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Research Article

Fuzzy Parameterized Soft Expert Set

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In 2011 Alkhazaleh and Salleh introduced the concept of soft expert set and gave an application in a decision-making problem. In this paper we introduce the concept of fuzzy parameterized soft expert set by giving an important degree for each element in the set of parameters. We also study its properties and define its basic operations, namely, complement, union, intersection, AND, and OR. Finally, we give an application in decision making.

1. Introduction

Many fields deal with uncertain data that may not be successfully modeled by classical math ematics. Molodtsov [1] proposed a completely new approach for modeling vagueness and uncertainty. This so-called soft set theory has potential applications in many different fields. After Molodtsov's work, some different operations and application of soft sets were studied by Chen et al. [2] and Maji et al. [3, 4]. Furthermore Maji et al. [5] presented the definition of fuzzy soft set as a generalization of Molodtsov's soft set, and Roy and Maji [6] presented an application of fuzzy soft sets in a decision-making problem. Majumdar and Samanta [7] defined and studied the generalised fuzzy soft sets where the degree is attached with the parameterization of fuzzy sets while defining a fuzzy soft set. Zhou et al. [8] defined and studied generalised interval-valued fuzzy soft sets where the degree is attached with the parameterization of interval-valued fuzzy sets while defining an interval-valued fuzzy soft set. Alkhazaleh et al. [9] defined the concepts of possibility fuzzy soft set and gave their applications in decision making and medical diagnosis. They also introduced the concept of fuzzy parameterized interval-valued fuzzy soft set [10] where the mapping of approximate function is defined from the set of parameters to the interval-valued fuzzy subsets of the universal set and gave an application of this concept in decision making. Salleh et al. [11]

introduced the concept of multiparameterized soft set and studied its properties and basic operations. In 2010 Çağman et al. introduced the concept of fuzzy parameterized fuzzy soft sets and their operations [12]. Also Çağman et al. [13] introduced the concept of fuzzy parameterized soft sets and their related properties. Alkhazaleh and Salleh [14] introduced the concept of a soft expert set, and Alkhazaleh [15] introduced fuzzy soft expert set, where the user can know the opinion of all experts in one model without any operations. In this paper we introduce the concept of fuzzy parameterized soft expert set which is a combination of fuzzy set and soft expert set. We also define its basic operations, namely, complement, union, intersection, and the operations AND and OR. Finally, we give an application of fuzzy parameterized soft expert set in decision-making problem.

2. Preliminaries

In this section, we recall some basic notions in soft expert set theory. Alkhazaleh and Salleh [14] defined soft expert set in the following way. Let U be a universe, let E be a set of parameters, and let X be a set of experts (agents). Let $O = \{1 = \text{agree}, 0 = \text{disagree}\}$ be a set of opinions, $Z = E \times X \times O$, and $A \subseteq Z$.

Definition 2.1. A pair (F, A) is called a *soft expert se*t over U, where F is a mapping $F : A \to P(U)$, and $P(U)$ denotes the power set of U.

Definition 2.2. For two soft expert sets (F, A) and (G, B) over U, (F, A) is called a *soft expert subset* of (G, B) if

(i) $A \subseteq B$,

(ii) for all $\varepsilon \in A, F(\varepsilon) \subseteq G(\varepsilon)$.

Definition 2.3. Two soft expert sets (F, A) and (G, B) over U are said to be *equal* if (F, A) is a soft expert subset of (G, B) and (G, B) is a soft expert subset of (F, A).

Definition 2.4. Let E be a set of parameters and let X be a set of experts. The *NOT* set of $Z = E \times X \times O$, denoted by $\sim Z$, is defined by

$$\sim Z = \left\{\left(\sim e_i, x_j, o_k\right), \forall i, j, k\right\} \quad \text{where } \sim e_i \text{ is not } e_i. \tag{2.1}$$

Definition 2.5. The *complement* of a soft expert set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \sim A)$ where $F^c :\sim A \to P(U)$ is a mapping given by $F^c(\alpha) = U - F(\sim \alpha)$, for all $\alpha \subset\sim A$.

Definition 2.6. An *agree-soft expert set* $(F, A)_1$ over U is a soft expert subset of (F, A) defined as follows:

$$(F, A)_1 = \{F_1(\alpha) : \alpha \in E \times X \times \{1\}\}. \tag{2.2}$$

Definition 2.7. A *disagree-soft expert set* $(F, A)_0$ over U is a soft expert subset of (F, A) defined as follows:

$$(F, A)_0 = \{F_0(\alpha) : \alpha \in E \times X \times \{0\}\}. \tag{2.3}$$

Definition 2.8. The *union* of two soft expert sets (F, A) and (G, B) over U, denoted by $(F, A) \tilde{\cup} (G, B)$, is the soft expert set (H, C) where $C = A \cup B$, and for all $\varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon) & \text{if } \varepsilon \in A - B, \\ G(\varepsilon) & \text{if } \varepsilon \in B - A, \\ F(\varepsilon) \cup G(\varepsilon) & \text{if } \varepsilon \in A \cap B. \end{cases} \tag{2.4}$$

Definition 2.9. The *intersection* of two soft expert sets (F, A) and (G, B) over U, denoted by $(F, A) \tilde{\cap} (G, B)$, is the soft expert (H, C) where $C = A \cup B$, and for all $\varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon) & \text{if } \varepsilon \in A - B, \\ G(\varepsilon) & \text{if } \varepsilon \in B - A, \\ F(\varepsilon) \cap G(\varepsilon) & \text{if } \varepsilon \in A \cap B. \end{cases} \tag{2.5}$$

Definition 2.10. If (F, A) and (G, B) are two soft expert sets, then (F, A) AND (G, B), denoted by $(F, A) \wedge (G, B)$, is defined by

$$(F, A) \wedge (G, B) = (H, A \times B), \tag{2.6}$$

where $H(\alpha, \beta) = F(\alpha) \tilde{\cap} G(\beta)$ for all $(\alpha, \beta) \in A \times B$.

Definition 2.11. If (F, A) and (G, B) are two soft expert sets, then (F, A) OR (G, B), denoted by $(F, A) \vee (G, B)$, is defined by

$$(F, A) \vee (G, B) = (H, A \times B), \tag{2.7}$$

where $H(\alpha, \beta) = F(\alpha) \tilde{\cup} G(\beta)$, for all $(\alpha, \beta) \in A \times B$.

3. Fuzzy Parameterized Soft Expert Sets

In this section, we introduce the definition of fuzzy parameterized soft expert set and its basic operations, namely, complement, union, intersection, and the operations AND and OR. We give examples for these concepts. Basic properties of the operations are also given.

Definition 3.1. Let U be a universe, let E be a set of parameters, let I^E denote the set of fuzzy subsets of E, X a set of experts (agents), and $O = \{1 = \text{agree}, 0 = \text{disagree}\}$ a set of opinions. Let $Z = D \times X \times O$ and $A \subseteq Z$ where $D \subset I^E$. Then the cartesian product $Z = D \times X \times O$ is defined as follows:

$$Z = \{(d, x, o) : d \in D, x \in X, o \in O\}. \tag{3.1}$$

Definition 3.2. A pair $(F, A)_D$ is called a *fuzzy parameterized soft expert set (FPSES)* over U, where F is a mapping given by $F_D : A \rightarrow P(U)$, and $P(U)$ denotes the power set of U.

Example 3.3. Suppose that a hotel chain is looking for a construction company to upgrade the hotels to keep pace with globalization and wishes to take the opinion of some experts concerning this matter. Let $U = \{u_1, u_2, u_3, u_4\}$ be a set of construction companies, let $E = \{e_1, e_2, e_3\}$ be a set of decision parameters where e_i $(i = 1, 2, 3)$ denotes the decision "good service," "quality," and "cheap," respectively, and $D = \{e_1/0.3, e_2/0.5, e_3/0.8\}$ a fuzzy subset of I^E, and let $X = \{p, q, r\}$ be a set of experts.

Suppose that the hotel chain has distributed a questionnaire to the three experts to make decisions on the construction companies, and we get the following information:

$$\begin{gathered}
F\left(\frac{e_1}{0.3}, p, 1\right) = \{u_2, u_3, u_4\}, \quad F\left(\frac{e_1}{0.3}, q, 1\right) = \{u_2, u_4\}, \quad F\left(\frac{e_1}{0.3}, r, 1\right) = \{u_3, u_4\}, \\
F\left(\frac{e_2}{0.5}, p, 1\right) = \{u_2, u_4\}, \quad F\left(\frac{e_2}{0.5}, q, 1\right) = \{u_2, u_3, u_4\}, \quad F\left(\frac{e_2}{0.5}, r, 1\right) = \{u_4\}, \\
F\left(\frac{e_3}{0.8}, p, 1\right) = \{u_2\}, \quad F\left(\frac{e_3}{0.8}, q, 1\right) = \{u_2, u_3\}, \quad F\left(\frac{e_3}{0.8}, r, 1\right) = \{u_2, u_4\}, \\
F\left(\frac{e_1}{0.3}, p, 0\right) = \{u_1\}, \quad F\left(\frac{e_1}{0.3}, q, 0\right) = \{u_1, u_3\}, \quad F\left(\frac{e_1}{0.3}, r, 0\right) = \{u_1, u_2\}, \\
F\left(\frac{e_2}{0.5}, p, 0\right) = \{u_1, u_3\}, \quad F\left(\frac{e_2}{0.5}, q, 0\right) = \{u_1\}, \quad F\left(\frac{e_2}{0.5}, r, 0\right) = \{u_1, u_2, u_3\}, \\
F\left(\frac{e_3}{0.8}, p, 0\right) = \{u_1, u_3, u_4\}, \quad F\left(\frac{e_3}{0.8}, q, 0\right) = \{u_1, u_4\}, \quad F\left(\frac{e_3}{0.8}, r, 0\right) = \{u_1, u_3\}.
\end{gathered} \tag{3.2}$$

Then we can view the FPSES (F, Z) as consisting of the following collection of approximations:

$$\begin{aligned}
(F, Z)_D = \Bigg\{ & \left\{\left(\frac{e_1}{0.3}, p, 1\right), \{u_2, u_3, u_4\}\right\}, \left\{\left(\frac{e_1}{0.3}, q, 1\right), \{u_2, u_4\}\right\}, \left\{\left(\frac{e_1}{0.3}, r, 1\right), \{u_3, u_4\}\right\}, \\
& \left\{\left(\frac{e_2}{0.5}, p, 1\right), \{u_2, u_4\}\right\}, \left\{\left(\frac{e_2}{0.5}, q, 1\right), \{u_2, u_3, u_4\}\right\}, \left\{\left(\frac{e_2}{0.5}, r, 1\right), \{u_4\}\right\}, \\
& \left\{\left(\frac{e_3}{0.8}, p, 1\right), \{u_2\}\right\}, \left\{\left(\frac{e_3}{0.8}, q, 1\right), \{u_2, u_3\}\right\}, \left\{\left(\frac{e_3}{0.8}, r, 1\right), \{u_2, u_4\}\right\}, \\
& \left\{\left(\frac{e_1}{0.3}, p, 0\right), \{u_1\}\right\}, \left\{\left(\frac{e_1}{0.3}, q, 0\right), \{u_1, u_3\}\right\}, \left\{\left(\frac{e_1}{0.3}, r, 0\right), \{u_1, u_2\}\right\}, \\
& \left\{\left(\frac{e_2}{0.5}, p, 0\right), \{u_1, u_3\}\right\}, \left\{\left(\frac{e_2}{0.5}, q, 0\right), \{u_1\}\right\}, \left\{\left(\frac{e_2}{0.5}, r, 0\right), \{u_1, u_2, u_3\}\right\}, \\
& \left\{\left(\frac{e_3}{0.8}, p, 0\right), \{u_1, u_3, u_4\}\right\}, \left\{\left(\frac{e_3}{0.8}, q, 0\right), \{u_1, u_4\}\right\}, \left\{\left(\frac{e_3}{0.8}, r, 0\right), \{u_1, u_3\}\right\}\Bigg\}.
\end{aligned} \tag{3.3}$$

Definition 3.4. For two FPFESs $(F,A)_D$ and $(G,B)_K$ over U, $(F,A)_D$ is called an *FPSE subset* of $(G,B)_K$, and we write $(F,A)_D \subseteq (G,B)_K$, if

(i) $A \subseteq B$,

(ii) for all $\varepsilon \in A$, $F_D(\varepsilon) \subseteq G_K(\varepsilon)$.

Definition 3.5. Two fuzzy FPSESs $(F,A)_D$ and $(G,B)_K$ over U are said to be *equal* if $(F,A)_D$ is an FPSE subset of $(G,B)_K$ and $(G,B)_K$ is a FPSE subset of $(F,A)_D$.

Example 3.6. Consider Example 3.3. Suppose that the hotel chain takes the opinion of the experts once again after the hotel chain has been opened.

Let $D = \{e_1/0.6, e_2/0.3, e_3/0.2\}$ be a fuzzy subset over E, and let $K = \{e_1/0.3, e_2/0.2, e_3/0.1\}$ be another fuzzy subset over E. Suppose

$$\begin{aligned} A_D = \Big\{ &\left(\frac{e_1}{0.6}, p, 1\right), \left(\frac{e_2}{0.3}, p, 1\right), \left(\frac{e_3}{0.2}, p, 0\right), \left(\frac{e_1}{0.6}, q, 1\right), \left(\frac{e_1}{0.6}, q, 0\right), \\ &\left(\frac{e_2}{0.3}, q, 1\right), \left(\frac{e_3}{0.2}, q, 1\right), \left(\frac{e_1}{0.6}, r, 1\right), \left(\frac{e_2}{0.3}, r, 0\right) \Big\}, \\ B_K = \Big\{ &\left(\frac{e_1}{0.3}, p, 1\right), \left(\frac{e_2}{0.2}, p, 1\right), \left(\frac{e_1}{0.3}, q, 1\right), \left(\frac{e_2}{0.2}, q, 1\right), \left(\frac{e_3}{0.1}, q, 1\right), \\ &\left(\frac{e_1}{0.3}, r, 1\right), \left(\frac{e_2}{0.2}, r, 0\right) \Big\}. \end{aligned} \tag{3.4}$$

Since K is a fuzzy subset of D, clearly $B_K \subset A_D$. Let $(F,A)_D$ and $(F,B)_K$ be defined as follows:

$$\begin{aligned} (F,A)_D = \Big\{ &\Big\{ \left(\frac{e_1}{0.6}, p, 1\right), \{u_2, u_3, u_4\} \Big\}, \Big\{ \left(\frac{e_1}{0.6}, q, 1\right), \{u_2, u_4\} \Big\}, \Big\{ \left(\frac{e_1}{0.6}, r, 1\right), \{u_3, u_4\} \Big\}, \\ &\Big\{ \left(\frac{e_2}{0.3}, p, 1\right), \{u_2, u_4\} \Big\}, \Big\{ \left(\frac{e_2}{0.3}, q, 1\right), \{u_2, u_3, u_4\} \Big\}, \Big\{ \left(\frac{e_3}{0.2}, q, 1\right), \{u_2, u_3\} \Big\}, \\ &\Big\{ \left(\frac{e_1}{0.6}, q, 0\right), \{u_4\} \Big\}, \Big\{ \left(\frac{e_2}{0.3}, r, 0\right), \{u_3\} \Big\}, \Big\{ \left(\frac{e_3}{0.2}, p, 0\right), \{u_2, u_3\} \Big\} \Big\}, \\ (G,B)_K = \Big\{ &\Big\{ \left(\frac{e_1}{0.3}, p, 1\right), \{u_3, u_4\} \Big\}, \Big\{ \left(\frac{e_1}{0.3}, q, 1\right), \{u_2, u_4\} \Big\}, \Big\{ \left(\frac{e_1}{0.3}, r, 1\right), \{u_4\} \Big\}, \\ &\Big\{ \left(\frac{e_2}{0.2}, p, 1\right), \{u_2\} \Big\}, \Big\{ \left(\frac{e_2}{0.2}, q, 1\right), \{u_2, u_4\} \Big\}, \Big\{ \left(\frac{e_3}{0.1}, q, 1\right), \{u_2\} \Big\}, \\ &\Big\{ \left(\frac{e_2}{0.2}, r, 0\right), \{u_3\} \Big\} \Big\}. \end{aligned} \tag{3.5}$$

Therefore $(G,B)_K \subseteq (F,A)_D$.

Definition 3.7. The *complement* of an FPSES $(F,A)_D$ is denoted by $(F,A)_D^c$ and is defined by $(F,A)_D^c = (F^c, \sim A)_D$ where $F_D^c : \sim A \rightarrow P(U)$ is a mapping given by $F_D^c(\alpha) = U - F_D(\sim \alpha)$, for all $\alpha \in \sim A$ and $\sim A \subset \{D^c \times X \times O\}$.

Example 3.8. Consider Example 3.3. By using the basic fuzzy complement, we have

$$\begin{aligned}(F,Z)_D^c = \Big\{&\Big\{\Big(\frac{e_1}{0.7},p,1\Big),\{u_1\}\Big\},\Big\{\Big(\frac{e_1}{0.7},q,1\Big),\{u_1,u_3\}\Big\},\Big\{\Big(\frac{e_1}{0.7},r,1\Big),\{u_1,u_2\}\Big\},\\ &\Big\{\Big(\frac{e_2}{0.5},p,1\Big),\{u_1,u_3\}\Big\},\Big\{\Big(\frac{e_2}{0.5},q,1\Big),\{u_1\}\Big\},\Big\{\Big(\frac{e_2}{0.5},r,1\Big),\{u_1,u_2,u_3\}\Big\},\\ &\Big\{\Big(\frac{e_3}{0.2},p,1\Big),\{u_1,u_3,u_4\}\Big\},\Big\{\Big(\frac{e_3}{0.2},q,1\Big),\{u_1,u_4\}\Big\},\Big\{\Big(\frac{e_3}{0.2},r,1\Big),\{u_1,u_3\}\Big\},\\ &\Big\{\Big(\frac{e_1}{0.7},p,0\Big),\{u_2,u_3,u_4\}\Big\},\Big\{\Big(\frac{e_1}{0.7},q,0\Big),\{u_2,u_4\}\Big\},\Big\{\Big(\frac{e_1}{0.7},r,0\Big),\{u_3,u_4\}\Big\},\\ &\Big\{\Big(\frac{e_2}{0.5},p,0\Big),\{u_2,u_4\}\Big\},\Big\{\Big(\frac{e_2}{0.5},q,0\Big),\{u_2,u_3,u_4\}\Big\},\Big\{\Big(\frac{e_2}{0.5},r,0\Big),\{u_4\}\Big\},\\ &\Big\{\Big(\frac{e_3}{0.2},p,0\Big),\{u_2\}\Big\},\Big\{\Big(\frac{e_3}{0.2},q,0\Big),\{u_2,u_3\}\Big\},\Big\{\Big(\frac{e_3}{0.2},r,0\Big),\{u_2,u_4\}\Big\}\Big\}.\end{aligned} \tag{3.6}$$

Definition 3.9. An *agree-FPSES* $(F,A)_{D_1}$ over U is an FPSE subset of $(F,A)_D$ where the opinions of all experts are agree and is defined as follows:

$$(F,A)_{D_1} = \{F_D(\alpha) : \alpha \in D \times X \times \{1\}\}. \tag{3.7}$$

Example 3.10. Consider Example 3.3. Then the agree-FPSES $(F,A)_{D_1}$ over U is

$$\begin{aligned}(F,A)_{D_1} = \Big\{&\Big\{\Big(\frac{e_1}{0.3},p,1\Big),\{u_2,u_3,u_4\}\Big\},\Big\{\Big(\frac{e_1}{0.3},q,1\Big),\{u_2,u_4\}\Big\},\Big\{\Big(\frac{e_1}{0.3},r,1\Big),\{u_3,u_4\}\Big\},\\ &\Big\{\Big(\frac{e_2}{0.5},p,1\Big),\{u_2,u_4\}\Big\},\Big\{\Big(\frac{e_2}{0.5},q,1\Big),\{u_2,u_3,u_4\}\Big\},\Big\{\Big(\frac{e_2}{0.5},r,1\Big),\{u_4\}\Big\},\\ &\Big\{\Big(\frac{e_3}{0.8},p,1\Big),\{u_2\}\Big\},\Big\{\Big(\frac{e_3}{0.8},q,1\Big),\{u_2,u_3\}\Big\},\Big\{\Big(\frac{e_3}{0.8},r,1\Big),\{u_2,u_4\}\Big\}\Big\}.\end{aligned} \tag{3.8}$$

Definition 3.11. A *disagree-FPSES* $(F,A)_{D_0}$ over U is an FPSE subset of $(F,A)_D$ where the opinions of all experts are disagree and is defined as follows:

$$(F,A)_{D_0} = \{F_D(\alpha) : \alpha \in D \times X \times \{0\}\}. \tag{3.9}$$

Example 3.12. Consider Example 3.3. Then the disagree-FPSES $(F,A)_{D_0}$ over U is

$$\begin{aligned}(F,A)_{D_0} = \Big\{ & \Big\{\Big(\frac{e_1}{0.3},p,0\Big),\{u_1\}\Big\},\Big\{\Big(\frac{e_1}{0.3},q,0\Big),\{u_1,u_3\}\Big\},\Big\{\Big(\frac{e_1}{0.3},r,0\Big),\{u_1,u_2\}\Big\},\\ & \Big\{\Big(\frac{e_2}{0.5},p,0\Big),\{u_1,u_3\}\Big\},\Big\{\Big(\frac{e_2}{0.5},q,0\Big),\{u_1\}\Big\},\Big\{\Big(\frac{e_2}{0.5},r,0\Big),\{u_1,u_2,u_3\}\Big\},\\ & \Big\{\Big(\frac{e_3}{0.8},p,0\Big),\{u_1,u_3,u_4\}\Big\},\Big\{\Big(\frac{e_3}{0.8},q,0\Big),\{u_1,u_4\}\Big\},\Big\{\Big(\frac{e_3}{0.8},r,0\Big),\{u_1,u_3\}\Big\}\Big\}.\end{aligned} \tag{3.10}$$

Proposition 3.13. *If $(F,A)_D$ is an FPSES over U, then $((F,A)_D^c)^c = (F,A)_D$.*

Proof. By using Definition 3.7, we have $F_D^c : \sim A \rightarrow P(U)$ is a mapping given by $F_D^c(\alpha) = U - F_D(\sim \alpha)$, for all $\alpha \in \sim A$ and $\sim A \subset \{D^c \times X \times O\}$. Now, $(F_D^c)^c : \sim(\sim A) \rightarrow P(U)$ is a mapping given by $(F_D^c)^c(\alpha) = U - F_D^c(\sim(\sim \alpha))$, for all $\alpha \in \sim(\sim A)$ and $\sim(\sim A) \subset \{(D^c)^c \times X \times O\}$, and since $(D^c)^c = D$ so the proof is complete. □

Definition 3.14. The *union* of two FPSESs $(F,A)_D$ and $(G,B)_k$ over U, denoted by $(F,A)_D \tilde{\cup} (G,B)_K$, is the FPSES $(H,C)_R$ such that $C = R \times X \times O$ where $R = D \cup K$ and for all $\varepsilon \in C$,

$$H_R(\varepsilon) = F_D(\varepsilon) \cup G_K(\varepsilon). \tag{3.11}$$

Example 3.15. Consider Example 3.3. Let $D = \{e_1/0.6, e_2/0.3, e_3/0.2\}$ be a fuzzy subset over E, and let $K = \{e_1/0.3, e_2/0.2, e_3/0.1\}$ be another fuzzy subset over E:

$$\begin{aligned}A_D = \Big\{ & \Big(\frac{e_1}{0.6},p,1\Big),\Big(\frac{e_2}{0.3},p,1\Big),\Big(\frac{e_3}{0.2},p,0\Big),\Big(\frac{e_1}{0.6},q,1\Big),\Big(\frac{e_1}{0.6},q,0\Big),\Big(\frac{e_2}{0.3},q,1\Big),\\ & \Big(\frac{e_3}{0.2},q,1\Big),\Big(\frac{e_1}{0.6},r,1\Big),\Big(\frac{e_2}{0.3},r,0\Big)\Big\},\\ B_K = \Big\{ & \Big(\frac{e_1}{0.3},p,1\Big),\Big(\frac{e_2}{0.2},p,1\Big),\Big(\frac{e_3}{0.1},p,0\Big),\Big(\frac{e_1}{0.3},q,1\Big),\Big(\frac{e_2}{0.2},q,1\Big),\Big(\frac{e_3}{0.1},q,1\Big),\\ & \Big(\frac{e_1}{0.3},r,1\Big),\Big(\frac{e_2}{0.2},r,1\Big),\Big(\frac{e_2}{0.2},r,0\Big)\Big\}.\end{aligned} \tag{3.12}$$

Suppose $(F,A)_D$ and $(G,B)_K$ are two FPSESs over the same U given by

$$\begin{aligned}(F,A)_D = \Big\{ & \Big\{\Big(\frac{e_1}{0.6},p,1\Big),\{u_2,u_3,u_4\}\Big\},\Big\{\Big(\frac{e_1}{0.6},q,1\Big),\{u_4\}\Big\},\Big\{\Big(\frac{e_1}{0.6},r,1\Big),\{u_3,u_4\}\Big\},\\ & \Big\{\Big(\frac{e_2}{0.3},p,1\Big),\{u_2,u_4\}\Big\},\Big\{\Big(\frac{e_2}{0.3},q,1\Big),\{u_1,u_2,u_3,u_4\}\Big\},\Big\{\Big(\frac{e_3}{0.2},q,1\Big),\{u_2,u_3\}\Big\},\\ & \Big\{\Big(\frac{e_1}{0.6},q,0\Big),\{u_1,u_3\}\Big\},\Big\{\Big(\frac{e_2}{0.3},r,0\Big),\{u_2,u_3\}\Big\},\Big\{\Big(\frac{e_3}{0.2},p,0\Big),\{u_2,u_3\}\Big\}\Big\},\end{aligned}$$

$$(G,B)_K=\left\{\left\{\left(\frac{e_1}{0.3},p,1\right),\{u_2,u_3\}\right\},\left\{\left(\frac{e_1}{0.3},q,1\right),\{u_2,u_3,u_4\}\right\},\left\{\left(\frac{e_1}{0.3},r,1\right),\{u_1,u_3,u_4\}\right\},\right.$$
$$\left\{\left(\frac{e_2}{0.2},p,1\right),\{u_2,u_4\}\right\},\left\{\left(\frac{e_2}{0.2},q,1\right),\{u_3,u_4\}\right\},\left\{\left(\frac{e_2}{0.2},r,1\right),\{u_4\}\right\},$$
$$\left.\left\{\left(\frac{e_3}{0.1},q,1\right),\{u_1,u_2,u_3\}\right\},\left\{\left(\frac{e_3}{0.1},p,0\right),\{u_2,u_3\}\right\},\left\{\left(\frac{e_2}{0.2},r,0\right),\{u_1,u_2,u_3\}\right\}\right\}. \tag{3.13}$$

By using the operator max which is the basic fuzzy union, we get

$$(F,A)_D\,\tilde{\cup}\,(G,B)_K=(H,C)_R=\left\{\left\{\left(\frac{e_1}{0.6},p,1\right),\{u_2,u_3,u_4\}\right\},\left\{\left(\frac{e_1}{0.6},q,1\right),\{u_2,u_3,u_4\}\right\},\right.$$
$$\left\{\left(\frac{e_1}{0.6},r,1\right),\{u_1,u_3,u_4\}\right\},\left\{\left(\frac{e_2}{0.3},p,1\right),\{u_2,u_4\}\right\},$$
$$\left\{\left(\frac{e_2}{0.3},q,1\right),U\right\},\left\{\left(\frac{e_2}{0.3},r,1\right),\{u_4\}\right\},$$
$$\left\{\left(\frac{e_3}{0.2},q,1\right),\{u_1,u_2,u_3\}\right\},\left\{\left(\frac{e_1}{0.6},q,0\right),\{u_1,u_3\}\right\},$$
$$\left.\left\{\left(\frac{e_2}{0.3},r,0\right),\{u_2,u_3,u_4\}\right\},\left\{\left(\frac{e_3}{0.2},p,0\right),\{u_2,u_3\}\right\}\right\}. \tag{3.14}$$

Proposition 3.16. *If $(F,A)_D$, $(G,B)_K$, and $(H,C)_R$ are three FPSESs over U, then*

(i) $(F,A)_D\,\tilde{\cup}\,(G,B)_K=(G,B)_K\,\tilde{\cup}\,(F,A)_D$,

(ii) $(F,A)_D\,\tilde{\cup}\,((G,B)_K\,\tilde{\cup}\,(H,C)_R)=((F,A)_D\,\tilde{\cup}\,(G,B)_K)\,\tilde{\cup}\,(H,C)_R$.

Proof. (i) By using Definition 3.14 we have the union of two FPSESs $(F,A)_D$, and $(G,B)_k$ is the FPSES $(H,C)_R$ such that $C=R\times X\times O$ where $R=D\cup K$ and for all $\varepsilon\in C$, $H_R(\varepsilon)=F_D(\varepsilon)\cup G_K(\varepsilon)$. Now, since union for fuzzy sets and crisp sets are commutative, then $R=K\cup D$ and $H_R(\varepsilon)=F_K(\varepsilon)\cup G_D(\varepsilon)$, and this gives the result.

(ii) We use the fact that union for fuzzy sets and crisp sets is associative. □

Definition 3.17. The *intersection* of two FPSESs $(F,A)_D$ and $(G,B)_K$ over U, denoted by $(F,A)_D\,\tilde{\cap}\,(G,B)_K$, is the FPSES $(H,C)_R$ such that $C=R\times X\times O$ where $R=D\cap K$ and for all $\varepsilon\in C$,

$$H_R(\varepsilon)=F_D(\varepsilon)\cap G_K(\varepsilon). \tag{3.15}$$

Example 3.18. Consider Example 3.15. Then by using the basic fuzzy intersection (minimum) we have

$$(F,A)_D\tilde{\cap}(G,B)_K=(H,C)_R$$
$$=\left\{\left\{\left(\frac{e_1}{0.2},p,1\right),\{u_2,u_3\}\right\},\left\{\left(\frac{e_1}{0.3},q,1\right),\{u_4\}\right\},\left\{\left(\frac{e_1}{0.3},r,1\right),\{u_3,u_4\}\right\},\right.$$

$$\left\{\left(\frac{e_2}{0.2},p,1\right),\{u_2,u_4\}\right\},\left\{\left(\frac{e_2}{0.2},q,1\right),\{u_3,u_4\}\right\},\left\{\left(\frac{e_2}{0.2},r,1\right),\{u_4\}\right\},$$
$$\left\{\left(\frac{e_3}{0.1},q,1\right),\{u_2,u_3\}\right\},\left\{\left(\frac{e_1}{0.6},q,0\right),\{u_1,u_3\}\right\},$$
$$\left\{\left(\frac{e_2}{0.2},r,0\right),\{u_2,u_3\}\right\},\left\{\left(\frac{e_3}{0.1},p,0\right),\{u_2,u_3\}\right\}\bigg\}. \tag{3.16}$$

Proposition 3.19. *If $(F,A)_D$, $(G,B)_K$ and $(H,C)_R$ are three FPSESs over U, then*

(i) $(F,A)_D\,\widetilde{\cap}\,(G,B)_k=(G,B)_k\,\widetilde{\cap}\,(F,A)_D$,

(ii) $(F,A)_D\,\widetilde{\cap}\,((G,B)_K\,\widetilde{\cap}\,(H,C)_R)=((F,A)_D\,\widetilde{\cap}\,(G,B)_K)\,\widetilde{\cap}\,(H,C)_R$.

Proof. (i) By using Definition 3.17 we have the intersection of two FPSESs $(F,A)_D$ and $(G,B)_k$ is the FPSES $(H,C)_R$ such that $C=R\times X\times O$ where $R=D\cap K$ and for all $\varepsilon\in C$, $H_R(\varepsilon)=F_D(\varepsilon)\cap G_K(\varepsilon)$. Now, since intersection for fuzzy sets and crisp sets is commutative, then $R=K\cap D$ and $H_R(\varepsilon)=F_K(\varepsilon)\cap G_D(\varepsilon)$, and this gives the result.

(ii) We use the fact that intersection for fuzzy sets and crisp sets is associative. □

Proposition 3.20. *If $(F,A)_D$, $(G,B)_K$ and $(H,C)_R$ are three FPSESs over U, then*

(i) $(F,A)_D\widetilde{\cup}((G,B)_K\widetilde{\cap}(H,C)_R)=((F,A)_D\widetilde{\cup}(G,B)_K)\widetilde{\cap}((F,A)_D\widetilde{\cup}(H,C)_R)$,

(ii) $(F,A)_D\widetilde{\cap}((G,B)_K\widetilde{\cup}(H,C)_R)=((F,A)_D\widetilde{\cap}(G,B)_K)\widetilde{\cup}((F,A)_D\widetilde{\cap}(H,C)_R)$.

Proof. We prove (i), and we can use the same method to prove (ii). □

Let $M_T(\varepsilon)=G_K(\varepsilon)\cap H_R(\varepsilon)$ where $T=K\cap R$, and let $N_S(\varepsilon)=F_D(\varepsilon)\cup M_T(\varepsilon)$ where $S=D\cup T$. So

$$\begin{aligned} S&=D\cup(K\cap R)=(D\cup K)\cap(D\cup R),\\ F_D(\varepsilon)\cup M_T(\varepsilon)&=F_D(\varepsilon)\cup(G_K(\varepsilon)\cap H_R(\varepsilon))\\ &=(F_D(\varepsilon)\cup G_K(\varepsilon))\cap(F_D(\varepsilon)\cup H_R(\varepsilon)). \end{aligned} \tag{3.17}$$

Definition 3.21. If $(F,A)_D$ and $(G,B)_K$ are two FPSESs over U, then $(F,A)_D$ AND $(G,B)_K$, denoted by $(F,A)_D\wedge(G,B)_K$, is defined by

$$(F,A)_D\wedge(G,B)_K=(H,A\times B)_R, \tag{3.18}$$

such that $H(\alpha,\beta)_R=F_D(\alpha)\cap G_R(\beta)$, for all $(\alpha,\beta)\in A\times B$, where $R=D\times K$.

Example 3.22. Consider Example 3.3. Let

$$\begin{aligned} A_D&=\left\{\left(\frac{e_1}{0.6},p,1\right),\left(\frac{e_3}{0.1},p,1\right),\left(\frac{e_1}{0.6},r,1\right),\left(\frac{e_2}{0.4},r,0\right)\right\},\\ B_K&=\left\{\left(\frac{e_1}{0.5},p,1\right),\left(\frac{e_1}{0.5},q,1\right),\left(\frac{e_2}{0.2},r,0\right)\right\}. \end{aligned} \tag{3.19}$$

Suppose $(F,A)_D$ and $(G,B)_K$ are two FPSESs over the same U such that

$$\begin{aligned}(F,A)_D = \Bigg\{&\left\{\left(\frac{e_1}{0.6},p,1\right),\{u_2,u_3,u_4\}\right\},\left\{\left(\frac{e_1}{0.6},r,1\right),\{u_2,u_4\}\right\},\left\{\left(\frac{e_2}{0.4},r,0\right),\{u_2\}\right\},\\ &\left\{\left(\frac{e_3}{0.1},p,1\right),\{u_2,u_3\}\right\}\Bigg\},\\ (G,B)_R = \Bigg\{&\left\{\left(\frac{e_1}{0.5},p,1\right),\{u_4\}\right\},\left\{\left(\frac{e_1}{0.5},q,1\right),\{u_2,u_3\}\right\},\left\{\left(\frac{e_2}{0.2},r,0\right),\{u_3\}\right\}\Bigg\}.\end{aligned} \tag{3.20}$$

By using the basic fuzzy intersection (minimum) we have

$$(F,A)_D \wedge (G,B)_K = (H,A\times B)_R, \tag{3.21}$$

where

$$\begin{aligned}(H,A\times B)_R = \Bigg\{&\left\{\left(\left(\frac{e_1}{0.6},p,1\right),\left(\frac{e_1}{0.5},p,1\right),\{u_4\}\right)\right\},\left\{\left(\left(\frac{e_1}{0.6},p,1\right),\left(\frac{e_1}{0.5},q,1\right),\{u_2,u_3\}\right)\right\},\\ &\left\{\left(\left(\frac{e_1}{0.6},p,1\right),\left(\frac{e_2}{0.2},r,0\right),\{u_3\}\right)\right\},\left\{\left(\left(\frac{e_1}{0.6},r,1\right),\left(\frac{e_1}{0.5},p,1\right),\{u_4\}\right)\right\},\\ &\left\{\left(\left(\frac{e_1}{0.6},r,1\right),\left(\frac{e_1}{0.5},q,1\right),\{u_2\}\right)\right\},\left\{\left(\left(\frac{e_1}{0.6},r,1\right),\left(\frac{e_2}{0.2},r,0\right),\emptyset\right)\right\},\\ &\left\{\left(\left(\frac{e_1}{0.6},p,1\right),\left(\frac{e_1}{0.5},p,1\right),\{u_4\}\right)\right\},\left\{\left(\left(\frac{e_2}{0.4},r,0\right),\left(\frac{e_1}{0.5},p,1\right),\emptyset\right)\right\},\\ &\left\{\left(\left(\frac{e_2}{0.4},r,0\right),\left(\frac{e_1}{0.5},q,1\right),\{u_2\}\right)\right\},\left\{\left(\left(\frac{e_2}{0.4},r,0\right),\left(\frac{e_1}{0.2},r,0\right),\emptyset\right)\right\},\\ &\left\{\left(\left(\frac{e_3}{0.1},p,1\right),\left(\frac{e_1}{0.5},p,1\right),\emptyset\right)\right\},\left\{\left(\left(\frac{e_3}{0.1},p,1\right),\left(\frac{e_1}{0.5},q,1\right),\{u_2,u_3\}\right)\right\},\\ &\left\{\left(\left(\frac{e_3}{0.1},q,1\right),\left(\frac{e_2}{0.2},n,0\right),\{u_3\}\right)\right\}\Bigg\}.\end{aligned} \tag{3.22}$$

Definition 3.23. If $(F,A)_D$ and $(G,B)_K$ are two FPSESs over U, then $(F,A)_D$ OR $(G,B)_K$, denoted by $(F,A)_D \vee (G,B)_K$, is defined by

$$(F,A)_D \vee (G,B)_K = (H,A\times B)_R, \tag{3.23}$$

such that $H(\alpha,\beta)_R = F_D(\alpha)\cup G_R(\beta)$, for all $(\alpha,\beta)\in A\times B$, where $R = D\times K$.

Example 3.24. Consider Example 3.22. By using the basic fuzzy union (maximum) we have

$$
\begin{aligned}
&(F,A)_D \vee (G,B)_K \\
&\quad = (H, A \times B)_R \\
&\quad = \Bigg\{\Bigg\{\Bigg(\left(\frac{e_1}{0.6},p,1\right),\left(\frac{e_1}{0.5},p,1\right),\{u_2,u_3,u_4\}\Bigg)\Bigg\},\Bigg\{\Bigg(\left(\frac{e_1}{0.6},p,1\right),\left(\frac{e_1}{0.5},q,1\right),\{u_2,u_3,u_4\}\Bigg)\Bigg\}, \\
&\qquad \Bigg\{\Bigg(\left(\frac{e_1}{0.6},p,1\right),\left(\frac{e_2}{0.2},r,0\right),\{u_2,u_3,u_4\}\Bigg)\Bigg\},\Bigg\{\Bigg(\left(\frac{e_1}{0.6},r,1\right),\left(\frac{e_1}{0.5},p,1\right),\{u_2,u_4\}\Bigg)\Bigg\}, \\
&\qquad \Bigg\{\Bigg(\left(\frac{e_1}{0.6},r,1\right),\left(\frac{e_1}{0.5},q,1\right),\{u_2,u_3,u_4\}\Bigg)\Bigg\},\Bigg\{\Bigg(\left(\frac{e_1}{0.6},r,1\right),\left(\frac{e_2}{0.2},r,0\right),\{u_2,u_3,u_4\}\Bigg)\Bigg\}, \\
&\qquad \Bigg\{\Bigg(\left(\frac{e_1}{0.6},p,1\right),\left(\frac{e_1}{0.5},p,1\right),\{u_2,u_4\}\Bigg)\Bigg\},\Bigg\{\Bigg(\left(\frac{e_2}{0.4},r,0\right),\left(\frac{e_1}{0.5},p,1\right),\{u_2,u_3,u_4\}\Bigg)\Bigg\}, \\
&\qquad \Bigg\{\Bigg(\left(\frac{e_2}{0.4},r,0\right),\left(\frac{e_1}{0.5},q,1\right),\{u_2,u_3\}\Bigg)\Bigg\},\Bigg\{\Bigg(\left(\frac{e_2}{0.4},r,0\right),\left(\frac{e_1}{0.2},r,0\right),\{u_2,u_3\}\Bigg)\Bigg\}, \\
&\qquad \Bigg\{\Bigg(\left(\frac{e_3}{0.1},p,1\right),\left(\frac{e_1}{0.5},p,1\right),\{u_2,u_3,u_4\}\Bigg)\Bigg\},\Bigg\{\Bigg(\left(\frac{e_3}{0.1},p,1\right),\left(\frac{e_1}{0.5},q,1\right),\{u_2,u_3\}\Bigg)\Bigg\}, \\
&\qquad \Bigg\{\Bigg(\left(\frac{e_3}{0.1},p,1\right),\left(\frac{e_2}{0.2},r,0\right),\{u_2,u_3\}\Bigg)\Bigg\}\Bigg\}.
\end{aligned}
\tag{3.24}
$$

4. An Application of Fuzzy Parameterized Soft Expert Set

Ahkhazaleh and Salleh [14] applied the theory of soft expert sets to solve a decision-making problem. In this section, we present an application of FPSES in a decision-making problem by generalizing Ahkhazaleh and Salleh's Algorithm to be compatible with our work. We consider the following problem.

Example 4.1. Assume that a hotel chain wants to fill a position for the management of the chain. There are five candidates who form the universe $U = \{u_1, u_2, u_3, u_4, u_5\}$. The hiring committee decided to have a set of parameters, $E = \{e_1, e_2, e_3\}$, where the parameters e_i $(i = 1, 2, 3)$ stand for "computer knowledge," "experience," and "good speaking," respectively. Let $X = \{p, q, r\}$ be a set of experts (committee members). Suppose

$$
\begin{aligned}
(F,Z) = \Bigg\{&\Bigg\{\left(\frac{e_1}{0.1},p,1\right),\{u_1,u_2,u_3,u_4\}\Bigg\},\Bigg\{\left(\frac{e_1}{0.1},q,1\right),\{u_2,u_3,u_4\}\Bigg\}, \\
&\Bigg\{\left(\frac{e_1}{0.1},r,1\right),\{u_3,u_4,u_5\}\Bigg\},\Bigg\{\left(\frac{e_2}{0.3},p,1\right),\{u_2,u_3,u_4,u_5\}\Bigg\}, \\
&\Bigg\{\left(\frac{e_2}{0.3},q,1\right),\{u_2,u_3,u_4\}\Bigg\},\Bigg\{\left(\frac{e_2}{0.3},r,1\right),\{u_5\}\Bigg\}, \\
&\Bigg\{\left(\frac{e_3}{0.4},p,1\right),\{u_2,u_5\}\Bigg\},\Bigg\{\left(\frac{e_3}{0.4},q,1\right),\{u_2,u_3\}\Bigg\},
\end{aligned}
$$

$$\left\{\left(\frac{e_3}{0.4},r,1\right),\{u_3,u_4\}\right\},\left\{\left(\frac{e_1}{0.1},p,0\right),\{u_5\}\right\},$$
$$\left\{\left(\frac{e_1}{0.1},q,0\right),\{u_1,u_5\}\right\},\left\{\left(\frac{e_1}{0.1},r,0\right),\{u_1,u_2\}\right\},$$
$$\left\{\left(\frac{e_2}{0.3},p,0\right),\{u_1\}\right\},\left\{\left(\frac{e_2}{0.3},q,0\right),\{u_1,u_5\}\right\},$$
$$\left\{\left(\frac{e_2}{0.3},r,0\right),\{u_1,u_2,u_3,u_4\}\right\},\left\{\left(\frac{e_3}{0.4},p,0\right),\{u_1,u_3,u_4\}\right\},$$
$$\left\{\left(\frac{e_3}{0.4},q,0\right),\{u_1,u_4,u_5\}\right\},\left\{\left(\frac{e_3}{0.4},r,0\right),\{u_1,u_2,u_5\}\right\}\right\}. \tag{4.1}$$

In Tables 1 and 2 we present the agree-FPSES and disagree-FPSES, respectively, such that

$$\text{if } u_i \in F_1(\varepsilon) \text{ then } u_{ij} = 1, \text{ otherwise } u_{ij} = 0,$$
$$\text{if } u_i \in F_0(\varepsilon) \text{ then } u_{ij} = 1, \text{ otherwise } u_{ij} = 0,$$

where u_{ij} are the entries in Tables 1 and 2.

The following Algorithm 4.2 may be followed by the hotel chain to fill the position.

Algorithm 4.2. (1) Input the FPSES (F, A).

(2) Find an agree-FPSES and a disagree-FPSES.

(3) Find $c_j = \sum_{x\in X}\sum_i u_{ij}(\mu_E(e_i))$ for agree-FPSES.

(4) Find $k_j = \sum_{x\in X}\sum_i u_{ij}(\mu_E(e_i))$ for disagree-FPSES.

(5) Find $s_j = c_j - k_j$.

(6) Find m, for which $s_m = \max s_j$.

Then s_m is the optimal choice object. If m has more than one value, then anyone of them could be chosen by the hotel chain using its option. Now we use Algorithm 4.2 to find the best choice for the hotel chain to fill the position.

Then $\max s_j = s_3$, as shown in Table 3, so the committee will choose candidate u_3 for the job.

5. Weighted Fuzzy Parameterized Soft Expert Set

In this section we introduce the notion of weighted fuzzy parameterized soft expert sets and discuss its application to decision-making problem.

Definition 5.1. Let $F(U)$ be the set of all fuzzy parameterized soft expert sets in the universe U. Let E be a set of parameters and $A \subseteq X$. A *weighted fuzzy parameterized soft expert set* is a triple $\zeta = (F, A, \omega)$ where (F, A) is a fuzzy parameterized soft expert set over U, and $\omega : X \to [0,1]$ is a weight function specifying $w_j = \omega(\varepsilon_j)$ for each attribute $\varepsilon_j \in X$.

By definition, every fuzzy parameterized soft expert set can be considered as a weighted fuzzy parameterized soft expert set. This is an extension of the weighted fuzzy soft sets discussed in [16]. The notion of weighted fuzzy parameterized soft expert set provides a mathematical framework for modeling and analyzing the decision-making problems in

Table 1: Agree-FPSES.

U	u_1	u_2	u_3	u_4	u_5
$(e_1/0.1,\ p)$	1	1	1	1	0
$(e_2/0.3,\ p)$	0	1	1	1	0
$(e_3/0.4,\ p)$	0	0	1	1	1
$(e_1/0.1,\ q)$	0	1	1	1	1
$(e_2/0.3,\ q)$	0	1	1	1	0
$(e_3/0.4,\ q)$	0	0	0	0	1
$(e_1/0.1,\ r)$	0	1	0	0	1
$(e_2/0.3,\ r)$	0	1	1	0	0
$(e_3/0.4,\ r)$	0	0	1	1	0
$c_j = \sum_{x\in X}\sum_i u_{ij}(\mu_E(e_i))$	$c_1 = 0.1$	$c_2 = 1.2$	$c_3 = 1.9$	$c_4 = 1.6$	$c_5 = 1$

Table 2: Disagree-FPSES.

U	u_1	u_2	u_3	u_4	u_5
$(e_1/0.1,\ p)$	0	0	0	0	1
$(e_2/0.3,\ p)$	1	0	0	0	1
$(e_3/0.4,\ p)$	1	1	0	0	0
$(e_1/0.1,\ q)$	1	0	0	0	1
$(e_2/0.3,\ q)$	1	0	0	0	1
$(e_3/0.4,\ q)$	1	1	1	1	0
$(e_1/0.1,\ r)$	1	0	1	1	0
$(e_2/0.3,\ r)$	1	0	0	1	1
$(e_3/0.4,\ r)$	1	1	0	0	1
$k_j = \sum_{x\in X}\sum_i u_{ij}(\mu_E(e_i))$	$k_1 = 2.3$	$k_2 = 1.2$	$k_3 = 0.5$	$k_4 = 0.8$	$k_5 = 1.5$

Table 3

$c_j = \sum_{x\in X}\sum_i u_{ij}(\mu_E(e_i))$	$k_j = \sum_{x\in X}\sum_i u_{ij}(\mu_E(e_i))$	$s_j = c_j - k_j$
$c_1 = 0.1$	$k_1 = 2.3$	$s_1 = -2.2$
$c_2 = 1.2$	$k_2 = 1.2$	$s_2 = 0$
$c_3 = 1.9$	$k_3 = 0.5$	$s_3 = 1.4$
$c_4 = 1.6$	$k_4 = 0.8$	$s_4 = 0.8$
$c_5 = 1$	$k_5 = 1.5$	$s_5 = -0.5$

which all the choice experts may not be of equal importance. These differences between the importance of experts are characterized by the weight function in a weighted fuzzy parameterized soft expert set.

Example 5.2. Suppose that a hotel chain has imposed the following weights for the experts in Example 4.1. For the expert "p," $w_1 = 0.7$; for the expert "q," $w_2 = 0.6$; for the expert "r," $w_3 = 0.4$. Thus we have a weight function $\omega : X \to [0,1]$, and the fuzzy parameterized soft expert set (F, A) in Example 4.1 is changed into a weighted fuzzy parameterized soft expert set (WFPSES) $\zeta = (F, A, \omega)$.

Table 4: Agree-WFPSES.

U	u_1	u_2	u_3	u_4	u_5
$(e_1/0.1,\ p/0.7)$	1	1	1	1	0
$(e_2/0.3,\ p/0.7)$	0	1	1	1	0
$(e_3/0.4,\ p/0.7)$	0	0	1	1	1
$(e_1/0.1,\ q/0.6)$	0	1	1	1	1
$(e_2/0.3,\ q/0.6)$	0	1	1	1	0
$(e_3/0.4,\ q/0.6)$	0	0	0	0	1
$(e_1/0.1,\ r/0.4)$	0	1	0	0	1
$(e_2/0.3,\ r/0.4)$	0	1	1	0	0
$(e_3/0.4,\ r/0.4)$	0	0	1	1	0
$c_j = \sum_{x\in X}\sum_i u_{ij}(\mu_E(e_i))\omega(x)$	$c_1 = 0.07$	$c_2 = 0.68$	$c_3 = 1.08$	$c_4 = 0.96$	$c_5 = 0.62$

Table 5: disagree-WFPSES.

U	u_1	u_2	u_3	u_4	u_5
$(e_1/0.1,\ p/0.7)$	0	0	0	0	1
$(e_2/0.3,\ p/0.7)$	1	0	0	0	1
$(e_3/0.4,\ p/0.7)$	1	1	0	0	0
$(e_1/0.1,\ q/0.6)$	1	0	0	0	1
$(e_2/0.3,\ q/0.6)$	1	0	0	0	1
$(e_3/0.4,\ q/0.6)$	1	1	1	1	0
$((e_1/0.1),\ r/0.4)$	1	0	1	1	0
$(e_2/0.3,\ r/0.4)$	1	0	0	1	1
$(e_3/0.4,\ r/0.4)$	1	1	0	0	1
$k_j = \sum_{x\in X}\sum_i u_{ij}(\mu_E(e_i))\omega(x)$	$k_1 = 1.29$	$k_2 = 0.68$	$k_3 = 0.28$	$k_4 = 0.4$	$k_5 = 0.8$

In Tables 4 and 5 we present the agree-WFPSES and disagree-WFPSES respectively such that

$$\text{if } u_i \in F_1(\varepsilon) \text{ then } u_{ij} = 1, \text{ otherwise } u_{ij} = 0,$$

$$\text{if } u_i \in F_0(\varepsilon) \text{ then } u_{ij} = 1, \text{ otherwise } u_{ij} = 0,$$

where u_{ij} are the entries in Tables 4 and 5.

The following Algorithm 5.3 may be followed by the hotel chain to fill the position.

Algorithm 5.3. (1) Input the WFPSES (F, A).

(2) Find an agree-WFPSES and a disagree-WFPSES.

(3) Find $c_j = \sum_{x\in X}\sum_i u_{ij}(\mu_E(e_i))\omega(x)$ for agree-WFPSES.

(4) Find $k_j = \sum_{x\in X}\sum_i u_{ij}(\mu_E(e_i))\omega(x)$ for disagree-WFPSES.

(5) Find $s_j = c_j - k_j$.

(6) Find m, for which $s_m = \max s_j$.

Then s_m is the optimal choice object. If m has more than one value, then any one of them could be chosen by the hotel chain using its option. Now we use Algorithm 5.3 to find the best choice for the hotel chain to fill the position.

Then $\max s_j = s_4$, as shown in Table 6, so the committee will choose candidate u_4 for the job.

Table 6

$c_j = \sum_{x\in X} \sum_i u_{ij}(\mu_E(e_i))\omega(x)$	$k_j = \sum_{x\in X} \sum_i u_{ij}(\mu_E(e_i))\omega(x)$	$s_j = c_j - k_j$
$c_1 = 0.07$	$k_1 = 1.29$	$s_1 = -1.22$
$c_2 = 0.68$	$k_2 = 0.68$	$s_2 = 0$
$c_3 = 1.08$	$k_3 = 0.28$	$s_3 = 0.8$
$c_4 = 1.96$	$k_4 = 0.4$	$s_4 = 1.56$
$c_5 = 0.62$	$k_5 = 0.8$	$s_5 = -0.18$

Remark 5.4. By comparing the results obtained using Algorithms 4.2 and 5.3 we can see that giving more consideration to the expert (weight) might affect the result.

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