Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2012, Article ID 762516, 13 pages
doi:10.1155/2012/762516

Research Article

Numerical Simulation of the FitzHugh-Nagumo Equations

A. A. Soliman[1, 2]

[1] *Department of Mathematics, Faculty of Education (AL-Arish), Suez Canal University, AL-Arish 45111, Egypt*
[2] *Department of Mathematics, Bisha Faculty of Science and Arts, King Khalid University, P.O. Box. 344, Bisha, 61922, Saudi Arabia*

Correspondence should be addressed to A. A. Soliman, asoliman_99@yahoo.com

Received 2 March 2012; Accepted 1 June 2012

Academic Editor: Irena Rachůnková

The variational iteration method and Adomian decomposition method are applied to solve the FitzHugh-Nagumo (FN) equations. The two algorithms are illustrated by studying an initial value problem. The obtained results show that only few terms are required to deduce approximated solutions which are found to be accurate and efficient.

1. Introduction

The pioneering work of Hodgkin and Huxley [1], and subsequent investigations, has established that good mathematical models for the conduction of nerve impulses along an axon can be given. These models take the form of a system of ordinary differential equations, coupled to a diffusion equation. Simpler models, which seem to describe the qualitative behavior, have been proposed by FitzHugh [2] and Nagumo [3]. This paper is devoted to the study of the FitzHugh-Nagumo (FN) system:

$$\begin{aligned} v_t(x,t) &= v_{xx}(x,t) - f(v(x,t)) - w(x,t), \\ w_t(x,t) &= bv(x,t) - \gamma w(x,t), \end{aligned} \tag{1.1}$$

where b and γ are positive constants and $f(v(x,t))$ is nonlinear function. Existence and uniqueness for this system is given in 1978 by Rauch and Smoller [4], in which they showed that small solutions $v(x,t)$ decay to 0 as $t \to \infty$ and large pulses produce a traveling wave. We consider the FN equations in the following form

$$\begin{aligned} v_t(x,t) &= v_{xx}(x,t) - f(v(x,t)) - w(x,t), \\ w_t(x,t) &= bv(x,t), \end{aligned} \tag{1.2}$$

and the function $f(v(x,t))$ is given by Mckean [5] such that:

$$f(v(x,t)) = v(x,t) - H(v(x,t) - a), \quad 0 \le a \le \frac{1}{2}, \tag{1.3}$$

where H is the Heaviside step function

$$H(s) = \begin{cases} 0 & s < 0, \\ 1 & s \ge 0. \end{cases} \tag{1.4}$$

The exact solution of this system is given by:

$$v(x,t) = \begin{cases} a\ e^{\alpha_1 z}, & z \le 0 \\ \left(a - \dfrac{1}{p_1'}\right)e^{\alpha_1 z} - \dfrac{1}{p_2'}e^{\alpha_2 z} - \dfrac{1}{p_3'}e^{\alpha_3 z}, & 0 \le z \le z_1 \\ \dfrac{e^{-\alpha_2 z_1} - 1}{p_2'}e^{\alpha_2 z} + \dfrac{e^{-\alpha_3\ z_1} - 1}{p_3'}e^{\alpha_3 z}, & z_1 \le z, \end{cases} \tag{1.5}$$

$$w(x,t) = v_{xx}(x,t) - v_t(x,t) - f(v(x,t)), \tag{1.6}$$

where $z = x + ct$, c is the speed of the traveling wave and $\alpha_i,\ i = 1,2,3$ are the zeros of the polynomial

$$\begin{aligned} p(\alpha) &= \alpha^3 - c\alpha^2 - \alpha - \frac{b}{c}, \\ p_i' &= p'(\alpha_i), \quad i = 1,2,3. \end{aligned} \tag{1.7}$$

A numerical scheme for FN equations [6] by collocation method and the "Hopscotch" finite difference scheme first proposed by Gordon [7], and further developed by Gourlay and McGuire [8, 9]. Other possible schemes which were considered are (i) finite difference schemes [10], (ii) Galerkin-type schemes [11], and (iii) collocation schemes with quadratic and cubic splines [6]. In this paper, we use the variational iteration and Adomian decomposition methods to find the numerical solutions of the FN equations which will be useful in numerical studies. In our numerical study we consider the case $b = 0.1$ and $a = 0.3$, also

$$\begin{aligned}
&c = 0.7122, \\
&\alpha_1 = 1.46192629534582, \\
&\alpha_2 = -0.1639653991443764, \\
&\alpha_3 = -0.5857608638090818, \\
&z_1 = 4.5976770121482735,
\end{aligned} \tag{1.8}$$

with these parameters now we can use the exact travelling wave solution (1.5) to test the suggested numerical methods.

2. The Formalism

We introduce the main points of each of the two methods, where details can be found in [12–37].

2.1. *The Variational Iteration Method (VIM)*

The VIM is the general Lagrange method, in which an extremely accurate approximation at some special point can be obtained, but not an analytical solution. To illustrate the basic idea of the VIM we consider the following general partial differential equation:

$$L_t u(x,t) + L_x u(x,t) + N u(x,t) + g(t,x) = 0, \tag{2.1}$$

where L_t and L_x are linear operators of t and x respectively, and N is a nonlinear operator. According to the VIM, we can expressed the following correction functional in t-, and x-directions, respectively, as follows:

$$u_{n+1}(x,t) = u_n(x,t) + \int_{t_0}^{t} \lambda\bigl(L_s u_n(x,s) + (L_x + N)\tilde{u}_n(x,s) + g(x,s)\bigr)ds, \tag{2.2a}$$

$$u_{n+1}(x,t) = u_n(x,t) + \int_{x_0}^{x} \mu\bigl(L_s u_n(t,s) + (L_t + N)\tilde{u}_n(t,x) + \mathrm{g}(t,s)\bigr)ds, \tag{2.2b}$$

where λ and μ are general Lagrange multipliers, which can be identified optimally via the variational theory, and $\tilde{u}_n(x,t)$ are restricted variations which mean that $\delta\tilde{u}_n(x,t) = 0$. By this method, it is required first to determine Lagrange multipliers λ and μ that will be identified optimally. The successive approximations $u_{n+1}(x,t)$, $n \geq 0$ of the solution $u(x,t)$ will be readily obtained upon using the determined Lagrange multipliers and any selective function $u_0(x,t)$. Consequently, the solution is given by

$$u(x,t) = \lim_{n\to\infty} u_n(x,t). \tag{2.3}$$

The above analysis yields the following theorem.

Theorem 2.1. *The VIM solution of the partial differential equation* (2.1) *can be determined by* (2.3) *with the iterations* (2.2a) *or* (2.2b).

2.2. Adomian Decomposition Method (ADM)

Applying the inverse operator $L^{-1}(\cdot) = \int_0^t (\cdot)dt$ to both sides of (2.1) and using the initial condition, we get

$$\begin{aligned} u_0(x,t) &= u(x,0), \\ u_{n+1}(x,t) &= \int_0^t \left(-L_x\, u_n(x,t) - A_n - g(x,t)\right)dt, \quad n \geq 0, \end{aligned} \tag{2.4}$$

where the nonlinear operator $N(u) = \sum_{n=0}^{\infty} A_n$ is the Adomian polynomial determined by

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left(\sum_{i=0}^{\infty}\lambda^i\, u_i(x,t)\right)\Bigg|_{\lambda=0}, \qquad n = 0,1,2\ldots. \tag{2.5}$$

We next decompose the unknown function $u(x,t)$ by a sum of components defined by the following decomposition series

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t). \tag{2.6}$$

The above analysis yields the following theorem

Theorem 2.2. *The ADM solution of the partial differential equation* (2.1) *can be determined by the series* (2.6) *with the iterations* (2.4).

3. Applications

We solve the FN equations using the two methods VIM and ADM.

3.1. The VIM for the FN Equations

Consider the FN equations in the form

$$\begin{gathered} v_t - v_{xx} + f(v) + w = 0, \\ w_t - bv = 0. \end{gathered} \tag{3.1}$$

Then the VIM formulae take the forms

$$\begin{gathered} v_{n+1}(x,t) = v_n(x,t) + \int_0^t \lambda\ \left(v_s(x,s) - \tilde{v}_{xx}(x,s) + f(\tilde{v}(x,s)) + \tilde{w}(x,s)\right)ds, \\ w_{n+1}(x,t) = w_n(x,t) + \int_0^t \mu(w_s(x,s) - b\tilde{v}(x,s)ds, \end{gathered} \tag{3.2}$$

where $v_0(x,t) = v(x,0)$, $w_0(x,t) = w(x,0)$ and $n \geq 0$. This yields the stationary conditions

$$\lambda'(s) = 0, \qquad \lambda + 1|_{s=t} = 0, \qquad \mu'(s) = 0, \qquad \mu + 1\big|_{s=t} = 0, \tag{3.3}$$

Hence, the Lagrange multipliers are

$$\lambda(s) = \ \mu(s) = -1. \tag{3.4}$$

Substituting these values of Lagrange multipliers into the functional correction (3.2) gives the iterations formulae

$$\begin{gathered} v_{n+1}(x,t) = v_n(x,t) + \int_0^t \lambda\left(v_s(x,s) - v_{xx}(x,s) + f(v(x,s)) + w(x,s)\right)ds, \\ w_{n+1}(x,t) = w_n(x,t) + \int_0^t \mu\ (w_s(x,s) - bv(x,s)ds. \end{gathered} \tag{3.5}$$

We start with initial approximations as follows

$$\begin{gathered} v_0(x,t) = \begin{cases} a\ e^{\alpha_1 x}, & x \leq 0, \\ \left(a - \dfrac{1}{p_1'}\right)e^{\alpha_1 x} - \dfrac{1}{p_2'}e^{\alpha_2 x} - \dfrac{1}{p_3'}e^{\alpha_3 x}, & 0 \leq x \leq z_1, \\ \dfrac{e^{-\alpha_2\ z_1} - 1}{p_2'}e^{\alpha_2 x} + \dfrac{e^{-\alpha_3 z_1} - 1}{p_3'}e^{\alpha_3 x}, & z_1 \leq x, \end{cases} \\ w_0(x,t) = (v_0(x,t))_{xx} - (v_0(x,t))_t - f(v_0(x,t)), \end{gathered} \tag{3.6}$$

and then the first iterations are

$$v_1(x,t) = \begin{cases} v11, & z \le 0, \\ v12, & 0 \le z \le z_1, \\ v13 & z_1 \le z, \end{cases}$$

$$\begin{aligned}
v11 &= 0.312355e^{1.46193x}(0.960445 + \ t), \\
v12 &= e^{-0.749726x}\Big(e^{0.585761x}(1.45816 - 0.170279t) + e^{2.21165\ x}(-0.000361868 - 0.000376771t) \\
&\qquad +e^{0.163965x}(-1.1578 + 0.483011t)\Big)\ , \\
v13 &= e^{-0.749726x}\Big(e^{0.163965x}(15.9522 - 6.65492t) + e^{0.585761x}(-1.64071 + 0.191596t)\Big),
\end{aligned}$$

$$w_1(x,t) = \begin{cases} w11, & z \le 0 \\ w12, & 0 \le z \le z_1 \\ w13 & z_1 \le z \end{cases}$$

$$\begin{aligned}
w11 &= 0.03\ e^{1.46193x}(0.960445 + \ t), \\
w12 &= e^{-0.749726x}\Big(e^{0.749726\ x} + e^{0.16396\ x}(0.277531 - 0.11578\ t) \\
&\qquad + e^{2.21165x}(-0.000034755 - 0.0000361868\ t) + e^{0.585761x}(-1.24868 + 0.145816\ t), \\
w13 &= e^{\substack{-0.749726 \\ x}}\ \Big(e^{0.585761x}(1.405 - 0.164071\ t) + e^{0.163965x}(-3.82383 + 1.59522t)\Big),
\end{aligned} \tag{3.7}$$

and so on.

The VIM produces the solutions $v(x,t),\ w(x,t)$ as follows

$$v(x,t) = \lim_{n\to\infty} v_n(x,t), \qquad w(x,t) = \lim_{n\to\infty} w_n(x,t), \tag{3.8}$$

where $v_n(x,t),\ w_n(x,t)$, will be determined in a recursive manner.

3.2. The ADM for the FN Equations

Consider the FN equations in the following form:

$$Lv = v_{xx} - f(v) - w, \tag{3.9}$$

$$Lw = bv, \tag{3.10}$$

where $L(\cdot) = \partial(\cdot)/\partial t$. Operating by $L^{-1}(\cdot) = \int_0^t (\cdot)dt$ on both sides of (3.9), we get

$$v(x,t) = v(x,0) + \int_0^t \left(v_{xx}(x,t) - f(v(x,t)) - w(x,t)\right)dt,$$
$$w(x,t) = w(x,0) + \int_0^t (bv(x,t)dt. \tag{3.11}$$

The ADM assumes that the unknown functions $v(x,t)$ and $w(x,t)$ can be expressed by an infinite series in the forms

$$v(x,t) = \sum_{n=0}^{\infty} v_n(x,t),$$
$$w(x,t) = \sum_{n=0}^{\infty} w_n(x,t), \tag{3.12}$$

where $v_n(x,t)$,and $w_n(x,t)$ can be determined by using the recurrence relations:

$$v_{n+1}(x,t) = \int_0^t \left(v_{nxx}(x,t) - f(v_n(x,t)) - w_n(x,t)\right)dt,$$
$$w_{n+1}(x,t) = \int_0^t (bv_n(x,t)dt, \quad n = 0, 1, \ldots, \tag{3.13}$$

where

$$f(v_n(x,t)) = \begin{cases} v_n(x,t), & v_n(x,t) < a, \\ v_n(x,t) - 1 & v_n(x,t) \geq a \end{cases} \tag{3.14}$$

such that

$$v_0(x,t) = v(x,0),$$
$$w_0(x,t) = w(x,0). \tag{3.15}$$

Then the first iterations are

$$v_1(x,t) = \begin{cases} 0.312355e^{1.46193}, & z \le 0 \\ t + \left(-1 + 0.483011e^{-0.585761x} - 0.170279e^{-0.163965x} \right. & \\ \qquad \left. -0.000376771e^{1.46193x}\right)t, & 0 \le z \le z_1 \\ \left(-6.65492e^{-0.585761x} + 0.191596e^{-0.163965x}\right) t, & z_1 \le z \end{cases} \tag{3.16a}$$

$$w_1(x,t) = \begin{cases} 0.03\ e^{1.46193x}\ , & z \le 0 \\ 0.1\left(-1.1578\ e^{-0.585761x} + 1.45816e^{-0.163965x} - 0.000361868e^{1.46193x}\ \right)t, & 0 \le z \le z_1\,, \\ 0.1\ \left(15.9522\ e^{-0.585761x} - 1.64071\ e^{-0.163965x}\right)t, & z_1 \le z \end{cases} \tag{3.16b}$$

and so on.

The ADM yields the solutions $v(x,t),\ w(x,t)$ as

$$v(x,t) = \sum_{n=0}^{\infty} v_n(x,t), \qquad w(x,t) = \sum_{n=0}^{\infty} w_n(x,t), \tag{3.17}$$

where $v_n(x,t),\ w_n(x,t)$, will be determined in a recursive manner.

4. A Test Problem for the FN Equations

We discuss the solutions of the FN equations using the two considered VIM and ADM methods.

4.1. The VIM

Solve the FN equations (1.2) using the VIM with finite iterations at time $T = 5$. A comparison between the computed solutions and the exact solutions at different values of x are given in Table 1. We note that the VIM solutions converge to the exact solutions specially when n is increased. We show in Figure 1 the behavior of the VIM solutions of FN equations at time $T = 5$. If the exact solutions are plotted on Figure 1 we will find that the VIM and exact solutions curves are indistinguishable.

4.2. The ADM

Consider the same problems and use the ADM with the same initial conditions and use the technique discussed in Section 2. A comparison between the exact solutions and ADM solutions are shown in Table 2 and it seems that the errors are very small. We show in Figure 2 the numerical solutions of the FN equations.

The results listed in Table 3 are representing the maximum errors at different times of VIM and ADM which shows that the VIM is better than ADM in the solutions of FN equations.

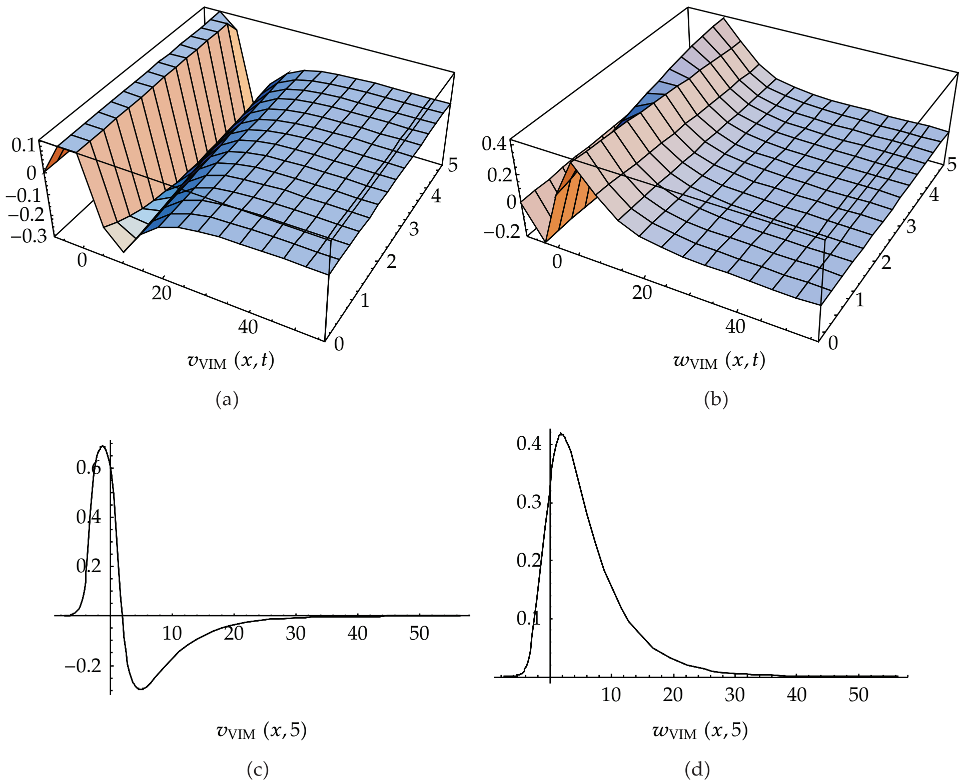

Figure 1: The approximated solutions for $v(x,t)$, $w(x,t)$ at time $T = 5$.

Table 1: Comparison between the exact and approximate (VIM) solutions for the FN equations at time $T = 5$.

x	v_{VIM}	v_{exact}	w_{VIM}	w_{exact}
−7.561	0.000865955	0.000865955	0.0000831702	0.0000831702
−3.561	0.3	0.3	0.0288134	0.0288134
−0.561	0.662823	0.662823	0.28156	0.28156
1.439	0.130074	0.130074	0.414489	0.414489
3.439	−0.25639	−0.25639	0.382524	0.382524
8.439	−0.215232	−0.215232	0.193024	0.193024
16.439	−0.0616494	−0.0616494	16.439	16.439
22.439	−0.023095	−0.023095	0.0197795	0.0197795
48.439	−0.000325199	−0.000325199	0.000278481	0.000278481

Now we show a comparison between our schemes and other methods as shown in Table 4.

It is clear that the suggested methods for solving FN equation are the best methods than all other methods. Also all other methods give the solution as a discrete solution but our methods give the solution as a function x and t.

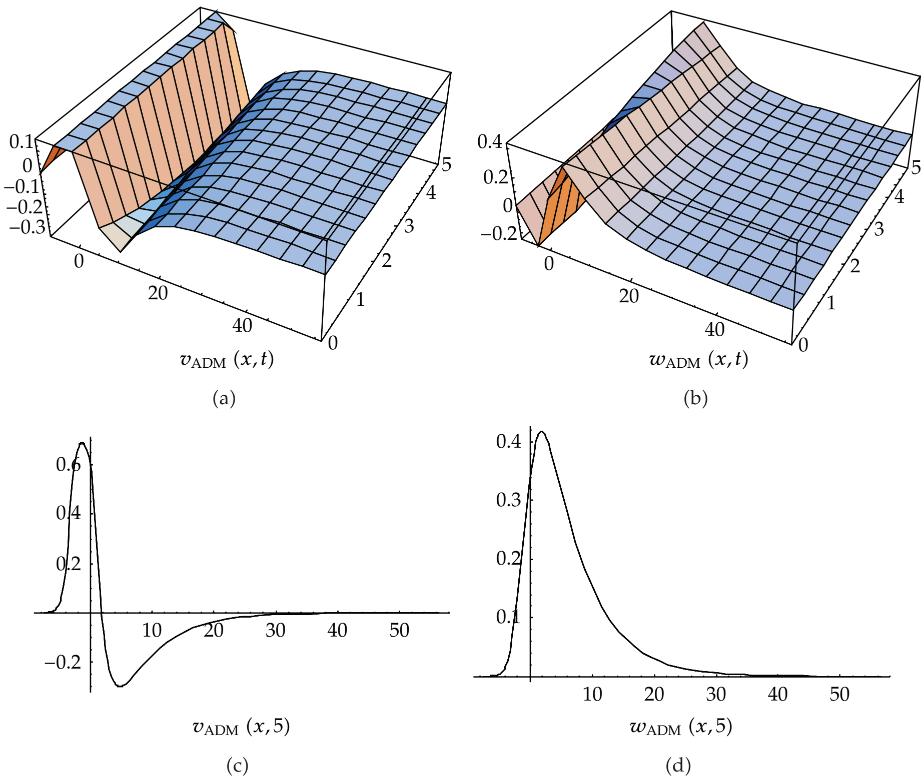

Figure 2: The approximation solutions $v(x,t),\ w(x,t)$.

Table 2: Comparison between the exact solutions and approximation solutions (ADM) for FN equations at time = 5.

x	v_{ADM}	v_{exact}	w_{ADM}	w_{exact}
−7.561	0.000865955	0.000865955	0.0000831702	0.0000831702
−3.561	0.3	0.3	0.0288134	0.0288134
−0.561	0.662823	0.662823	0.28156	0.28156
1.439	0.130074	0.130074	0.414489	0.414489
3.439	−0.25639	−0.25639	0.382524	0.382524
8.439	−0.215232	−0.215232	0.193024	0.193024
16.439	−0.0616494	−0.0616494	16.439	16.439
22.439	−0.023095	−0.023095	0.0197795	0.0197795
48.439	−0.000325199	−0.000325199	0.000278481	0.000278481

Table 3: The maximum errors of our suggested methods VIM and ADM.

Time	VIM		ADM	
	Max. errors for $v\ (x,t)$	Max. errors for $w\ (x,t)$	Max. errors for $v\ (x,t)$	Max. errors for $w\ (x,t)$
2.0	$3.66374E-15$	$4.02456E-16$	$3.71925E-15$	$4.71845E-16$
4.0	$1.14429E-9$	$1.09902E-10$	$1.15225E-9$	$1.10668E-10$
6.0	$3.37523E-7$	$3.24191E-8$	$3.37523E-7$	$3.24191E-8$

Table 4: Comparison between VIM, ADM, and other methods by maximum errors.

Method	$T = 1.60$	$T = 10.0$
Finite difference		
C-N	$0.848E-2$	0.189
Hopscotch [9]	$0.557E-2$	0.0506
Collocation method		
Quadratic [6]	$0.758E-2$	0.138
Cubic [6]	$0.589E-2$	0.12
VIM	$3.33067E-16$	0.000316341
ADM	$4.44089E-16$	0.000316341

5. Conclusion

In this paper the solutions for the FN equations using VIM and ADM methods have been generated. All numerical results obtained using few terms of the VIM and ADM show very good agreement with the exact solutions. Comparing our results with those of previous several methods shows that the considered techniques are more reliable, powerful, and promising mathematical tools. We believe that the accuracy of the VIM and ADM recommend it to be much wider applicability and also we find that the VIM more accurate than ADM.

Acknowledgment

This paper has been supported by the research support program, King Khalid University, Saudi Arabia, Grand no. KKU-COM-11-001.

References

[1] A. L. Hodgkin and A. F. Huxley, "A quantitative description of membrane current and its application to conduction and excitation in nerve," *The Journal of Physiology*, vol. 117, no. 4, pp. 500–544, 1952.

[2] R. Fitzhugh, "Impulse and physiological states in models of nerve membrane," *Biophysical Journal*, vol. 1, no. 6, pp. 445–466, 1961.

[3] J. S. Nagumo, S. Arimoto, and S. Yoshizawa, "An active pulse transmission line simulating nurve axon," *Proceedings of the Institute of Radio Engineers*, vol. 50, no. 10, pp. 2061–2070, 1962.

[4] J. Rauch and J. Smoller, "Qualitative theory of the FitzHugh-Nagumo equations," *Advances in Mathematics*, vol. 27, no. 1, pp. 12–44, 1978.

[5] H. P. McKean, "Nagumo's equation," *Advances in Mathematics*, vol. 4, no. 3, pp. 209–223, 1970.

[6] A. K. A. Khalifa, *Theory and application of the collocation method with splines for ordinary and partial differential equations [Ph.D. thesis]*, Heriot-Watt University, 1979.

[7] P. Gordon, "Nonsymmetric difference equations," *Journal of the Society For Industrial and Applied Mathematics*, vol. 13, no. 3, pp. 667–673, 1965.

[8] A. R. Gourlay, "Hopscotch: a fast second-order partial differential equation solver," *Journal of the Institute of Mathematics and Its Applications*, vol. 6, pp. 375–390, 1970.

[9] A. R. Gourlay, "Some recent methods for the numerical solution of time-dependent partial differential equations," *Proceedings of the Royal Society*, vol. 323, pp. 219–235, 1971.

[10] J. Rinzel, "Repetitive nerve impulse propagation: numerical results and methods," in *Nonlinear Diffusion*, W. E. Fitzgibbon III and H. F. Walker, Eds., Pitman, New York, NY, USA, 1977.

[11] J. R. Cannon and R. E. Ewing, "Galerkin procedures for systems of parabolic partial differential equations related to the transmission of nerve impulses," in *Nonlinear Diffusion*, W. E. Fitzgibbon III and H. F. Walker, Eds., Pitman, New York, NY, USA, 1977.

[12] J. H. He and X. H. Wu, "Exp-function method for nonlinear wave equations," *Chaos, Solitons & Fractals*, vol. 30, no. 3, pp. 700–708, 2006.

[13] J. H. He, "Variational approach for nonlinear oscillators," *Chaos, Solitons & Fractals*, vol. 34, no. 5, pp. 1430–1439, 2007.

[14] J. H. He, "Variational principles for some nonlinear partial differential equations with variable coefficients," *Chaos, Solitons & Fractals*, vol. 19, no. 4, pp. 847–851, 2004.

[15] J. H. He, "Some asymptotic methods for strongly nonlinear equations," *International Journal of Modern Physics B*, vol. 20, no. 10, pp. 1141–1199, 2006.

[16] J. H. He and X.-H. Wu, "Construction of solitary solution and compacton-like solution by variational iteration method," *Chaos, Solitons & Fractals*, vol. 29, no. 1, pp. 108–113, 2006.

[17] J. H. He, "Variational iteration method for autonomous ordinary differential systems," *Applied Mathematics and Computation*, vol. 114, no. 2-3, pp. 115–123, 2000.

[18] J. H. He, "Variational iteration method—a kind of non-linear analytical technique: aome examples," *International Journal of Non-Linear Mechanics*, vol. 34, no. 4, pp. 699–708, 1999.

[19] J. H. He, "Approximate analytical solution for seepage flow with fractional derivatives in porous media," *Computer Methods in Applied Mechanics and Engineering*, vol. 167, no. 1-2, pp. 57–68, 1998.

[20] Z. M. Odibat and S. Momani, "Application of variational iteration method to nonlinear differential equations of fractional order," *International Journal of Nonlinear Sciences and Numerical Simulation*, vol. 7, no. 1, pp. 27–34, 2006.

[21] N. Bildik and A. Konuralp, "The use of variational iteration method, differential transform method and adomian decomposition method for solving different types of nonlinear partial differential equations," *International Journal of Nonlinear Sciences and Numerical Simulation*, vol. 7, no. 1, pp. 65–70, 2006.

[22] E. Yusufoglu, "Variational iteration method for construction of some compact and noncompact structures of Klein-Gordon equations," *International Journal of Nonlinear Sciences and Numerical Simulation*, vol. 8, no. 2, pp. 153–158, 2007.

[23] N. H. Sweilam and M. M. Khader, "Variational iteration method for one dimensional nonlinear thermoelasticity," *Chaos, Solitons and Fractals*, vol. 32, no. 1, pp. 145–149, 2007.

[24] H. Tari, D. D. Ganji, and M. Rostamian, "Approximate solutions of K (2,2), KdV and modified KdV equations by variational iteration method, homotopy perturbation method and homotopy analysis method," *International Journal of Nonlinear Sciences and Numerical Simulation*, vol. 8, no. 2, pp. 203–210, 2007.

[25] M. A. Abdou and A. A. Soliman, "Variational iteration method for solving Burger's and coupled Burger's equations," *Journal of Computational and Applied Mathematics*, vol. 181, no. 2, pp. 245–251, 2005.

[26] A. A. Soliman and M. A. Abdou, "Numerical solutions of nonlinear evolution equations using variational iteration method," *Journal of Computational and Applied Mathematics*, vol. 207, no. 1, pp. 111–120, 2007.

[27] A. A. Soliman, "Numerical simulation of the generalized regularized long wave equation by He's variational iteration method," *Mathematics and Computers in Simulation*, vol. 70, no. 2, pp. 119–124, 2005.

[28] M. A. Abdou and A. A. Soliman, "New applications of variational iteration method," *Physica D*, vol. 211, no. 1-2, pp. 1–8, 2005.

[29] A.-M. Wazwaz, "A comparison between the variational iteration method and Adomian decomposition method," *Journal of Computational and Applied Mathematics*, vol. 207, no. 1, pp. 129–136, 2007.

[30] A.-M. Wazwaz, "The variational iteration method for rational solutions for KdV, $K(2,2)$, Burgers, and cubic Boussinesq equations," *Journal of Computational and Applied Mathematics*, vol. 207, no. 1, pp. 18–23, 2007.

[31] A.-M. Wazwaz and A. Gorguis, "Exact solutions for heat-like and wave-like equations with variable coefficients," *Applied Mathematics and Computation*, vol. 149, no. 1, pp. 15–29, 2004.

[32] G. Adomian, *Solving Frontier Problems of Physics: The Decomposition Method*, Kluwer Academic, Boston, Mass, USA, 1994.

[33] A. A. Soliman, "A numerical simulation and explicit solutions of KdV-Burgers' and Lax's seventh-order KdV equations," *Chaos, Solitons & Fractals*, vol. 29, no. 2, pp. 294–302, 2006.

[34] M. T. Darvishi, F. Khani, and A. A. Soliman, "The numerical simulation for stiff systems of ordinary differential equations," *Computers and Mathematics with Applications*, vol. 54, no. 7-8, pp. 1055–1063, 2007.

[35] A. A. Soliman, "On the solution of two-dimensional coupled Burgers' equations by variational iteration method," *Chaos, Solitons & Fractals*, vol. 40, no. 3, pp. 1146–1155, 2009.

[36] A. A. Soliman and M. A. Abdou, "The decomposition method for solving the coupled modified KdV equations," *Mathematical and Computer Modelling*, vol. 47, no. 9-10, pp. 1035–1041, 2008.

[37] J. H. He, "A short remark on fractional variational iteration method," *Physics Letters A*, vol. 375, no. 38, pp. 3362–3364, 2011.