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Erratum

Erratum to "Fixed-Point Theorems for Multivalued Mappings in Modular Metric Spaces"

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1. On the Results in the Original Paper

In the original paper, the authors have studied and introduced some fixed-point theorems in the framework of a modular metric space. We will first state the main result and then discuss some small gap herewith.

Theorem 1.1 (see the original paper). *Let X_ω be a complete modular metric space and F a multivalued self-mapping on X_ω satisfying the inequality*

$$\Omega_\lambda(Fx, Fy) \leq k\omega_\lambda(x, y), \tag{1.1}$$

for all $x, y \in X_\omega$, where $k \in [0, 1)$. Then, F has a fixed point in X_ω.

We now claim that the conditions in this theorem are not sufficient to guarantee the existence of the fixed points. We state a counterexample to Theorem 1.1 in the single-valued case as follows.

Example 1.2. Let $X := \{0, 1\}$ and ω be given by

$$\omega_\lambda(x, y) = \begin{cases} \infty, & \text{if } 0 < \lambda < 1 \text{ and } x \neq y, \\ 0, & \text{if } \lambda \geq 1 \text{ or } x = y. \end{cases} \tag{1.2}$$

Thus, the modular metric space $X_\omega = X$. Now let f be a self-mapping on X defined by

$$f(0) = 1, \quad f(1) = 0. \tag{1.3}$$

Then, f satisfies the hypothesis of Theorem 1.1 with any $k \in [0,1)$ but it possesses no fixed point after all. Notice that this gap flaws the theorem only when ∞ is involved. Also, this gap is also found along the rest of the paper.

2. Revised Theorems

In this section, we will now give the corrections of our theorems in the original paper. Every theorem in Section 3 the original paper can be corrected by adding the following condition:

$$\text{there exists } x_0 \in X_\omega \text{ such that } \omega_\lambda(x_0, y) < \infty \quad \forall y \in Fx_0,\ \lambda > 0. \tag{2.1}$$

For the proofs, take the initial point x_0 satisfying the condition (2.1). The rest of the proofs run the same lines.

Further, Theorems 4.1 and 4.3 in Section 4 of the original paper must be corrected by adding the following two conditions.

(H1) There exists $x_\star \in X_\omega$ such that $\omega_\lambda(x_\star, y) < \infty$ for all $y \in Fx_\star$ and $\lambda > 0$.

(H2) If $x_0 \in \mathrm{Fix}(F)$, then $\omega_\lambda(x_0, y) < \infty$ for all $y \in Gx_0$ and $\lambda > 0$.

To prove, conditions (H1) and (H2) imply the existence of the fixed points of F and G. Take $x_0 \in \mathrm{Fix}(F)$ and follow the proof lines in the original paper to obtain the results.

Similarly Corollaries 4.2 and 4.4 in Section 4 of the original paper, must be corrected by adding the following three conditions:

(H3) There exists $x_0 \in X_\omega$ such that $\omega_\lambda(x_0, y) < \infty$ for all $y \in F_1 x_0$ and $\lambda > 0$.

(H4) If $x_n \in \mathrm{Fix}(F_n)$, then $\omega_\lambda(x_n, y) < \infty$ for all $y \in F_{n+1} x_n$ and $\lambda > 0$.

(H5) There exists $x_\infty \in X_\omega$ such that $\omega_\lambda(x_\infty, y) < \infty$ for all $y \in Fx_\infty$ and $\lambda > 0$.

For the proof, conditions (H3) and (H4) guarantee the existence of fixed points of each F_n, while condition (H5) implies the existence of fixed points of F. The rest of the proofs are as illustrated in the original paper.