisfies all the hypotheses of Theorem 3.4. Hence F has a unique fixed point. Here 0 is the unique fixed point of F. □

As an application of our results, we introduce some fixed-point theorems of integral type.

Denote by Φ the set of functions $\phi : [0, +\infty) \to [0, +\infty)$ satisfying the following hypotheses:

(1) ϕ is a Lebesgue integrable function on each compact subset of $[0, +\infty)$,

(2) for every $\epsilon > 0$, we have $\int_0^\epsilon \phi(s)ds > 0$.

It is an easy matter, to see that the mapping $\psi : [0, +\infty) \to [0, +\infty)$ defined by

$$\psi(t) = \int_0^t \phi(s)ds \tag{4.23}$$

is an altering distance function. Now, we have the following result.

Corollary 4.2. *Let (X, G) be a complete G-metric space. Let $F : X \to X$ be a mapping. Suppose that there exist $L \geq 0$ such that for $\phi, \lambda, \mu \in \Phi$, we have*

$$\begin{aligned}\int_0^{G(Fx,Fy,Fy)} \phi(s)ds \leq & \int_0^{\max\{G(x,y,y),G(x,Fx,Fx),G(y,Fy,Fy),(1/3)(G(x,Fy,Fy)+G(y,Fy,Fy)+G(y,Fx,Fx))\}} \phi(s)ds \\ & - \int_0^{\max\{G(x,y,y),G(y,Fy,Fy)\}} \lambda(s)ds \\ & + L\int_0^{\min\{G(x,Fx,Fx),G(y,Fy,Fy),G(y,Fx,Fx)\}} \mu(s)ds,\end{aligned} \tag{4.24}$$

for $x, y \in X$. Then F has a unique fixed point.

Proof. Follows from Theorem 3.4 by taking

$$\psi(t) = \int_0^t \phi(s)ds, \qquad \phi(t) = \int_0^t \lambda(s)ds, \qquad \gamma(t) = \int_0^t \mu(s)ds. \tag{4.25}$$

□

Corollary 4.3. *Let (X, G) be a complete G-metric space. Let $F : X \to X$ be a mapping. Suppose that there exist $L \geq 0$ such that for $\phi, \lambda, \mu \in \Phi$, we have*

$$\begin{aligned}\int_0^{G(Fx,Fx,Fy)} \phi(s)ds \leq & \int_0^{\max\{G(x,x,y),G(x,x,Fx),G(y,y,Fy),(1/3)(G(x,x,Fy)+G(y,y,Fy)+G(y,y,Fx))\}} \phi(s)ds \\ & - \int_0^{\max\{G(x,x,y),G(y,y,Fy)\}} \lambda(s)ds \\ & + L\int_0^{\min\{G(x,x,Fx),G(y,y,Fy),G(y,y,Fx)\}} \mu(s)ds,\end{aligned} \tag{4.26}$$

for $x, y \in X$. Then F has a unique fixed point.

Proof. Follows from Theorem 3.10 by taking

$$\psi(t) = \int_0^t \phi(s)ds, \qquad \phi(t) = \int_0^t \lambda(s)ds, \qquad \gamma(t) = \int_0^t \mu(s)ds. \tag{4.27}$$

□

5. Conclusion

In this paper we introduced contractive conditions of two kinds, independent of the existing ones, to establish new results on nonlinear analysis on G-metric spaces. More accurately, we established fixed point results for two kinds of G-weak contraction mappings, in complete

G-metric spaces. We illustrated our theory with nontrivial example. As applications of our main theorems, we introduced fixed point results for mappings satisfying some contractive conditions of integral type in G-metric spaces.

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