RALPH HENSTOCK, 1923-2007

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Abstract

Influential analyst Ralph Henstock died January 17 2007. His best-known achievement is the gauge integral or generalized Riemann integral.

Integration theorist Ralph Henstock died on January 17 2007 after a short illness. He was born in the coal-mining village of Newstead, near Nottingham, England, on June 2 1923; the only child of mineworker and former coalminer William Henstock and Mary Ellen Henstock (nee Bancroft). On the Henstock side he was descended from 17th century Flemish immigrants called Hemstok.

Because of his early academic promise it was expected that Henstock would attend Nottingham University where his father and uncle had received technical education, but as it turned out he won scholarships which enabled him to study mathematics at St. John's College Cambridge from October 1941 until November 1943, when he was assigned to the Ministry of Supply's department of Statistical Method and Quality Control in London.

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This work did not satisfy him, and he enrolled at Birkbeck College London where he joined the weekly seminar of Professor Paul Dienes which was then a focus for mathematical activity in London. Henstock wanted to study divergent series but Dienes prevailed upon him to get involved in the theory of integration and its relation to the summability of series and integrals, thereby setting him on course for his life's work.

He was awarded the Cambridge B.A. in 1944 and began research for the Ph.D. in London, which he gained in December 1948 with a thesis entitled *Interval Functions and their Integrals*, an extension of J.C. Burkill's theory. His Ph.D. examiners were J.C. Burkill and H. Kestelman. In 1947 he returned briefly to Cambridge to complete the undergraduate mathematical studies which had been truncated by his Ministry of Supply work.

Henstock was a distinguished analyst who specialized in the theory of integration. From initial studies of the Burkill and Ward integrals he formulated an integration process whereby the domain of integration is suitably partitioned for Riemann sums to approximate the integral of a function. His methods led to an integral on the real line that was very similar in construction and simplicity to the Riemann integral but which included the Lebesgue integral and, in addition, allowed non-absolute convergence.

These ideas were developed from the late 1950's. Independently, Jaroslav Kurzweil developed a similar Riemann-type integral on the real line. The resulting integral is now known as the Henstock-Kurzweil integral. On the real line it is equivalent to the Denjoy-Perron integral, but has a much simpler definition and is generally much easier to work with. An absolutely convergent version of the integral, equivalent to the Lebesgue integral on the real line, was developed by E.J. McShane.

In the following decades, Henstock developed extensively the distinctive features of his theory, inventing the concepts of division spaces or integration bases to demonstrate in general settings the essential properties and characteristics of mathematical integration in all its forms. His theory provides a unified approach to many problems which were considered earlier by different methods using different types of non-absolute integrals. Now many of them can be solved using different kinds of Henstock integral, just choosing an appropriate integration basis (or division space in Henstock's own terminology).

The underlying simplicity of the Henstock-Kurzweil integral has revivified the subject of mathematical integration and the theory now has many practitioners and exponents. It has proved useful in differential and integral equations, harmonic analysis, probability theory and quantum mechanics, where the random variables of Feynman integration are not absolutely integrable and therefore are not amenable to the methods of Lebesgue integration and classical probability theory. Numerous monographs and texts have appeared since 1980 and there have been several conferences devoted to the theory. The simplicity of the underlying concept can give rise to naive expectations of the subject, which is in reality deep and subtle. Initially a research specialism, it is nowadays increasingly taught in standard courses in mathematical analysis.

Henstock was author of 42 journal papers in the period 1946 to 2006. He

published four books on analysis (Theory of Integration, 1963; Linear Analysis, 1967; Lectures on the Theory of Integration, 1988; and The General Theory of Integration, 1991). He wrote 171 reviews for MathSciNet. In 1994 he was awarded the Andy Prize of the XVIII Summer Symposium in Real Analysis. His academic career began as Assistant Lecturer, Bedford College for Women, 1947-48; then Assistant Lecturer at Birkbeck, 1948-51; Lecturer, Queen's University Belfast, 1951-56; Lecturer, Bristol University, 1956-60; Senior Lecturer and Reader, Queen's University Belfast, 1960-64; Reader, Lancaster University, 1964-70; Chair of Pure Mathematics, New University of Ulster, 1970-88; and Leverhulme Fellow 1988-91. In 1958 he was appointed Fellow of the Royal Statistical Society.

Henstock married Marjorie Jardine in 1949 and is survived by their son John. A devoted Methodist, with an abiding interest in poetry, the lasting impression he made was one of gentle sincerity, kindness and amiability. The integrity and conscientiousness he displayed in his scientific work were mirrored in his generous relationships with colleagues, collaborators and students. As a mathematician and as a man, his loss is deeply felt by them all.